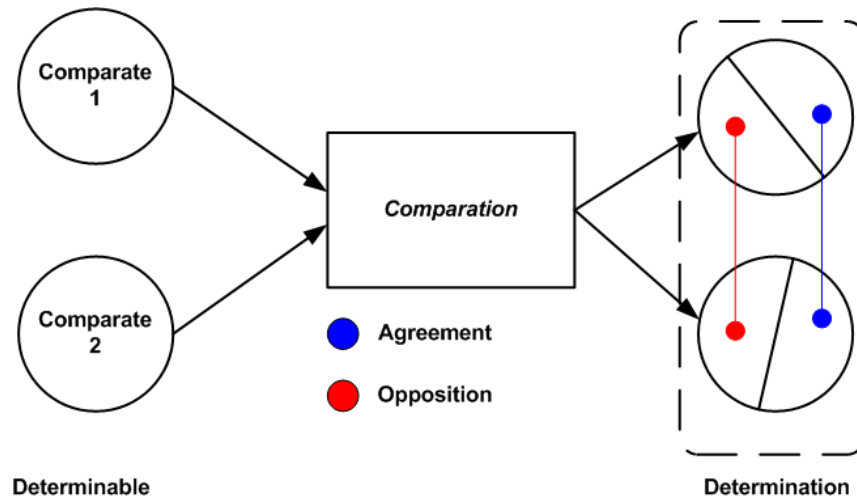


**Comparison in apprehensive imagination<sup>i</sup>**

1. The practical problem of Comparison.<sup>ii</sup> In *The Principles of Mental Physics* [Wells (2009), chapter 11]<sup>iii</sup>, I explained that *Comparison* is the synthesis of mathematical equivalence relations. This mathematical definition is an abstract definition begging the question of equivalence. All objects are equivalent in some contexts and inequivalent in others. The definition of mathematical equivalence does not specify a context for the relation and, therefore, does not speak to the manner in which two comparands are to be regarded as being in equivalence. A father and son are "equivalent" in the context of belonging to the same family and "inequivalent" in the context of individuality. Thus we have both  $\langle \text{father} = \text{son} \rangle$  and  $\langle \text{father} \neq \text{son} \rangle$  (likeness *and* non-likeness) depending on the context of comparison.



**Figure 1:** Kant's description of *Comparison* [Wells (2006), chap. 3, §4.2].

I explained the operational context of equivalence in *The Critical Philosophy and the Phenomenon of Mind* (chap. 3, §4.2). This is illustrated in figure 1 above. The general idea of comparison (*Vergleichung*) had not been adequately treated by metaphysics prior to this and had been left as an undefined primitive. General comparison divides into logical comparison (*Comparison*) and material comparison (*Reflexion*). Although *Comparison* is logical comparison, that does not remove the need for *practical* context in the act of comparison.

2. The transcendental Logic of Comparison in composition. The Organized Being has two processes of *Comparison* in its structure, one in the synthesis in sensibility and the second in the synthesis of appetite in pure practical Reason. I call the latter *practical* Comparison. These notes deal only with *Comparison* in sensibility. First, in *Critique of Pure Reason* [Kant (1787), *Kritik der reinen Vernunft*, 2nd ed., B317-319], Kant tells us that the synthesis makes a complete representation, which means it is a synthesis of Quantity, Quality, Relation and Modality. All four titles are represented implicitly in figure 1. Kant also tells us,

The *Verstandes-Actus* [acts of understanding], through which concepts are begat as to their form, are:

1. *Comparison*, i.e. the comparison of representations among one another in relationship to unity of consciousness;
2. *Reflexion*, i.e. reconsideration as to how various representations can be comprehended in one consciousness; and finally
3. *abstraction* or separation of everything else in which the given representations differ. [Kant (1800), *Logik*, 9: 94]

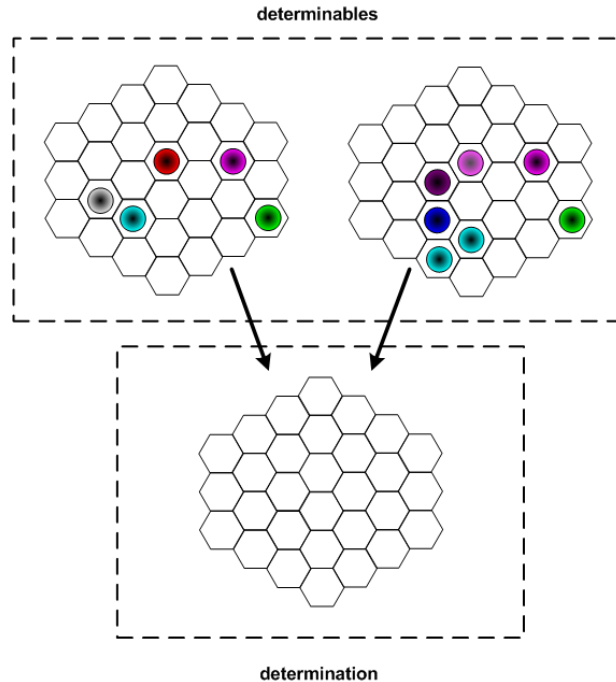
Figure 1, which is an abstract illustration of *Comparison* and not its definition, depicts two different kinds of equivalence *likenesses* in the depiction of the determination. The first is the depiction of equivalence in agreement, the second of equivalence in opposition. We are accustomed to thinking of equivalence in terms of agreement, but "likeness" in terms of opposition is not something we typically consider. In what way are opposites "alike"? The answer, of course, is in their *non-likeness*. We cannot say that *a* is unlike *b* without also, and at the same time, saying *b* is unlike *a*. Mathematicians call this the *symmetric* relation. The phrase "is unlike" is a *mathematical* relation. Stated formally,

**Definition 1 (Symmetry).** A relation  $\alpha$  on a set  $A$  is said to have the symmetric property (to be symmetric) if, for  $a$  and  $b$  in  $A$ ,  $a \alpha b$  implies  $b \alpha a$ .

Figure 1 depicts in the determination not one but two outcomes of the act of Comparison, the "is like" and the "is unlike" outcome. Furthermore, the relationship denoted "opposition" is the English rendering of Kant's term *Widerstreit*, a word that carries connotations of conflict, clash and antagonism. Two moving powers standing in a relationship of *Widerstreit* with one another have intensive magnitudes that are regarded as relatively negative such that each moving power acts to negate the effect of the other. Two moving powers standing in a relationship of agreement have intensive magnitudes that are regarded as relatively positive such that each moving power combines with the other to produce an increase in their overall effect. Two co-existing comparands can stand, overall, in both relationships at the same moment in time and so Quality in the act of *Comparison* is understood under the idea of subcontrariness, i.e., "*a* and *b* are in agreement AND *a* and *b* are in opposition." This relationship of Quality is understood by the category of limitation in determining judgment, but the *Verstandes-Actus* of *Comparison* is not a judgment. Rather, it denotes merely syncretism in comparison. It does not alter either comparand but merely depicts a subcontrariness relationship in combining the two if an attempt at combining the comparands were to be enacted.

For specificity of context, the comparands in figure 1, regarded in the context of the *Verstandes-Actus* in the synthesis of apprehension, are *materia ex qua* ("matter from which") in mental representation that correspond to somatic activity fields in *soma* [see Wells, "The applied metaphysics of the somatic code" and "On Critical representation in brain theory, part II: General schema of knowledge representation"]. Without over-specificity, we can regard these mental representations as one-to-one images of their somatic counterparts insofar as they are comparands depicted by receptivity in *psyche* or by the synthesis of reproduction in imagination. (Because the mental representations as objects are supersensible – mathematical – objects, we are free to so picture them). Figure 2 illustrates this idea of noetic comparand images of two somatic activity fields comprising the determinables of the act of *Comparison*.

The determination, as a noetic representation, is a *parástase* ("depiction") of some sort of unity of the comparands. Figure 2 depicts no details of this *parástase* because it is premature to do so at this point in the discussion. Figure 1 must do for the moment as a depiction of the idea of what is represented by the determination. It is, however, clear that figure 2 conveys to us an idea of Quantity in the composition of *Comparison*. Composition in Quantity is the synthesis of a manifold of what does not necessarily belong to each other yet for which there is a *homogeneity* in representation such that the things being combined can be composed as an *aggregate* [Kant (1787), B201-202 fn]. Just as the comparands themselves are each regarded as an aggregate (a *set*), the determination is likewise to be regarded as an aggregate (*another set*). An *aggregation* is a composition of *extensive* magnitude, and extensive magnitude means representation by repeated positing of homogeneous parts in which the representation of the parts precedes the representation of the whole and makes the latter depiction possible. What we are bound to ask here is this: (1) what are these parts? and (2) in what manner or way are these parts regardable as being homogeneous?



**Figure 2:** Illustration of activity field determinables and *Comparison* determination.

The answers to both questions are more or less obvious in figure 2. The parts are the noetic images of *somatic places* because it is the aggregation of somatic places that makes up an activity field. How are the parts homogeneous? The answer to this is they have *phoronomic* homogeneity, i.e., corresponding places in each activity field depict *the same place* in *soma*. It follows from this that the act of *Comparison* is an act of place-by-place logical comparison and, more particularly, the *aggregation* of logical place-by-place comparisons insofar as the depiction of extensive magnitude is concerned. *Comparison* does not change the comparands; it begets a *new* aggregation (set). This new set is what the determination depicts in figure 2.

This does not yet tell us everything we need to know about *Comparison*. I have so far only talked about Quantity and Quality, which Kant called "mathematical" relationship. The headings of Relation and Modality in *Comparison* (which Kant denoted as "dynamical" relationship) also have to be explained. Both headings pertain to the synthesis of a manifold of what is *inhomogeneous* yet regarded as belonging to each other *necessarily* and combined *a priori* [*ibid.*]. What is inhomogeneous in the figure 2 depiction of the comparands? This is not so obvious at first glance. However, only a little transcendental deduction is required to bring this into the light.

Homogeneity in composition requires us to regard the places in both comparands as the *same* organic places. Yet even a brief glance at the figure illustrates that these same places do not all depict the same intensive magnitudes of moving power. How can this be? How can the same somatic place have a different degree of activity at the same objective time? The answer, of course, is quite easy: *it cannot*. Here, then, is an immediate encounter with the *mathematical* nature of the concept of activity fields. *Semantic* Critical field theory, and therefore somatic and noetic field theory, is a relativistic theory of material *space-time*, not space *and* time.

I find this notion easier to explain to someone who has at least an introductory acquaintance with Einstein's theory, although even in that case it is hard to bring another person to a proper understanding of the concept if his thinking is ontology-centered. I find it difficult to make the

explanation clearer than this:

(1) We are talking here of objective space and objective time, and these are not things-in-physical-nature but things-in-mathematical-Nature. They belong to facet B (the mathematical world), not facet A (the physical world) in human understanding [Wells (2009), chap. 1 §4];

(2) In this facet B world there is no *ontological* difference between objective space and objective time, only an *epistemological* difference. Einstein's 1905 and 1915 papers on the theory of relativity could hardly have been more explicit about this. This Critical point is one that I find the great majority of even very renowned physicists since Einstein failing to grasp because of ontology-centered metaphysical prejudices. Einstein *said*, "Space is not a thing."<sup>1</sup> In their defense, Einstein himself is partially to blame for the misunderstanding. By 1934<sup>2</sup>, after the success of the general theory of relativity, Einstein had slipped back into a form of Platonism, a prejudicial change in his thinking that coincided with the end of his great work in physics. His fellow physicists have followed him in this blunder and continue this following to this day. Mathematician Hermann Minkowski stated the real epistemology very well in 1908:

The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth, space by itself and time by itself are doomed to fade away into mere shadows, and only a kind of union between the two will preserve an independent reality. [Minkowski, "Space and Time"]<sup>3</sup>

The "mere shadows" of which Minkowski speaks are the shadows of mathematical secondary quantities in facet B (the mathematical world). The objective validity of objective space-time is vested in facet B and connects with physical objectivity only at its principal quantities;

(3) The depicted determinables in figure 2 therefore must (epistemologically) be regarded as occupying *different places* in mathematical objective space-time. Once we see them in this correct Critical context, the contradiction pointed out above appears as what it is: a mere transcendental illusion;

(4) This same deduction of transcendental Logic also tells us that the noetic depictions in figure 2 **are not copies of a somatic activity field**. Recall that I called them *images*, not *copies*, earlier. *H. sapiens* possesses no mystical copy-of-reality mechanism, a fact that Piaget *et al.* have demonstrated empirically with an abundance of evidence. Metaphysically, the unreality of all copy-of-reality notions is a basic acroam of Critical epistemology<sup>4</sup>. The comparand depictions of figure 2 (and also the determination depiction) are secondary quantities of hypothetical mathematics. For them to have any objectively valid context, though, these depictions do require a connection to principal quantities that can be associated with phenomena of facet A. This is a point to which I will return later. When I do, we will see the proper relationship between noetic state space representation and somatic state space representation and acquire deeper and more distinct concepts of these *materia ex qua*. As for the principal quantities, the explanation I will be presenting has for its linchpin Critical semantics, and we will see illustrations of the embedding field structures that take us towards

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<sup>1</sup> Quoted by Jeremy Bernstein in his biography, *Einstein*, NY: The Viking Press, 1973, pg. 127.

<sup>2</sup> See Einstein, *Essays on Science*, NY: Philosophical Library, 1934, pg. 17.

<sup>3</sup> Minkowski, Hermann (1908), "Space and time," an address delivered at the 80th Assembly of German Natural Scientists and Physicians, Cologne, Sept. 21, 1908. An English translation is given in Einstein *et al.*, *The Principle of Relativity*, NY: Dover Publications, 1952, pp. 72-91.

<sup>4</sup> All ontology-centered systems of metaphysics require some sort of copy-of-reality mechanism at or near their foundations. This is enough, all by itself, to guarantee that all ontology-centered systems of metaphysics will fail the test of science, as all have done with numbing regularity for over two millennia.

the principal quantities of facet B and their relationships with somatic state space.

3. The transcendental Logic of Comparison in nexus. Combination in regard to *nexus* is connection, i.e. Relation and Modality. Because the depicted manifold of the combination is the determination of figure 1, we can see directly from that figure what sort of Relation pertains to *Comparison*. Because the determination is neither the one comparand nor the other, nor even a homogeneous synthesis of both at once (the determination is "something other" than the determinables), what figure 1 tells us is that the Relation pertinent to this act is internal Relation. Internal Relation is the form of connection in representation in which the connections have no reference to anything other than the object of the representation. The category of Relation that is the *a priori* notion of the scheme of representation for such a concept is the category of substance & accident [Wells (2009), *The Principles of Mental Physics*, chap. 5 §4.2.3].

This category has the temporal character (in *subjective* time) of persistence-in-time. I emphasize here that I am talking now about *subjective* time, which is the pure form of inner sense in organized being. Now, when we look at an illustration of the synthesis in sensibility (figure 3), one thing to immediately note is that the *Verstandes-Actus* all stand in parallel, i.e., outside, the synthesis of pure intuition. This is to say these acts are "not in subjective time" and, consequently, all notions of order or succession we find ourselves necessarily attaching to their representations (as secondary quantities of facet B) are ideas of *logical ordering*, not of *temporal sequencing*. The idea of logical ordering is an idea of a type of objective time. The practical implication of this is that the comparands logically presented to *Comparison* have no *necessary* objective time reference nor any *actual* subjective time positioning. This is as much as to say the comparands and the determination are bound by no *temporal* constraints.

This notion underlies what was said earlier about the comparands occupying different places in space-time, especially insofar as the comparands are regarded as sensory data effects of receptivity in *psyche*. Receptivity in coordination with reproductive imagination is sufficient to permit different space-places in the noetic fields, but different time-places is grounded only

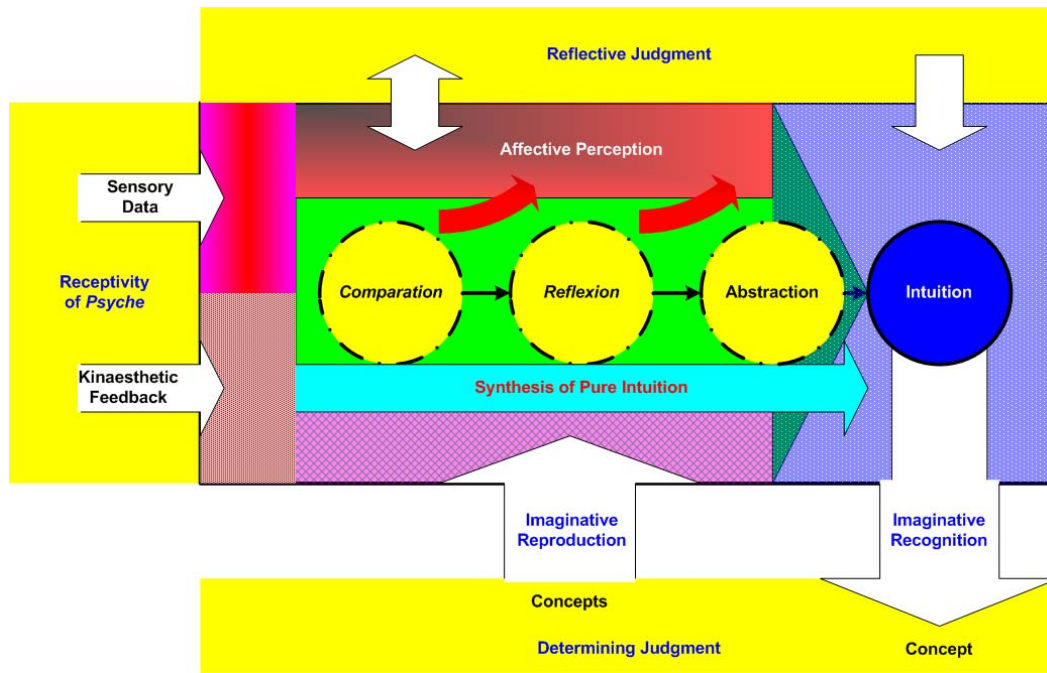


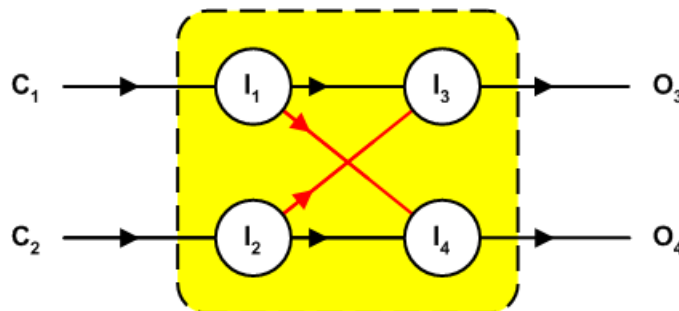
Figure 3: Illustration of the synthesis of apprehension in sensibility.

by having *Comparison* stand "outside the synthesis of the pure intuition of subjective time." This, by the way, means that the comparands and the determination in figures 1 and 2, while they are noetic representations, can never themselves be *conscious* representations. We might call them *pre-perceptions* but they can never be regarded as *perceptions*. They are what Kant called *obscure* representation and belong to what Freud called "the unconscious system of the mind." Empirical consciousness is "the representation that another representation is in me and is to be attended to." If we think of the *parastase* of the first representation in this description as being a kind of logical "pointer," the comparands and the determination can never be the second representation (the one that is "pointed at" by the first).

This character of Relation in *Comparison* is the Critical ground justifying the employment of: a number of mathematical objective-time techniques, e.g. Hamilton's principle in its integral form (Hamilton-Jacobi-Bellman optimization); backpropagation-in-time approximate dynamic programming methods; and ideas of divers types of *memory*, e.g., short-term, long-term, lexical, procedural, reconstructive, recovered, reproductive, semantic, and whatever other labels empirical psychology chooses for tagging various cognitive and affective behaviors (see Reber's *Dictionary of Psychology* for descriptions of these terms). Internal Relation in *Comparison* is, from one wry point of view, an open invitation to romp through hypothetical mathematics and "let the fur fly" at will provided that *in the final analysis* all such mathematical "flying fur" lands on principal quantities, which are subject to all the acroams of Critical metaphysics with their concomitant implications for the mathematical axioms that can be used with objective validity in their deduction and construction<sup>5</sup>.

As for Modality, we have already named and illustrated two of the ideas. The comparands are the determinable matter, the logical outcome of *Comparison* is the determination of the matter of representing in logical comparison. This leaves only the idea of the determining factor, which is, of course, the idea of the act of *Comparison* itself. I have already named this determining factor *en passant* above: all the mathematical construction must conclude at principal quantities – those mathematical objects that can be practically placed in direct correspondence with experience. Thus far I have talked about what *Comparison* does; next I must explain how it does it (or, more precisely, how to look at how it *could* do it). We must examine the mathematical *Existenz* of the act of *Comparison*, and this is where embedding fields enter the picture.

4. Dipole embedding fields. The presentation of subcontrarity in the determinable calls for a dual representation of some sort. This is readily accomplished in a dipole embedding field graph. Figure 4 illustrates the simplest minimal form of a dipole network graph, a feedforward graph.



**Figure 4:** Simplest form of dipole embedding field graph. This is a 2-stage feedforward graph.  $C_1$  and  $C_2$  are comparand 1 and 2 places, respectively. Black lines denote intensive magnitudes in agreement (excitation), red lines denote intensive magnitudes in opposition (*Widerstreit* regarded as inhibition).

<sup>5</sup> See, e.g., *The Critical Philosophy and the Phenomenon of Mind*, chapter 23.

There are many different types of embedding field graphs in the general class of dipole networks. The graph of figure 4 appeared very early [Grossberg (1970)<sup>6</sup>] although the general term "dipole network" did not come into use until around 1972 [Grossberg (1972a)<sup>7</sup>, (1972b)<sup>8</sup>, (1975)<sup>9</sup>]. There is in fact a very close similarity – in some cases even an identity – between dipole network structures and the on-center off-surround neuronal organization found to exist throughout the central nervous system (CNS).

The mathematical appearance of dipole networks in the *Comparison* function in sensibility raises a question of considerable importance. If the vertices in a graph depict somatic activities then why are these vertices not part of the somatic activity fields imaged in the comparates? Homeomorphism between noetic state space and somatic state space, which is mediated through the construct of the topological *semantic* space, requires that every noetic *parástase* have a corresponding somatic one. The depictions in figures 1 and 2 are clearly depictions of noetic representations, but if the *Comparison* function is depicted as a network graph, do not these graph vertices likewise require a somatic counterpart?

The answer, of course, is yes *insofar* as the degree of intensive magnitude of a noetic vertex is a *parástase* or, likewise, a somatic activity is a *signal* representation. Here it is important to bear in mind that what is not a noetic or somatic *comparand* place *can be* a place in a *semantic* field. Not every somatic activity is a *signal* [Wells (2011a)<sup>10</sup>]. A somatic activity, in and of itself, is the appearance of a physical phenomenon, and to be regarded as a signal it must have those properties by which temporal variations of that phenomenon can be said to "carry information." For example, the cells involved in the growth of hair certainly exhibit metabolic rates but hair growth is not a phenomenon that is said to carry information. Similarly, not every noetic quantity is said to represent knowledge and, therefore, such a noetic construct is not said to carry information. Information is the substance of representations and is the idea of inner Relation in the transcendental sensorimotor idea of *psyche* in mental physics. Appearances, whether somatic or noetic, that stand as accidents of this substance are *data* representations. Information is that which is persistent from one data representation to another and, in particular, from a somatic signal to a noetic *parástase* and vice versa.

This is one of the subtleties of *reflective perspective* in Critical epistemology. Something is or is not a representation only in the context of its relationship to the *Existenz* of the Organized Being, not in the context of its relationship to a theory or to a theorist. An appearance is a representation only in regard to the transcendental reflective perspective of Critical epistemology<sup>11</sup>.

To state this point precisely in theoretical terms we must state it mathematically. This calls us back to one of the subtleties of topology theory that is easy to overlook or neglect when one is involved in the whirl of mathematical construction. In any topological space  $(X, \mathfrak{T})$ , the set of places  $X$  is regarded as a subset of some mathematical *universe set*,  $X$ , i.e.,  $X \subset X$ . We call  $X$  "the space for which  $\mathfrak{T}$  is the topology." The Critical semantics of representation theory tells

<sup>6</sup> Grossberg, Stephen (1970), "Neural pattern discrimination," *Journal of Theoretical Biology*, **27**, 291-337.

<sup>7</sup> Grossberg, Stephen (1972a), "A neural theory of punishment and avoidance, I: Qualitative theory," *Mathematical Biosciences* **15**, 39-67.

<sup>8</sup> Grossberg, Stephen (1972b), "A neural theory of punishment and avoidance, II: Quantitative theory," *Mathematical Biosciences* **15**, 253-285.

<sup>9</sup> Grossberg, Stephen (1975), "A neural model of attention, reinforcement and discrimination learning," *International Review of Neurobiology*, **18**, 263-327.

<sup>10</sup> Wells, Richard B. (2011a), "On Critical representation in brain theory, part II: General schema of knowledge representation," Wells Laboratory web site.

<sup>11</sup> Transcendental reflective perspective is perspective in regard to Rational Psychology.

us that in representing the *Existenz* of the Organized Being there will be multiple semantic topological spaces and, therefore, a corresponding multiplicity of both somatic and noetic topological state spaces. *The space of the comparands and the space of a particular embedding field graph are not necessarily the same space* although the *place sets* of each are contained in one and the same universe set  $X$ . It is incumbent upon the theorist to keep track of his multiplicity of place sets and to delimit the differences between them. Mathematics is a language for saying things precisely, every symbol in its expression means something, and the theorist must keep up an awareness of the *practical* meanings of every symbol he uses.

We introduce the dipole field graph concept to precisely explain the synthetic *process* of *Comparison*. *Comparison* is a *transformation* process, so we should take a closer look at this idea of a "transformation."

5. Transformations and Embedding Field Graphs. A **transformation** is an action by which one representation is changed into another representation. This Critical *Realerklärung* tells us at the outset that the transformation itself is not necessarily a representation (even though, in order for us to theoretically understand it, it must itself *be* represented to us<sup>12</sup>). Furthermore, the great majority of all useful transformations are *information lossy* and it is the carefully prescribed way in which information is lost that *makes* them useful. A transformation is information lossy if one cannot tell from the transformed result what the transformed operand or operands were prior to the transformation. For example, consider the squaring transformation  $y = x^2$ . If you know  $y = 9$ , you do not know if  $x = +3$  or  $-3$ . The transformation is information lossy.

Homeomorphisms are the important exception to this. If transformation  $f:A \rightarrow B$  and transformation  $g:B \rightarrow A$  comprise a homeomorphic pair of transformations, then by the definition of homeomorphism we have  $g[f(a)] = a$  and  $f[g(b)] = b$  for all  $a \in A$  and all  $b \in B$  for which the transformations are defined. This provides us with a *practical tenet* for deciding if some theoretical construct depicts a representation. It depicts a noetic representation  $n$  with a corresponding somatic signal  $s$  if and only if a homeomorphic pair of transformations exists such that  $g[f(s)] = s$  and  $f[g(n)] = n$  (and likewise for a somatic transformation).

This is the tenet and a standard for judgment by which we can understand whether a graph, such as figure 4, is itself or contains within itself any semantic representation (and, therefore, depicts both a noetic *parástase* and somatic signal). We can generally anticipate that this judgment will depend on the level of representation depicted by the mathematical construct. A particularly good example of this is provided in Grossberg (1978)<sup>13</sup> by an embedding field graph called an avalanche network. Figure 5 depicts such a network at a relatively high (which is to say "more abstract") level. Figure 6 depicts a lower level (more distinct) representation of the  $F_2$  Field vertices,  $V^{(2)}$ , of figure 5. The mathematical details and explanations of this network are not particularly important for the present discussion. They are reported in full detail in Wells and MacPherson (2009)<sup>14</sup>.

<sup>12</sup> The Object of the science is the Organized Being. The scientist is an *observer* of this Object. The observer must never confuse and mistake himself for the Object he observes or vice versa. This sounds like a mere bromide, but the habits of ontology-centered thinking (which we all have) make it a challenging maxim to put into practice consistently. Even Kant fell victim to this when he mistook the categorical imperative of pure practical Reason for an ontological thing, namely, "the moral law within me." The scientist who tells himself, "I'd never make that mistake!" is more than halfway *en route* to making it.

<sup>13</sup> Grossberg, Stephen (1978), "A theory of human memory," *Progress in Theoretical Biology*, vol. 5, pp. 235-374, NY: Academic Press.

<sup>14</sup> Wells, Richard B. and Quinn MacPherson (2009), "The Martian Program 2009: Theory of the stage I infant model," available from the Wells Laboratory web site.



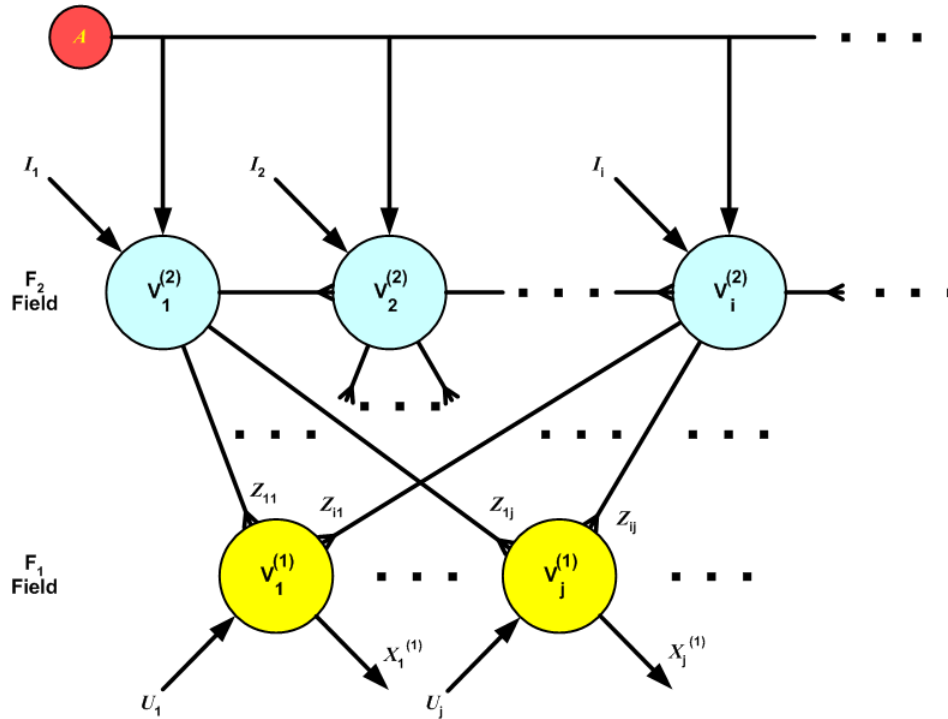


Figure 5: Embedding field graph of an avalanche network.

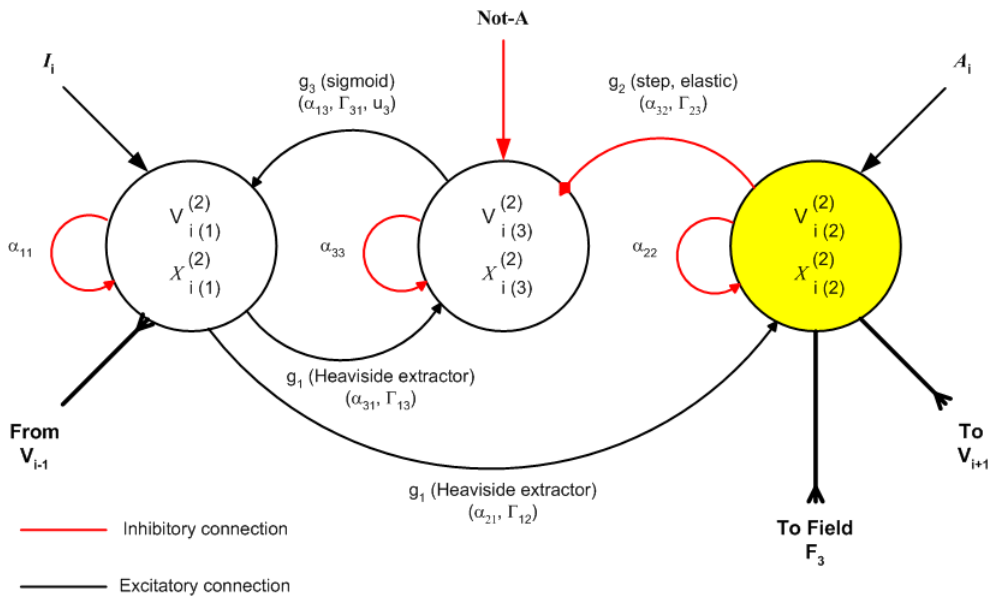


Figure 6: Detailed embedding field graph of vertices  $V^{(2)}$  of figure 5.

It is quite conceivable that in figure 5 the  $F_2$  Field vertices might depict noetic representations but that the non-colored vertices of figure 6 might not. This will depend, among other things, on the capacity to measure and identify corresponding somatic places. If this is the case, then the activity of the yellow-colored vertex in figure 6 would either be a principal quantity or else lie very close to one in the mathematical plane of facet B but the other vertices in the figure

would only depict secondary quantities (and, therefore, would not be representations with semantic import). The  $F_2$  Field vertices of figure 5, in contrast, would have to be principal quantities because we are assuming for this example that they are noetic representations and, therefore, have semantic import and, likewise, a somatic signal correspondent. The *rational* (theory) part of a proper science *always* resides in the mathematical world of facet B (whereas the empirical part is concerned with the facet A of actual experience)<sup>15</sup>. Objective validity for any theory depends upon the theorist to always be aware of when he is presenting secondary quantities and when he intends for one of his constructs to be a principal quantity. I say "intends" here because it ultimately is not up to the theorist but, rather, the empiricist to determine if a mathematical construct is or is not observable in somatic appearances. If it is not, then it is a secondary quantity regardless of how much one wants it to be a principal one and regardless of how scrupulously its construction conforms to mathematical axioms deduced from the Critical acroams and principles of mental physics<sup>16</sup>.

6. Mathematical requirements of the Comparison transformation. The metaphysics of *Comparison* do not go beyond its Critical mental physics principles and its identification as a mathematical equivalence relation. Going further than this is a task for natural science under the constraints imposed by the applied metaphysic of *psyche* and that of the somatic code. What is an equivalence relation? To understand this we need a few more mathematical definitions.

**Definition 2.** The *Cartesian product*  $A \times B$  of two sets,  $A = \{a_1, a_2, \dots, a_n\}$  and  $B = \{b_1, b_2, \dots, b_m\}$ , is the set  $C$  consisting of ordered pairs  $(a_i, b_j)$  for every  $a_i \in A$  and every  $b_j \in B$ .<sup>17</sup>

Set  $C$  is basically just a big table containing every possible pairing of the members of the two sets arranged in the order denoted by the Cartesian product symbol  $A \times B$ .  $C$  is most commonly used as a universe set for mathematical relations.

**Definition 3.** A relation  $\alpha$  *from* a set  $A$  to a set  $B$  is a subset  $R_\alpha$  of the Cartesian product  $A \times B$ . The *relation set* is written  $R_\alpha \subset C = A \times B$ .

The relation  $\alpha$  is basically defined by deleting entries in the table  $C$ . The undeleted entries give you the relation. If an ordered pair  $(a_i, b_j)$  is deleted from  $C$  then the relation does not hold between  $a_i$  and  $b_j$ . For example, if the relation is "is the son of" and  $B$  is the set of English/British kings and queens then (Richard, Henry II), (Geoffrey, Henry II), (Henry, Henry V) and (Edward, Victoria) are all in the relation set but (Eleanor, Henry II) and (Elizabeth, Henry VIII) are not because Eleanor and Elizabeth were daughters, not sons. Likewise, (Henry of Monmouth, Henry V) is not in the relation set because Henry of Monmouth is the same person as Henry V and no person is ever his own son.

**Definition 4.** A relation  $\alpha$  *on* a set  $A$  is a subset  $R_\alpha$  of the Cartesian product  $A \times A$ . The *relation set* is written  $R_\alpha \subset C = A \times A$ .

Other than for making  $B = A$ , a relation *on* a set is the same as a relation *from* a set  $A$  to a set  $B$ . In a mathematical graph it is quite common (although not actually *required*) that all the vertices of the graph are regarded as belonging to the same set  $V$ . On those occasions where I

<sup>15</sup> Slepian, David (1976), "On bandwidth," *Proceedings of the IEEE*, vol. 64, no. 3, pp. 292-300.

<sup>16</sup> If a mathematical construct does *not* conform to objectively valid axioms of Critical mathematics, then it lacks objective validity and can never be anything more than a secondary quantity.

<sup>17</sup> I should have provided this definition, and the next two, earlier (before definition 1). Sorry. I'm used to the hieroglyphics and I sometimes briefly forget how generally poor mathematics education in the United States has become since the first half of the 1960s. I usually don't forget for very long because my students are pretty good at reminding me of it. (Or, at any rate, my students who are not from mainland China).

am working with a graph that has distinct sets of vertices, I usually use a circle symbol to draw vertices from one of the sets and another symbol, e.g. a square, to draw vertices from the other.

**Definition 5.** A relation  $\alpha$  on a set  $A$  is said to have the *reflexive property* (to be reflexive) if, for every  $a \in A$ , we have  $a \alpha a$ .

If the relation  $\alpha$  is "equals," then the notation  $a \alpha a$  reads " $a$  equals  $a$ ." If the relation is "divides" then the hieroglyph reads " $a$  divides  $a$ " (or " $a$  divides itself"). If the relation is "excites" then the hieroglyph reads " $a$  excites  $a$ " (or " $a$  excites itself"). In graphical depiction, a vertex  $v$  is reflexive if that vertex projects an arc depicting the relation to itself. The graph as a whole depicts a reflexive relation on its set of vertices if and only if every vertex in it is reflexive.

**Definition 6.** A relation  $\alpha$  on a set  $A$  is said to have the *transitive property* (to be transitive) if, for members  $a$ ,  $b$ , and  $c \in A$ , the condition  $\langle a \alpha b$  and  $b \alpha c \rangle$  implies  $a \alpha c$ .

This string of definitions sets up the one we're actually after:

**Definition 7.** A relation  $\alpha$  on a set  $A$  is an equivalence relation if  $\alpha$  has the reflexive, symmetric and transitive properties.

*Comparison* is a process that synthesizes equivalence relations (note that this is plural; there are multiple ways in which two comparands can properly be said to be "equivalent"). Precisely what sort of equivalence relation in the particular depends on something else, namely, the process of reflective judgment. This is because reflective judgment provides the *context* for a particular type of "equivalence" in sensibility. No depiction that lacks a context can *mean* anything (and thus such a depiction is a semantic null). All meanings are ultimately practical, which is to say that a meaning implication implicates an action of some kind, and the process of reflective judgment is the bridge between sensibility and the expression of actions. A theorist is not free to simply make up whatever *ad hoc* mathematical equivalence relations he chooses without taking the process of reflective judgment into account.

The next important thing to now note is that neither figure 1 nor figure 2 explicitly depicts this mathematical definition of equivalence relation. Figure 1 does not explicitly depict a relation *on* a set at all. It explicitly depicts a relation *from* a set of comparand pairs (set  $A$ ) *to* a set of determinations (set  $B$ ). That is not a relation *on* a set. Figure 2 depicts (or could be constructed to depict) a relation on a set, but the figure does not explicitly depict any notion of reflexive property. Both figures claim to illustrate the *Comparison* process but *Comparison* is understood to be the process of synthesizing equivalence relations. Do we have a contradiction?

7. Construction of the mathematical relations and processes by minimal anatomies. Who would guess that the "obvious" idea of "comparison" could run into thorny issues like this?<sup>18</sup> As I have remarked elsewhere numerous times, figuring out *what* you want to do is always where the biggest challenges lie; figuring out *how* to do it is almost always, "by comparison," the easy part unless what you want to do is build a flying carpet or anything else that does not belong to the natural world. Mental physics tells us human beings learn *from* particular examples to get *to* abstract concepts. Let us set up a non-trivial example equivalence relation and see what goes into it. Let the equivalence relation  $\alpha$  be "is rightful heir to a throne." Our equivalence set  $H$  will be "the set of people who are rightful heirs to a throne." If person  $a$  and

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<sup>18</sup> Perhaps Bertrand Russell and Alfred North Whitehead. In their unreadable masterpiece, *Principia Mathematica* (Cambridge University Press, 1910) they formally and rigorously *proved*  $1 + 1 = 2$ . It took them *362 pages* to establish the proof. After such a labor to establish such a trivial fact, I suspect Russell and Whitehead probably *would* appreciate how non-obvious the obvious can be.

person  $b$  are both people who are rightful heirs to a throne, we'll write this as  $a \alpha b$ . By symmetry  $b \alpha a$ , and by reflexivity,  $a \alpha a$ , etc. If  $a$  is a rightful heir to a throne but  $b$  is not, we'll write this as  $a \sim \alpha b$ . By transitivity, if  $a \alpha b$  and  $b \alpha c$  then  $a \alpha c$ . The relation basically states "all heirs to a throne are equal."

Actual rules of succession appearing in history are fairly complicated so let us use a simpler set of rules for this example. One thing rules of succession have in common is that they use orders of precedence. Suppose the rules are as follows:

1. If  $a$  is the oldest living legitimate son of king  $k$  then  $a$  is first in line of succession and we will denote this condition by the notation  $p_1(a)$ ;
2. If king  $k$  has no living legitimate son and  $b$  is the oldest living daughter of  $k$  then  $b$  is first in line of succession and we will denote this condition by the notation  $p_2(b)$ ;
3. If king  $k$  has no living legitimate children and  $c$  is the oldest living grandchild of  $k$  then  $c$  is first in line of succession and we will denote this condition as  $p_3(c)$ ;
4. If king  $k$  has no living legitimate children or grandchildren and  $d$  is the oldest living nephew, niece, great-nephew or great-niece of  $k$  then  $d$  is first in line of succession and we will denote this condition as  $p_4(d)$ .

Let us use the following hieroglyphs:  $\exists$  means "there is some";  $\wedge$  means "logical conjunction" ("and");  $\vee$  means "logical disjunction" ("or");  $\Rightarrow$  means "implies"; s.t. means "such that." Let  $K$  denote the set of all kings,  $k$ . Now let us compare persons  $a$  and  $b$  according to the equivalence relation. In symbolic logic notation,

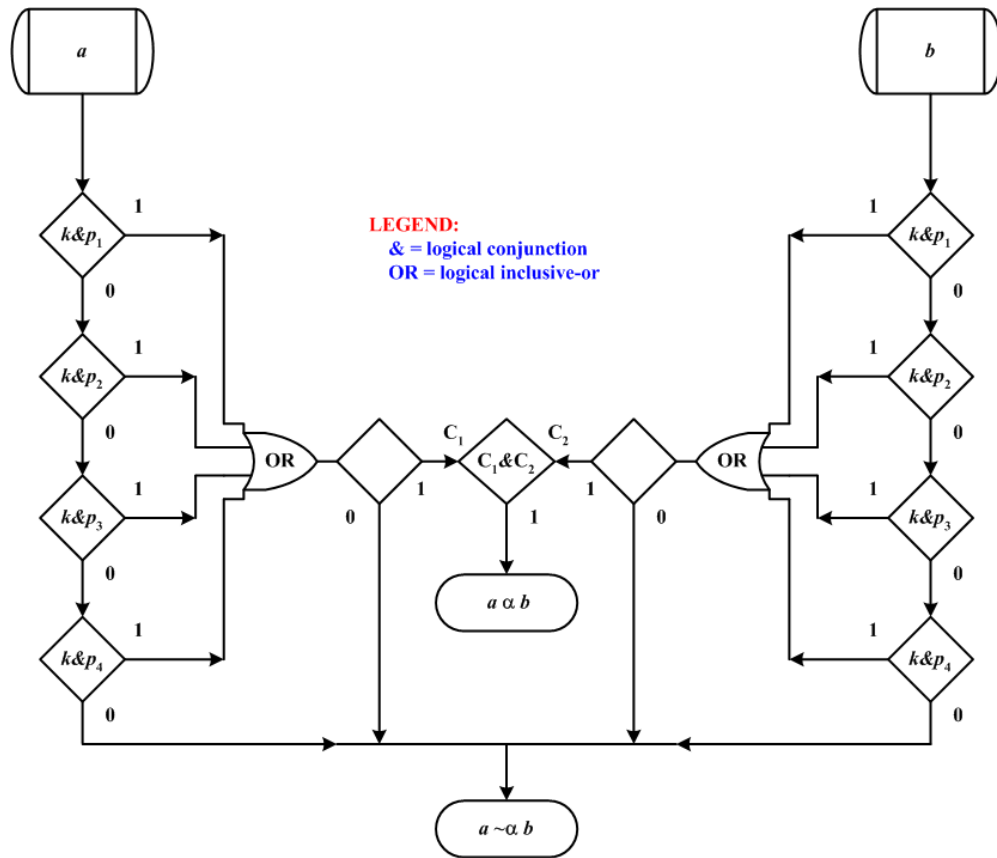
$$[\exists k_a \in K \text{ s.t. } p_1(a) \vee p_2(a) \vee p_3(a) \vee p_4(a)] \wedge [\exists k_b \in K \text{ s.t. } p_1(b) \vee p_2(b) \vee p_3(b) \vee p_4(b)] \\ \Rightarrow a \alpha b.$$

This is how a symbolic logician would say "If  $a$  is first in line of succession of king  $k_a$  and  $b$  is first in line of succession of king  $k_b$  then  $a$  is equivalent to  $b$ ." The mutually exclusive way we have defined the four properties ensures that the two kings are different people unless  $a$  and  $b$  are the same person (in which case we have the reflexive property) and the commutativity of the logical conjunction operation ensures the symmetry property. Proof of the transitive property is easy. The equivalence set  $H$  is the set of all people  $a$  such that

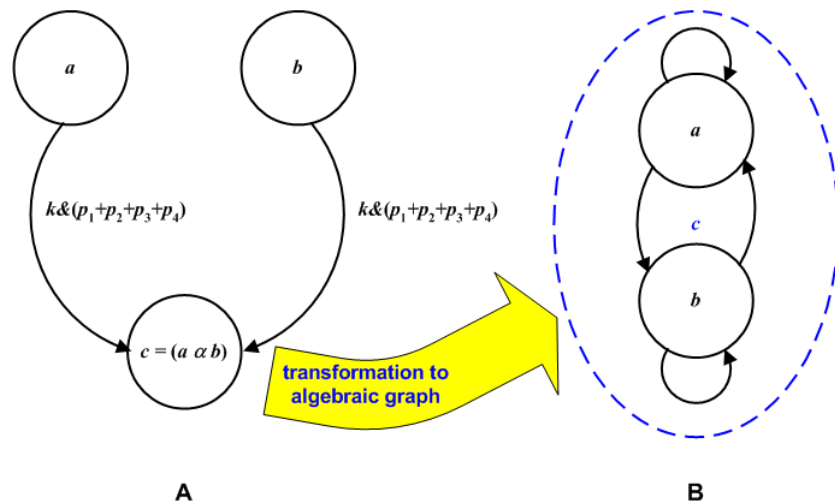
$$H = \{a \text{ s.t. } \exists k_a \in K \text{ and } p_1(a) \vee p_2(a) \vee p_3(a) \vee p_4(a)\}.$$

That the scrawl of symbolic logic notation above has anything to do with a graph is probably not obvious to most people. However, the logic sentence can be re-presented in the form of the logic flowchart shown in figure 7 and, because a flowchart is a type of graph, the logic flowchart is isomorphic to a graph. Figure 8A re-presents figure 7 in the form of graph called a signal flow graph or *network* graph. Figure 8B is an *algebraic* graph that re-presents what vertex  $c$  in figure 8A depicts.

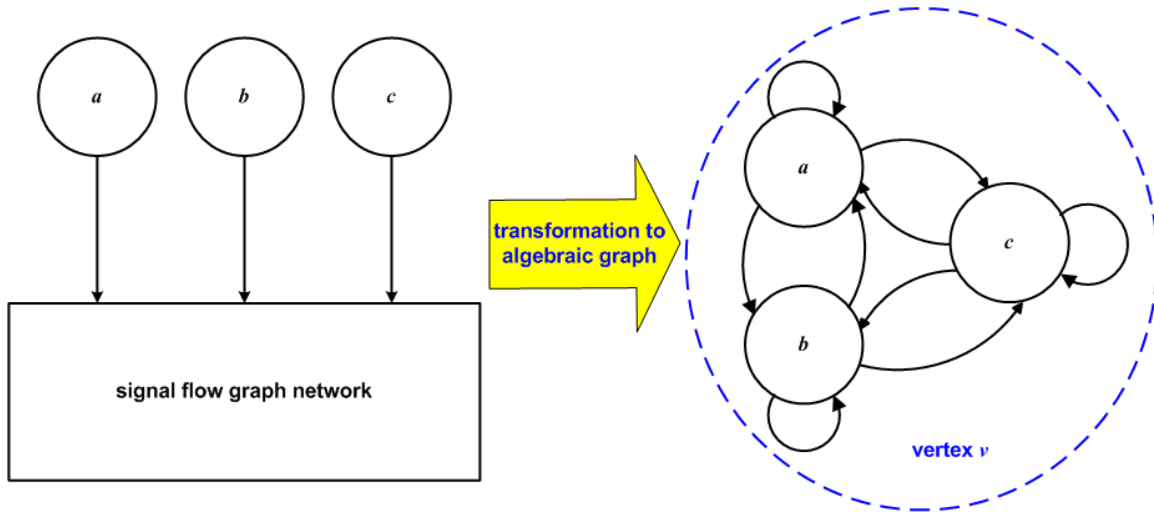
Figures 7 and 8A are data representations of precisely the same *information* carried in the symbolic logic notation above. This means all three are ***semantically equivalent*** depictions. What differs from one to the others is the *context* of how the depiction is put to use by the theorist. This is to say the difference is practical rather than in any way ontological. In the terminology of Critical epistemology, these different depictions are said to be equivalent representations made from different *Standpoints* of judgment. Each depicts the same ***matter of semantics*** but in a different ***form of applied context***. Likewise, the transformation from vertex  $c$  to figure 8B is an isomorphic transformation re-presenting vertex  $c$  in its *algebraic* context. It is often very useful to employ all three depictions in figures 7 and 8 in theoretical work.



**Figure 7:** Logic flowchart equivalent to the symbolic logic expression for the "heir to a throne" example. This form of representation is popular with computer engineers for designing algorithms to be implemented either in microelectronic hardware or as computer programs.



**Figure 8:** Alternate graphical representations for the "heir to a throne" example. A: Signal flow graph (also called a network graph) equivalent to the flowchart of figure 7. B: Algebraic graph equivalent to vertex  $c$  in the A figure. The transformation is isomorphic, i.e., one can go unambiguously from vertex  $c$  to 8B or from 8B to vertex  $c$ . This transformation is called a *meaning implication*.  $+$  denotes the logical inclusive-or operation,  $\&$  denotes the logical conjunction operation,  $k$  is some member of the set of kings  $K$ .



**Figure 9:** An example exercise for you to work out as a corollary. The exercise is to design a signal flow graph network depicting a 3-way relation  $a \alpha b \alpha c$  and describe the transformation from some vertex  $v$  in the signal flow graph to the algebraic graph for the "heir to a throne" example.

One subtle but important point I can bring out in this example is provided by the transformation from vertex  $c$  to the oval  $c$  of figure 8B. The oval is the depiction of a principal quantity in the facet B of mathematical theory. Vertex  $c$  in 8A is a depiction of an embedding field vertex *that can be used to represent an empirical object of appearance*. The transformation is nothing less than a description of how the principal quantity is to be set against its object in facet A. This is to say that the transformation stands in the place of the semantic representation of the graphico-mathematical depictions.

There is a lot of content in this example and it is probably not enough for you to just read what I have written here and look at the figures. To anchor your understanding, you should try your hand at working an example problem yourself. Figure 9 defines a good exercise for you to try. The solution is not very difficult (once you start to get used to the method). I need to mention, by the way, that there are multiple correct solutions (all of which are semantically equivalent). I also want to emphasize that to work with these "pictorial" representations is *to be doing mathematics*. It is not written anywhere that mathematics must only be symbolic algebra and/or symbolic logic.<sup>19</sup> Indeed, if you try this exercise beginning with symbolic logic or symbolic algebra the odds are quite good that you will fail to figure out a correct answer. This very stiff formal language manner of expression occasionally has its uses just as a knowledge of Latin or a familiarity with Australian geography occasionally has its uses. But it is a "thee and thou" stilted form of expression – the kind whose usage ends up taking 362 pages to prove  $1 + 1 = 2$ . Mathematicians Davis and Hersch<sup>20</sup> tell us,

Why are textbook and monograph presentations of mathematics so difficult to follow? The layman might get the idea that a skillful mathematician can sight-read a page of mathematics in the way that Liszt sight-read a page of difficult piano music. This is rarely the case. The absorption of a page of mathematics on the part of the professional is often a slow, tedious, and painstaking process.

<sup>19</sup> Come to think of it, it actually is. Bertrand Russell said exactly this in his *Introduction to Mathematical Philosophy* in 1919. However, the entire "logicist program" he labored to try and bring about eventually collapsed and he eventually came to admit publicly that the entire endeavor had been in vain.

<sup>20</sup> Philip J. Davis and Reuben Hersch (1981), *The Mathematical Experience*, Boston, MA: Houghton Mifflin, pp. 281-282.

The presentation in textbooks is often "backward." The discovery process is eliminated from the description and is not documented. After the theorem and its proof have been worked out, by whatever path and by whatever means, the whole verbal and symbolic presentation is rearranged, polished, and reorganized according to the canons of the logico-deductive method. . . Fuller explanations are regarded as tedious.

Tedious to write perhaps. It's always harder to explain something clearly than to wallow in esoteric obscurity. But when you're explaining something *to yourself*, don't you think you are worth the extra effort? Professional *mathematicians* think so. You should, too. They do favors for themselves that are "too tedious" to do for us. Do yourself the same favor.

Getting back on topic now, the construction of the signal flow graph, as an embedding field graph, is carried out by the method of minimal anatomies. Note how in figure 8A the *overall* operational statements are simply listed in the vertex *c* and the connecting arcs. This is the first, not the last, step in the procedure. The next step is to present these mathematical operations in more detail using more distinct embedding field graph representations (refer again to figures 5 and 6). This is continued until one reaches the point where he understands *precisely* how to carry out all the desired calculations needed to obtain computable results.

8. Property sets and feature detection. The example in (7) illustrates two key factors that are involved in the embedding field graph construction. The first is that *properties* ( $p_1$ - $p_4$  in the "heir to a throne" example) are used *to put in the context by which "equivalence" is to be understood*. In the language of set membership theory (SMT), these are called "property sets." Kant called them "marks of recognition of the object." The second factor is that activation of the representing vertex (vertex *c* in figure 8A) is based upon the comparands *each exhibiting this specific set of properties* as the basis (determining factor) of evaluating them to be "equivalent." In the terminology of neural network theory, this determination is called **feature detection**. The property sets *define* the features to be detected by the embedding field anatomy. The specific manner of evaluation is what figure 7 depicts for the "heir to a throne" example.

Psychology uses the word "feature" as an abbreviation for the term *distinctive feature*.<sup>21</sup> In general, a distinctive feature is an attribute ("feature") of some object or event that is critical in distinguishing that object or event from others. In phonetics, a distinctive feature is an aspect of a phoneme that distinguishes it from another. Phonetic features are always depicted as binary pairs, i.e., a phoneme either exhibits a feature (+) or it does not exhibit it (-). For example, "voicing" is a distinctive feature that distinguishes the phonemes /s/ and /z/, as in *sue* (voicing = -) and *zoo* (voicing = +). In non-Critical semantics, features are categorized as either *defining features* (attributes that are essential in the *Existenz* of an object, e.g., "having feathers" for *bird*) or *characteristic features* (typically attributed to an object, e.g., "flying" for *bird*, but not necessary for *defining* the object – as, e.g., in the case of an ostrich).

In these terminological usages, "feature" is basically used as a synonym for "attribute." This, however, confines the connotation to one of logical Quality, i.e., affirmative (+), negative (-) or infinite ( $\diamond$ )<sup>22</sup> and often overlooks or omits this last idea (an omission that frequently starts a speculator or theorist on the road to transcendental illusion and antinomy). The depiction of

<sup>21</sup> Reber's *Dictionary of Psychology*, 3rd ed., 2001.

<sup>22</sup> The infinite logical *momentum* of Quality does not pertain to mathematics' infinity (which is denoted by the symbol  $\infty$ ). The infinite function (symbolized by  $\diamond$ ) excludes predicate terms without specifying anything more about the object. For example, the infinite predication "Mel is not-a-man" merely specifies that "being-a-man" is incongruent with "being-the-object-'Mel' " and tells us nothing further about "Mel." The general form of an infinite predication is *x is not-y*. It is not correct to think of the infinite momentum as an "indefinite" function because it is quite definite about attribute *y* being incongruent with being object *x*.

any logical *property* must include Quantity (singular, particular, or universal) and Relation (categorical, hypothetical, or disjunctive) as well, and for a logical property to be a logical *feature* the depiction must further include a reference to *nexus* in the Organized Being, which is Modality (problematic, assertoric, or apodictic). In this technical logical context, **a feature is a property with Modality function.**<sup>23</sup>

Neurologist A.R. Damasio has presented an hypothesis on the architectural organization of the brain in which the concept of "distinctive features" is central<sup>24</sup>. Damasio's hypothesis states four central assertions:

1. neuron ensembles located in multiple and separate regions of primary and first-order sensory association cortices ("early cortices") and motor cortices contain representations of feature fragments inscribed as patterns of [neural] activity originally engaged by perceptuomotor interactions;
2. neuron ensembles located downstream from the former throughout single modality cortices (local convergence zones) inscribe amodal records of the combinatorial arrangement of feature fragments that occurred synchronously during the experience of entities or events in sector (1);
3. neuron ensembles located downstream from the former throughout higher-order association cortices (non-local convergence zones) inscribe amodal records of the synchronous combinatorial arrangements of local convergence zones during the experience of entities and events in sector (1);
4. feedforward and feedback projections reciprocally interlock the neuron ensembles in (1) with those in (2) according to a many-to-one (feedforward) and one-to-many (feedback) principle.

Damasio's hypothesis is not-incongruent with mental physics. It is completely in accord with adaptive resonance theory (ART), which grew out of embedding field theory and was first discovered and published by Grossberg in 1976.<sup>25</sup> Figure 10 illustrates the general organization of a minimal ART anatomy. Each of the field and other blocks depicted in the figure contain embedding field sub-graphs carrying out specialized transformations that collectively determine the overall transformation effected by the ART network. Different embedding field anatomies within the blocks define different kinds of basic ART subsystems.

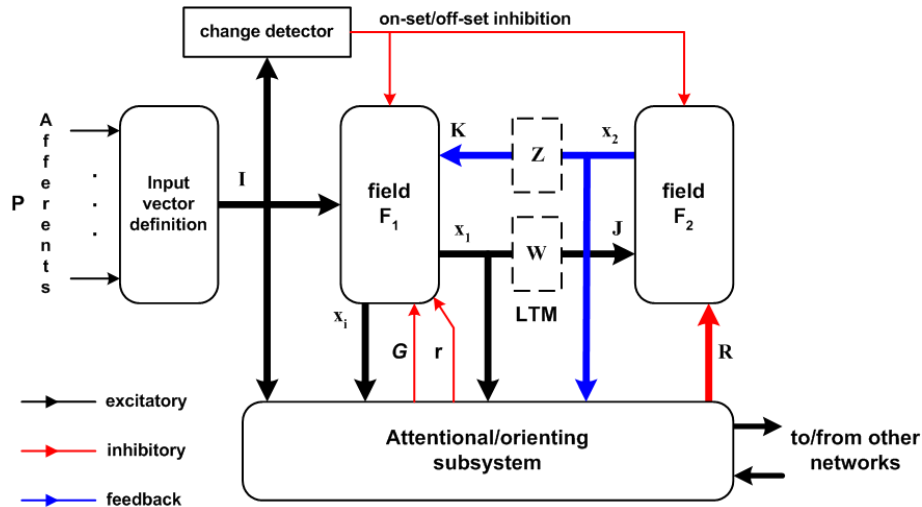
Neural network theorists frequently describe the ART subsystem of figure 10 as a "classifier." This description is not inaccurate but it falls a bit short of conveying the functional capabilities of the ART structure. If the input vector  $I$  in figure 10 is comprised as a partitioned vector in which the partitions correspond to places in two comparands, then "classification" of input vector  $I$  corresponds to the implementation of a binary (two-comparand) equivalence relation provided: (1) the classification is insensitive to permutation of the partitions holding the comparand data within vector  $I$  (symmetric property); (2) the partitions are equal-sized such that both comparands can be identical sub-vectors (reflexive property); and (3) a classification  $\alpha$  is self-consistent, by which I mean if the classification of a vector  $I = [a \mid b]^T$  is  $a \alpha b$  and that of a vector  $I = [b \mid c]^T$  is  $b \alpha c$  then  $I = [a \mid c]^T$  produces classification  $a \alpha c$  (transitive property). How the ART network "classifies" *determines* the type of mathematical relation that

<sup>23</sup> For more details on the logical *momenta* of understanding refer to Wells (2009), *The Principles of Mental Physics*, available on the Wells Laboratory web site.

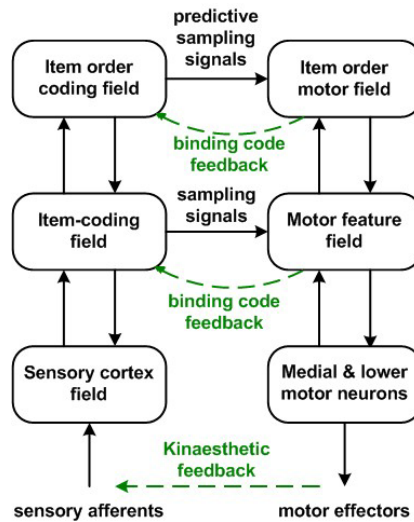
<sup>24</sup> Damasio, Antonio R. (1989), "Time-locked multiregional retroactivation: A systems-level proposal for the neural substrates of recall and recognition," *Cognition*, 33: 25-62.

<sup>25</sup> Grossberg, Stephen (1976), "Adaptive pattern classification and universal recoding: II. Feedback, expectation, olfaction, illusions," *Biological Cybernetics*, 23, 187-202.





**Figure 10:** Minimal anatomy of an ART subsystem. Field  $F_1$  corresponds to one of Damasio's "upstream" neuron ensemble aggregates. Field  $F_2$  corresponds to one of Damasio's downstream convergence zones. Feedforward associational strengths  $W$  make many-to-one projections into  $F_2$  and feedback associational strengths  $Z$  make one-to-many projections into  $F_1$ . Fields  $F_1$  and  $F_2$  are themselves comprised of non-trivial embedding field networks, and  $F_2$  contains a dipole network subsystem. Not shown in the figure are projections to other anatomical subsystems. Also not explicitly shown in the figure are details of how the input vector  $I$  is defined. It seems a likely-true conjecture that this function has for its neurological substrate the organization of thalamocortical interconnections [Wells (2011a)<sup>26</sup>, figure 13, and Wells (2011b)<sup>27</sup>, figure 6]. The interconnections to/from the attentional/orienting subsystem would include interconnections to network anatomies in the limbic system associated with functions of reflective judgment.



**Figure 11:** Higher-order structural organization, expressed in semantics terms, composed of interconnected ART subsystems. The direction bottom-to-top in this figure corresponds to dorsal-to-rostral direction in the brain of *H. sapiens*. The figure is a modification of an anatomical organization proposed earlier by Grossberg modified in order to make Grossberg's structure more reflect Damasio's hypothesis.

<sup>26</sup> Wells, Richard B. (2011a), "On Critical representation in brain theory, part II: General schema of knowledge representation," available on the Wells Laboratory web site.

<sup>27</sup> Wells, Richard B. (2011b), "Preliminary discussion of the Martian 2 Program," available on the Wells Laboratory web site.

it is implementing.

9. The ART network of figure 10 is insufficient by itself to determine semantic context. The minimal ART subsystem of figure 10 makes its classification outcomes based on nothing else than the relative degree of activities in the places represented in vector I. Its operating principle, in other words, is simply the principle of contrast enhancement. Its mathematical behavior is based on no other context than this and, in point of fact, usually does not take into account any partitioning structure within vector I. This means that the bare equations of the ART system, as these are presently employed, lacks semantic context. Any semantic context the system might have (e.g., list chunking) must be put there externally by the network designer by, e.g., his specification of what input afferent tracts are and are not routed to the ART network.

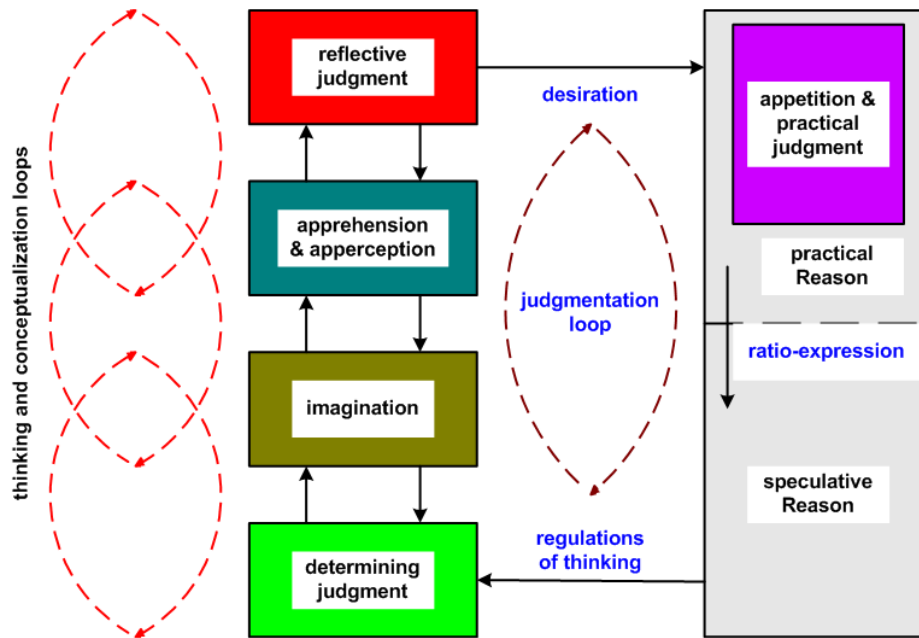
This at best addresses only Quality in the functioning of the ART subsystem of figure 10. By itself this is not enough to establish full property sets by which the mathematical relation the network realizes (e.g., equivalence) are either defined or provided with semantic context. The ART literature uses a too-vaguely-defined (too Platonic) concept of "feature." To provide for Quantity and Relation (which completes the structure of a property set) and for Modality (which completes the definition of a feature), the anatomy of figure 10 must be augmented by the architectural organization in which it is embedded.

Grossberg provided some discussion of the architectural aspect in Grossberg (1976)<sup>25</sup>, but this discussion was rather too broad and abstract to be of much practical service. He elaborated somewhat more on this topic in Grossberg (1978)<sup>13</sup>, but again this discussion was on the whole too abstract for practical purposes. Even so, one point that did clearly come out was a general schema, depicted in figure 11, in which one finds forward-and-backward (rostral-and-ventral) interactions among ART subsystems that, in principle, might provide some of (likely not all) the "missing pieces" I spoke of a moment ago. I say "likely not all" because Grossberg (1976)<sup>25</sup> also touched upon some "three-way" interaction structures (cf. figure 5 of that citation) that by and large have not been explored sufficiently in the ART literature (or, at least, have not been explained clearly enough in that literature; it is an unfortunate but nonetheless true statement that most ART papers since the 1990s have presented networks and findings but failed to provide a sufficient level of detailed description for other researchers to reproduce the results they report)<sup>28</sup>. The network organization depicted in figure 11 has been modified from its original presentations in order to make the idea better reflect Damasio's hypothesis and to illustrate the presence of that "third connection" briefly introduced in 1976.

The schema of figure 11 also shows up in the organization of *nous*. I think this unlikely to be a

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<sup>28</sup> I think one part of this issue is simply bad technical writing, but a much bigger part of the problem is the ever-more-restrictive page limitation rules the professional journals have been enforcing over the past three decades. While any competent administrator of a journal-publishing organization pays attention to his publication production costs, when it reaches the point where papers become uninformative the economics have overshadowed the purpose of producing the publication. The majority of technical journals today have truly become "archival" journals in the sense that they are merely shelved and rarely looked at again. The retread businessmen most professional organizations have hired in the past few decades to administer the business side of the organization's publications are rendering those journals *scientifically* useless. I suspect strongly that present day business practices have already made the tradition of the scientific-communication-by-journal method obsolete. The professional societies ought to rethink why they exist in the first place if they wish to continue to exist. It isn't so professors can go on junkets to far-away places on someone else's dime. (These junkets are called "conferences" but there isn't actually all that much conferring that goes on there). It isn't to provide a contentless counting mechanism (publication counts) for university committees to use to mechanize tenure and promotion decisions. To serve *science*, not to serve administrating or social networking, is the reason why any *healthy* professional scientific society exists. As Santayana once said, "Fanaticism consists of redoubling your effort after you have forgotten your purpose."



**Figure 12:** Thinking and judgmentation organization structured in *nous*. Not depicted in this figure are the connections to and from *psyche* (receptivity and motoregulatory expression) [cf. Wells (2011b)<sup>27</sup>, figure 1].

a coincidence. Figure 12 illustrates the structure of thinking and judgmentation in *nous*. The similarities between this figure and figure 11 are more or less obvious. Now, this structure is deduced in mental physics quite independently of both Damasio's hypothesis and Grossberg's ART research findings. The fact that this same schema is found in three such completely independent researches (where the only commonality is the Object, *H. sapiens*) I think is very significant. A term applicable to this, and made rather popular since the publication of Mandelbrot's book [Mandelbrot (1983)<sup>29</sup>], is *self-similarity*. While I am very far from being swayed by the aura of mysticism that attends much of what I read on the subject of fractals, Mandelbrot did make a point that is worth noting here:

Until now we stressed that coastlines' geometry is complicated, but there is also a great degree of order in their structure. Although maps drawn at different scales differ in their specific details, they have the same generic features. In a rough approximation, the small and the large details of coastlines are geometrically identical except for scale.

One may think of such a shape as drawn by a sort of fireworks, with each stage creating details smaller than those of the preceding stages. However, a better term is suggested by our Lewis Richardson's noted work on turbulence<sup>30</sup>: the generating mechanism may be called a *cascade*.

When each piece of a shape is geometrically similar to the whole, both the shape and the cascade that generates it are called *self-similar*. [Mandelbrot (1983), pg. 34]

I hold no opinions at all regarding whether coastlines have or have not a fractal quality, but it is a different matter when we're talking about mind-body phenomena. There is nothing in mental physics that grants any special status to the subject of fractal geometry, but when an interesting general schema begins showing up across greatly diverse scales and appears in findings or hypotheses sharing little with each other than a common scientific topic, paying

<sup>29</sup> Mandelbrot, Benoit B. (1983), *The Fractal Geometry of Nature*, NY: W.H. Freeman and Co.

<sup>30</sup> Mandelbrot is apparently referring to Richardson, L.F. (1926), "Atmospheric diffusion shown on a distance-neighbor graph," *Proceedings of the Royal Society of London*, **A**, **110**, 709-737.

heed to it at least as a possibly useful empirical heuristic seems quite prudent as a practical maxim of research and theory. When mental physics, empirical psychiatry, neurology and mathematics all seem to be converging on a common schema, it is wise to take that schema seriously. In regard to the matter at hand, I call this practical heuristic *the maxim of layered semantic embedding field constructs*.

Without dignifying it with a name, Grossberg has been arguing for many years now that "there is something fundamental in ART" pertinent to neuroscience and neuropsychology. Personally I tend to prefer a little broader view than one exclusively focused on ART graphs, but in all except for some quibbling minutia I am of the opinion that Grossberg has been right over those many years. Let us therefore look at this notion.

10. Layered and bi-directional embedding field construction of semantic relations. Research and theory construction for *Comparison* (and, indeed, for all the other functions, processes, and capacities of the Organized Being) is a formative process. It is more likely to succeed when the research proceeds in a step-wise progression of building up embedding field structures. I think nothing better illustrates this than the history of how Grossberg *et al.* built up to ART from the 1960s to 1980s and to systematic ART since the 1980s. Powerful mathematical relations are always built up from less powerful ones and built in precisely this way when it is done efficiently. For example, consider the following definition:

**Definition 8.** A *compatibility relation*  $\gamma$  on a set  $A$  is a relation that has the reflexive and symmetric properties.

It is easy to see from this definition that an equivalence relation is a compatibility relation to which additional property sets have been added to produce transitive relation as an additional characteristic of the relation. The process of *Reflexion* in sensibility is an equivalence relation with additional properties. In particular, it is an equivalence relation in which we also have an algebraic structure called a semigroup (a set  $A$  with an operation  $*$  that has the properties of closure and associativity) such that the overall structure constitutes what is called a *congruence* relation:

**Definition 9.** Given a semigroup  $[A, *]$ , an equivalence relation  $\gamma$  on  $A$  is called a *congruence relation* if, for arbitrary elements  $a, b, c$ , and  $d$  in  $A$ , the relation  $\gamma$  has the substitution property with respect to the operation  $*$ , i.e.,

$$a \gamma b \text{ and } c \gamma d \text{ imply } (a * c) \gamma (b * d).$$

The equivalence relation inherent in *Reflexion* is semantically different from that in *Comparison*, which means *Reflexion* has a property set different from *Comparison*. But it is nonetheless clear that in order to produce the *Reflexion* process in embedding field form, we will need to know how to "embed" an equivalence relation within it. To understand how to construct embedding fields for *Comparison* is to already understand much about *Reflexion*.

I have been speaking of "making constructs" throughout these notes. This seems to be a good place to observe that all construction requires construction material and construction tools. To begin to understand this fairly simple idea in a proper context with organized being is to begin by understanding that the "construction material and tools" with which we must deal *are those that are related to the property sets we require for semantic definition* of the mathematical relations actualized by embedding fields.

In facet A of organized being this material and these tools can only be sought in the somatic *materia* of the phenomenon of body and the physiological processes that govern body-matter. In this sensible aspect of organized being we find a logical division into two general classes of neural and endocrine system structures: (1) those structures characterized as stereotyped; and

(2) those structured characterized as non-stereotyped. Most of the neural systems in brainstem and spinal cord appear to be of the stereotyped class, as also appears to be the case for most of the peripheral nervous system. Most of the endocrine system likewise appears to fall into this class. This is significant because the stereotyped structures provide those innate capacities necessary for the possibility of forming later developed structures. The stereotyped structures supply the Organized Being with what Piaget would likely call its *constitutive functions*. Those structures that form in the post-natal development of experience (in contrast to those that are expressed through biological maturation) would then be called the *constituted functions* of organized being. To seek for the *fundamental* building blocks of semantic relations on the side of *soma*, we must begin with the appearances of constitutive structures.

So too it is on the side of *nous* except that here we seek for principal quantities of facet B to overlay those somatic appearances. The class names of these principal quantities are likely to provoke an habituated negative affective reaction from psychologists, although it is only the training they have received, especially in America, that is responsible for the habit. This training has been conditioned by the history of *misuse* of these terms by psychologists of the late nineteenth and early twentieth centuries. I'm going to ask psychologists at this point to put their acquired schoolmen prejudices on hold and maintain an open mind long enough for me to explain the Nature of those phenomena labeled by the names *instincts and preferences*.

11. Semantic character of instinct and preference. Reber's *Dictionary* calls instinct "a term with a tortured history indeed." Reber provides four usages of the term "instinct" and sketches a brief outline of the history of the psychology of instinct. I have provided a bit more historical detail in Wells (2006)<sup>31</sup>. There is a very close kinship between the Critical term "instinct" and the idea of *reflexes*. Reber's *Dictionary* tells us, "Reflexes are usually regarded as species-specific, innate behaviors which are largely outside of volition and choice and show little variability from instance to instance." Reflex *behavior* can be properly regarded as *a class of appearances of instinct* in a context I will now discuss.

What is the *practical significance* of the idea of "instinct" and how is it related to the idea of "intelligence"? One of the best expositions of this that I have seen was presented by Bergson<sup>32</sup> and merits being quoted at length here:

[Intelligence] and instinct, having originally been interpenetrating, retain something of their common origin. Neither is ever found in a pure state. . . There is no intelligence in which some traces of instinct are not to be discovered, more especially no instinct that is not surrounded with a fringe of intelligence. . . In reality, they accompany each other only because they are complementary, and they are complementary only because they are different, what is instinctive in instinct being opposite to what is intelligent in intelligence. [Bergson (1911)<sup>32</sup>, pp. 135-136]

No doubt, there is intelligence wherever there is inference; but inference, which consists in an inflection of past experience in the direction of present experience, is already a beginning of invention. Invention becomes complete when it is materialized in a manufactured instrument. Towards that achievement the intelligence of animals tends as towards an ideal. . . In short, *intelligence, considered in what seems to be its original feature, is the faculty of manufacturing artificial objects, especially tools to make tools, and of indefinitely varying the manufacture.*

Now, does an unintelligent animal also possess tools or machines? Yes, certainly, but here the instrument forms a part of the body that uses it; and, corresponding to this instrument, there is an *instinct* that knows how to use it. True, it cannot be maintained that

<sup>31</sup> Wells, Richard B. (2006), *The Critical Philosophy and the Phenomenon of Mind*, chap. 15, pp. 1342-1362, available on the Wells Laboratory web site.

<sup>32</sup> Bergson, Henri (1911), *Creative Evolution*, Mineola, NY: Dover Publications, 1998.

*all instincts consist in a natural ability to use an inborn mechanism. . . . But this definition, like that which we have provisionally given of intelligence, determines at least the ideal limit toward which the very numerous forms of instinct are traveling. Indeed, it has often been pointed out that most instincts are only the continuance, or rather the consummation, of the work of organization itself. . . . We may say, as we will, either that instinct organizes the instruments it is about to use, or that the process of organization is continued in the instinct that has to use the organ. . . . Thus, if we consider only those typical cases in which the complete triumph of intelligence and of instinct is seen, we find this essential difference between them: *instinct perfected is a faculty of using and even of constructing organized instruments; intelligence perfected is the faculty of making and using unorganized instruments.* . . .*

Instinct finds the appropriate instrument at hand: this instrument, which makes and repairs itself, which presents, like all the works of nature, an infinite complexity of detail combined with a marvelous simplicity of function, does at once, when required, what it is called upon to do, without difficulty and with a perfection that is often wonderful. In return, it retains an almost invariable structure, since a modification of it involves a modification of the species. Instinct is therefore necessarily specialized, being nothing but the utilization of a specific instrument for a specific object. The instrument constructed intelligently, on the contrary, is an imperfect instrument. It costs an effort. It is generally troublesome to handle. But, as it is made of unorganized matter, it can take any form whatsoever, serve any purpose, free the living being from every new difficulty that arises and bestows on it an unlimited number of powers. Whilst it is inferior to the natural instrument for the satisfaction of immediate wants, its advantage over it is the greater, the less urgent the need. Above all, it reacts on the nature of the being that constructs it; for in calling on him to exercise a new function, it confers on him, so to speak, a richer organization, being an artificial organ by which the natural organism is extended. . . . But this advantage of intelligence over instinct only appears at a late stage, when intelligence, having raised construction to a higher degree, proceeds to construct constructive machinery. . . . [*ibid.*, pp. 138-141]

It may be inferred from this that intelligence is likely to point towards consciousness, and instinct towards unconsciousness. For, where the implement to be used is organized by nature, the material furnished by nature, and the result to be obtained willed by nature, there is little left to choice; the consciousness inherent in the representation is therefore counterbalanced, whenever it tends to disengage itself, by the performance of the act, identical with the representation, which forms its counterweight. Where consciousness appears, it does not so much light up the instinct itself as the thwartings to which instinct is subject; it is the *deficit* of instinct, the distance, between the act and the idea, that becomes consciousness so that consciousness, here, is only an accident. . . . In short, while instinct and intelligence both involve knowledge, this knowledge is rather *acted* and unconscious in the case of instinct, *thought* and conscious in the case of intelligence. But it is a difference rather of degree than of kind. . . . [*ibid.*, pg. 145]

Now, if we look at intelligence . . . we find that it also [like instinct] knows certain things without having learned them. But the knowledge in the two cases is of a very different order. . . . [In] this sense intelligence, like instinct, is an inherited function, therefore an innate one. But this innate intelligence, although it is a faculty of knowing, knows no object in particular. When the new-born babe seeks for the first time its mother's breast, so showing that it has knowledge (unconscious, no doubt) of a thing it has never seen, we say, just because the innate knowledge is in this case of a definite object, that it belongs to *instinct* and not to *intelligence*. Intelligence does not then imply the innate knowledge of any object. And yet, if intelligence knows nothing by nature, it has nothing innate. What, then, if it be ignorant of all things, can it know? Besides *things*, there are *relations*. The new-born child, so far as intelligent, knows neither definite objects nor a definite property of any object; but when, a little later on, he will hear an epithet being applied to a substantive, he will immediately understand what it means. The relation of attribute to

subject is therefore seized by him naturally, and the same might be said of the general relation expressed by the verb, a relation so immediately conceived by the mind that language can leave it to be understood, as is instanced in rudimentary languages which have no verb. Intelligence, therefore, naturally makes use of relations of like with like, of content to container, of cause to effect, etc. . . . Let us say, therefore, that *whatever, in instinct and intelligence, is innate knowledge, bears in the first case on things and in the second on relations. . . . Intelligence, in so far as it is innate, is the knowledge of a form; instinct implies the knowledge of a matter.* [*ibid.*, pp. 147-149]

Bergson lays before us here the semantic root of the idea of "instinct." As for the idea of "preference," it will do for now to look at this term as the subjective counterpart in affectivity to "instinct" regarded in the Bergsonian context of *motoregulatory expression*.

Unfortunately, almost all the early twentieth century psychologists were eager disciples of positivism and embraced with enthusiasm its *ex cathedra* dogma of *ignorance* that: (1) science could learn nothing from philosophy; and (2) physics was "the queen of all the sciences." Crippled by their ontology-centered prejudices and baselessly subjective pseudo-metaphysics, they managed to turn the idea of "instinct" into such a mishmash of vague and Platonic notions that the subsequent usages of the term "instinct" actually became counterproductive. They then, like Aesop's fox, declared the whole idea to be sour grapes and, like medieval inquisitors prosecuting heresy, burned the idea at the stake and scattered its ashes on unhallowed grounds. Psychology has never recovered from the trauma of this historic episode and preserves the transcripts of those ecclesiastical court proceedings in the prosecution of "instinct" in school doctrines today.

Lost among these scattered ashes were the germs of some promising conjectures. Two in particular I will point out here were Freud's conjectures on the idea of "impulse" (*Triebe*)<sup>33</sup> and James' theory of instincts<sup>34</sup>. I discuss these in some detail in the earlier Wells (2006) citation<sup>31</sup> and will bring up James' theory again later in these notes. The business-at-hand for the moment is to set Bergson's semantic explanation of "instinct" in its proper mental physics context and then to discuss how this idea is used to produce property sets for defining the *Verstandes-Actus* of *Comparison*.

12. Comparison, instinct and the process of teleological reflective judgment. The logical division of the general idea of "comparison" (*Vergleichung*) into subjective comparison (*Reflexion*) and logical comparison (*Comparison*) mirrors the first ILAR division of the process of reflective judgment into the matter of a process of aesthetical reflective judgment and the form of a process of teleological reflective judgment [Kant (1790)<sup>35</sup>, 5:188-194]. The synthesis of apprehension in sensibility performs no acts of judgment and sensible representations are adjudicated by the process of reflective judgment. *Comparison*, as the first part of the three-step synthesis of the acts of understanding, does not pertain to immediate perception but, rather, to the preliminary preparation of matters-of-perception (sensation and feeling). Because the comparison here is *logical*, *Comparison* is a comparison in terms of *form* and, therefore, its adjudicating process is teleological reflective judgment.

<sup>33</sup> In the case of Freud, the damage caused is exhibited by the fact that psychologists to this day argue, sometimes with heat, over how Freud's word *Triebe* should be rendered in English. The most popular current preference among psychologists seems to be to render it as "drive" although most translations render it as "instinct." Actually, neither rendering is true to the overall context of how Freud used the term. The epistemologically correct rendering for *Triebe* is "impulse."

<sup>34</sup> James, William (1890), *The Principles of Psychology*, vol. 2, chap. XXIV, NY: Dover Publications, 1950.

<sup>35</sup> Kant, Immanuel (1790), *Kritik der Urtheilskraft*, in *Kant's gesammelte Schriften, Band V*, pp. 165-485, Berlin: Druck und Verlag von Georg Reimer, 1913.

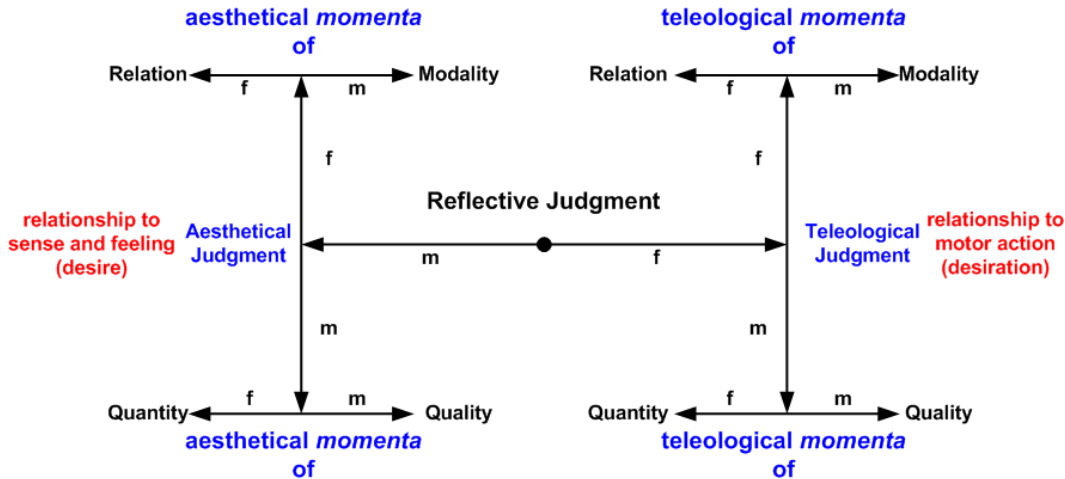


Figure 13: The structure of the process of reflective judgment.

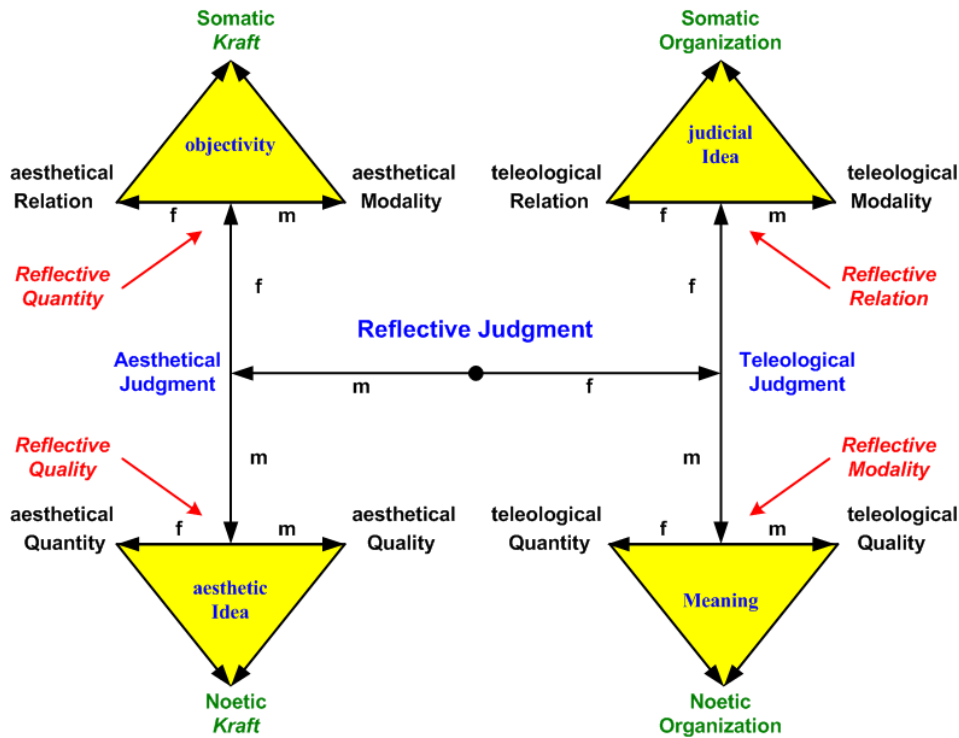


Figure 14: The relationship of reflective judgment and the structure of the divisions of *psyche*.

Figure 13 illustrates the 3LAR structure of the process of reflective judgment. Reflective judgment in general is charged with the task of organizing the understanding of a *system* of Nature, which is an undertaking beyond the capability of the process of determining judgment [Kant (c. 1789)<sup>36</sup>, 20: 202-205]. The governing Critical acroam of reflective judgment is the principle of formal expedience of Nature. Now, in constructing semantic embedding field graphs, we necessarily require a connection of meaning implications, all meanings are at root

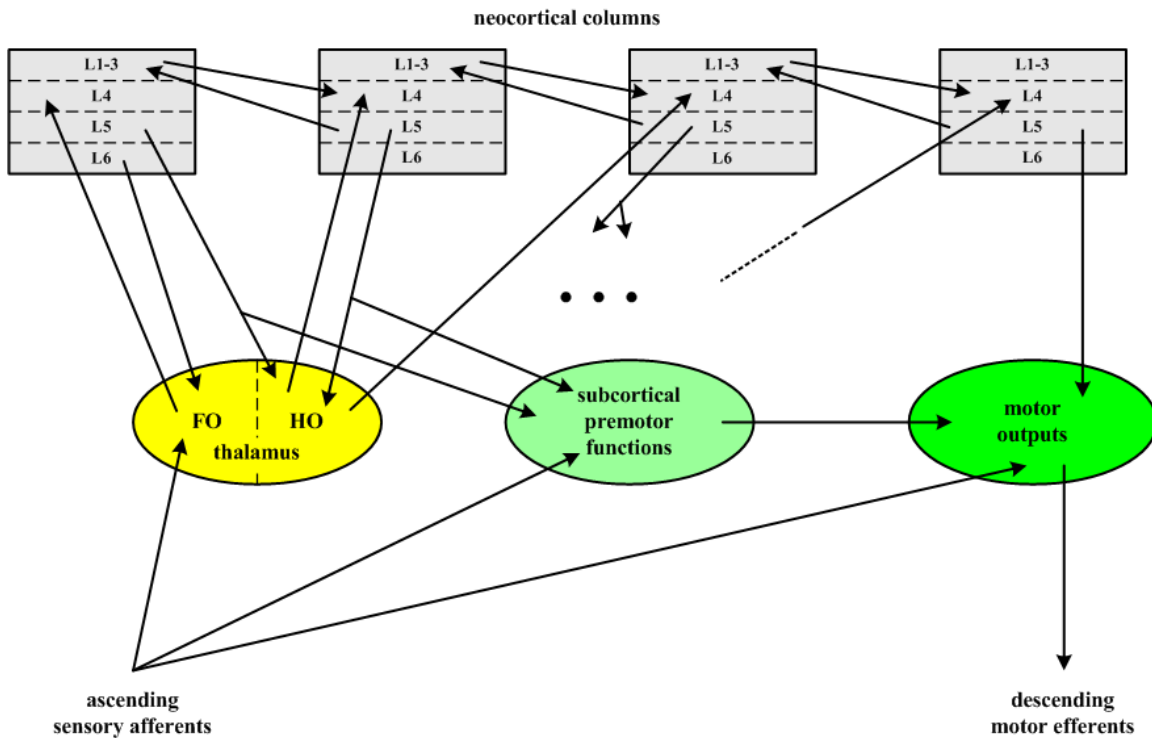
<sup>36</sup> Kant, Immanuel (c. 1789), *Erste Einleitung in die Kritik der Urtheilskraft*, in *Kant's gesammelte Schriften, Band XX*, pp. 193-251, Berlin: Walter de Gruyter & Co., 1942.



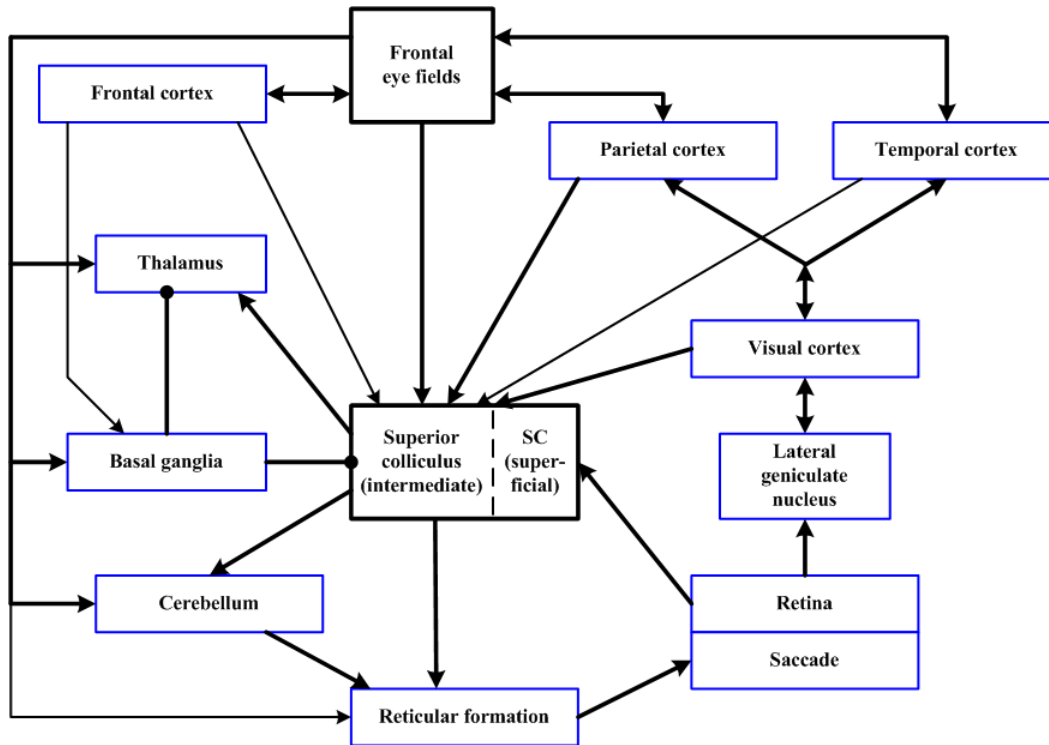
practical, and this practical character means that our Critical understanding of *Comparison* is one that stands in a relationship to motoregulatory expression in *psyche* (because the somatic code belongs to the applied metaphysic of *psyche*). However, this relationship with *psyche* is not an immediate relationship but, rather, is mediated through reflective judgment in its relationship to the general structure of the logical division of *psyche* in organized being. Figure 14 illustrates this relationship, which mental physics calls *the synthesis in continuity* [Wells (2009)<sup>23</sup>, chap. 7 §3.2]. As *logical* comparison, *Comparison* pertains to somatic and noetic *co-organization*, as figure 14 shows, whereas the *subjective* process of comparison (*Reflexion*) pertains to somatic and noetic *Kraft* (actualized power of *psyche*).

It is from this transcendental relationship that *Comparison* obtains its context with respect to reflective judgment. It is from this context that we deduce the property sets that are to be used in fixing the contextual meaning implication of *equivalence* in mathematical representations of the synthesis in *Comparison*. The logical comparison of comparands in sensibility *makes* them equivalent *in terms of possible motor actions*. It is in this context that we find *instinct* (as Bergson's "innate knowledge of a matter") placed in association with a principal quantity of mathematical *equivalence*. The "matter" to which this speaks is *the matter of motoregulatory meaning implications* as these implications are *expressed* by specific sensorimotor behavior.

To make all this a bit less abstract and a bit more concrete, let us stand our conception of the somatic activity field comparands of figure 2 up next to what we know empirically about the biological organization of sensory afferents and thalamo-cortico-motor structures in *H. sapiens*. Figure 15 illustrates a schematic of this interconnection as we understand it today. Some somatic places in the activity field correspond to ensembles of sensory cells in *soma*, others correspond to ensembles of motor cells, and the embedding field problem is to understand the arcs (associational strengths) linking them.



**Figure 15:** Schematic illustration of sensory afferents and their empirical connections in thalamo-cortico-motor structures in the brain as neural science currently understands them.



**Figure 16:** Schematic of brain areas and pathways involved in saccade generation (modified from Munoz and Schall). The superior colliculus (SC) is part of the brainstem (more specifically, the midbrain).

The schematic of figure 15 was derived from anatomical findings presented by Sherman and Guillery (2006)<sup>37</sup>. Like pretty much every such schematic deduced from empirical biology, it is a hugely simplified depiction of an organic system of enormous complexity. As is likewise the typical case for empirical anatomical illustrations, the scope of this qualitative model is limited by the scope of the specialists investigating that particular part of the anatomy. For Sherman and Guillery, the center-of-focus is the thalamus and everything else in the CNS orbits around it in their book. One always finds such a Ptolemaic view of the depicted biology of the CNS that varies from specialist to specialist. Munoz and Schall (2004)<sup>38</sup> presented an analogous Ptolemaic "CNS solar system," centered in their case around a star in the brainstem called the superior colliculus. A modified version of their schematic is shown in figure 16.

Now, it is obvious that all this relies on possession of *empirical* knowledge of a very complicated system, and so embedding field construction properly belongs in empirical science and not in metaphysics. This is one reason why a simplified agent model, e.g. Martian 2, is a useful computational neuroscience research vehicle. On the other hand, if the agent is to have any real bearing with regard to *H. sapiens*, we turn to mental physics for the general organization of the structure of property sets employed in developing the agent model. This tells us genus and species of the property set characteristics. Metaphorically speaking, it's easier to study elephants if you know elephants are land mammals and not fish: you don't end up investigating if ears are or are not gills. In a similar way, the same is true of property sets.

<sup>37</sup> Sherman, S. Murray and R. W. Guillery (2006), *Exploring the Thalamus and Its Role in Cortical Function*, 2nd ed., Cambridge, MA: The MIT Press.

<sup>38</sup> Munoz, Douglas P. and Jeffrey D. Schall (2004), "Concurrent, distributed control of saccade initiation in the frontal eye field and superior colliculus," in *The Superior Colliculus*, William C. Hall and Adonis Moschovakis (eds.), Boca Raton, FL: CRC Press, chap. 3, pp. 55-82.

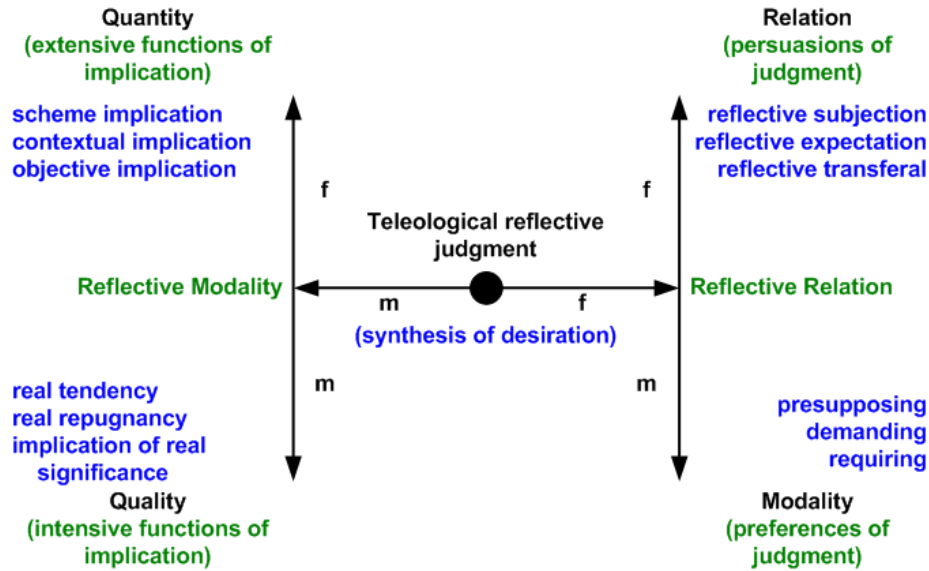


Figure 17: 2LAR structure of teleological reflective judgment.

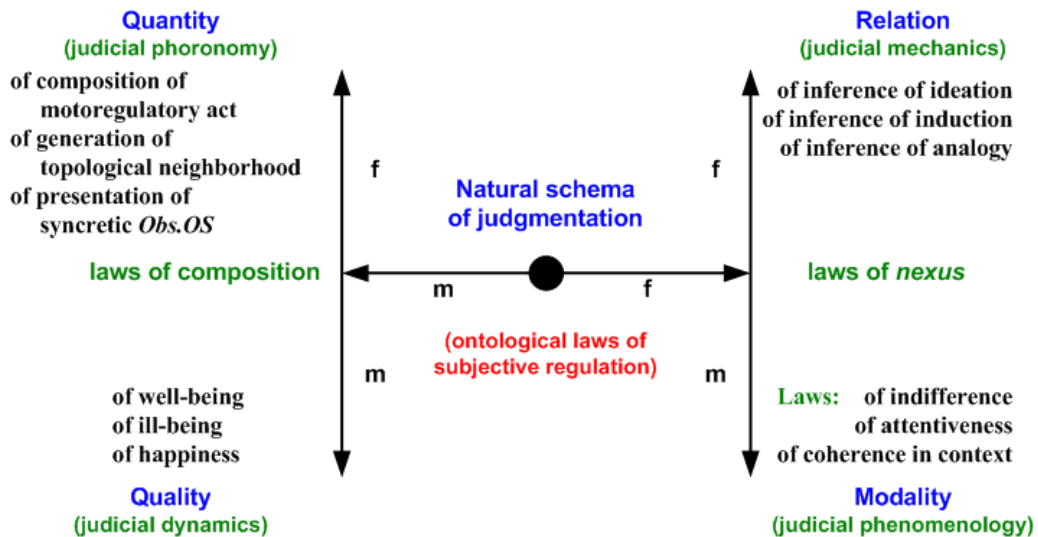


Figure 18: 2LAR structure of the natural schema of judgmentation in the metaphysics of affectivity.

13. Teleological judgment and the natural schema of judgmentation. The contextual intersect between *Comparison* and teleological reflective judgment (TRJ) having been established, the next topic is to examine what the general *momenta* of TRJ imply for *Comparison*. Figure 17 presents the 2LAR structure of TRJ. The twelve *momenta* are explained in Wells (2009)<sup>23</sup>, chapter 8. These functions in their turn take their context from the Quantity heading in the metaphysics of affectivity [Wells (2009), chapter 7 §3], the *natural schema of judgmentation* (NSJ). Figure 18 illustrates the NSJ. The NSJ is the metaphysics of the Nature of the Organized Being's capacity to organize knowledge into a systematic unity of Nature insofar as the OB constructs for itself its "world model" *by which it comes to know Nature*. Put another way, the NSJ is the metaphysics for *how the OB formally composes notions of physics in general*.

The ontology-centered thinker usually feels disturbed by this aspect of Critical epistemology. It might help to relieve this discomfort if you remind yourself that, regardless of whatever

transcendent notions you might hold in regard to "reality-regarded-as-a-thing-in-itself" and "nature-regarded-as-a-thing-in-itself," the simple fact is that all physics (including the special science officially known by that name) *is science-of-Nature* and every science is the *invention* (or, if you prefer, "the discovery") *of scientists*. Every science is a systematic doctrine for *comprehending* some aspect or aspects of the world and *how this comprehension is built is determined by the mental physics of the phenomenon of mind*. The special science we today call "physics" is *not* "the queen of the sciences" because *as a special science* its structure, organization *and practices* answer to the laws of *mental physics*. This is not a return to 18th century rationalism, however. That philosophy was ontology-centered (therefore its meta-physics were erroneous) and failed to take into account that that circumstance and situation in the part of Nature which is called "not-me" is contingent in relationship to one's capacity for knowledge. Viewed with this in mind, every special science is organized and systematic comprehension of the contingent, and we call such comprehension *empirical*.

Many people, and, especially, scientists and engineers, are also likely to feel uncomfortable with the notion that the judicial capacity which grounds a human being's understanding of Nature lies within the logical division of *affectivity* – which is to say the capacity is subjective. It might be that only poets and romanticists would greet this idea with favor. We scientists and engineers are trained (psychologically conditioned by the education we receive) to dislike and distrust the subjective. "Nature doesn't *care* how *you* feel about things" is almost an unofficial motto in science and engineering (and a motto that betrays the ontology-centeredness of this education). But this acquired and carefully cultivated prejudice against affectivity is gainsaid by direct and unrefuted empirical evidence. Neurologist A. R. Damasio works with medical patients who have suffered various kinds of brain damage from, e.g., accidents, tumors, stroke, etc. What he has found from this work is quite telling:

There has never been any doubt that, under certain circumstances, emotion disrupts reasoning. The evidence is abundant and constitutes the source for the sound advice with which we have been brought up. Keep a cool head, hold emotions at bay! Do not let your passions interfere with your judgment. As a result, we usually conceive of emotion as a supernumerary mental faculty, an unsolicited, nature-ordained accompaniment to our rational thinking. . .

There is much wisdom in this widely held belief, and I will not deny that uncontrolled or misdirected emotion can be a major source of irrational behavior. Nor will I deny that seemingly normal reason can be disturbed by subtle biases rooted in emotion. . . Nonetheless, what the traditional account leaves out is a notion that emerges from the study of patients such as Elliot<sup>39</sup> and from other observations I discuss below: *Reduction in emotion may constitute an equally important source of irrational behavior*. The counter-intuitive connection between absent emotion and warped behavior may tell us something about the biological machinery of reason. . .

My first concern was to verify that our observations about Elliot held firm in other patients. That proved to be the case. To date we have studied twelve patients with prefrontal damage of the type seen in Elliot, and in none have we failed to encounter a combination of decision-making defect and flat emotion and feeling. The powers of reason and the experience of emotion decline together, and their impairment stands out in a neuro-psychological profile within which basic attention, memory, intelligence, and language appear so intact that they could never be invoked to explain the patients' failures in

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<sup>39</sup> Elliot was a patient in one of Dr. Damasio's case studies. He had suffered bilateral damage to a part of the frontal lobe of his brain as the result of a tumor. It left him unable to "feel emotions" although his other capacities of judgment were unaffected. He was left, quite literally, an "unemotional man" and one who afterwards consistently made very logical bad decisions that proved to be unflinchingly disastrous to his own well-being and that of others. Quite literally, he could no longer successfully function in everyday life.

judgment. [Damasio (1994)<sup>40</sup>, pp. 52-54]

Damasio's work shows us the empirical facts. Mental physics explains these facts.

The *momenta* of teleological reflective judgment, which are synthesizing functions, align one-to-one with the *outcome principles* of the natural schema of judgmentation. Put another way, the principles of the NSJ identify the *species* of the result of a judicial act, the *momenta* of TRJ tells us *how* ("in what way") that act is performed. There are, consequently,  $3^4 = 81$  distinct species of outcome at the second level of Critical analysis of teleological judgmentation (one of three *momenta* under each heading and all four headings combined in one act). The Critical principles of the NSJ are provided for ease of reference in the first postscript at the end of these notes.

The *momenta* of teleological reflective judgment are summarized for ease of reference in the second postscript at the end of these notes. For the purpose of making these notes concise, I am omitting detailed explanations of these *momenta* here and instead advising you to consult chapter 8 of *The Principles of Mental Physics* [Wells (2009)] when you wish to get down to these details. In compensation for this brevity, I do cover in these notes what these functions (and the principles of the NSJ) mean for *Comparison*. I am likewise omitting detailed explanations of the principles of the NSJ and advising you instead to refer to chapter 7 of *The Principles of Mental Physics* when you come to want to know these details in depth.

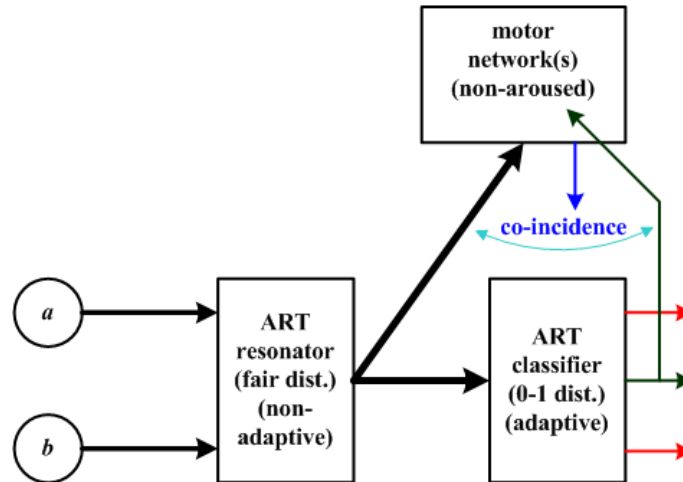
However, I think it prudent to briefly discuss key terms contained in the postscript summaries because otherwise these summaries will probably not say much to you. **Expedience** is a property of representation regarded as possible only with respect to some purpose. That-which-is-depicted by a representation is called *the object of the representation* and it may be a theoretical object (if the representation is a cognition), a judicial object (if the representation is an affective perception) or a practical object (if the representation is a practical rule of pure Reason). The object of a representation is a *purpose* insofar as the depiction (*parástase*) of the representation is taken to be the ground for the possibility of realizing (making actual) the object. If I have a perception of "being hungry" (a feeling of *Unlust*), a cognition that eating a snack will negate that feeling, and a practical representation of acting to change my condition by fixing a snack for myself, my purpose is the object "fixing-a-snack." An Organized Being has one innate pure purpose of practical Reason. It is called the categorical imperative and it is a practical formula for acting to achieve a state of complete equilibrium. Ultimately, all special purposes are grounded in the categorical imperative through actions.

**Desiration** is the *form* of unity in affective perception in relationship to the capacities of the Organized Being. The connections produced by acts of teleological reflective judgment are connections of desiration. These connections include, by means of meaning implications, relationship with the somatic organization of *psyche* (figure 14) for motoregulatory expression of actions. Put another way, every judgment of desiration implicates some possible sensorimotor action which, when actualized, appears in *soma*. All acts of TRJ pertain to possible real actions in the *homo phaenomenon* aspect of being a human being. All acts of TRJ are (1) judgments of expedience in their relationships with practical Reason and (2) judgments of specific motoregulatory actions in their relationships with *psyche*.

Even a superficial examination of the *momenta* of TRJ in the second postscript below shows that all these judicial functions pertain to expedience and desiration. Because the context for *Comparison* in sensibility is drawn from the relationship between the synthesis of apprehension and TRJ, the implication we are to draw is more or less straightforward: *the root*

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<sup>40</sup> Damasio, Antonio R. (1994), *Descartes' Error: Emotion, Reason, and the Human Brain*, NY: Avon Books.



**Figure 19:** Initial minimal anatomy for the equivalence relation of *Comparison*.

*context for equivalence in sensible Comparison is connection of perception to motor action.*

14. Fundamental property set characteristics for *Comparison*. We can now proceed directly from this to a *real explanation* of "equivalence" in the context of *Comparison* synthesis. **Two comparands of Comparison are equivalent if both implicate the same possible motor action.** Property sets for semantic embedding field graphs (and, therefore, somatic embedding field networks) are those properties necessary for the possibility of representing this real relationship mathematically. What, then, would these be?

In deducing or discovering property sets, it is prudent to begin as close as possible to the real explanation (*Realerklärung*, in the terminology of Critical epistemology). Figure 19 illustrates what I claim is a putative *initial* minimal anatomy for our real idea of *Comparison*. By "initial" I mean a "starting point" for the embedding field network research. It is *not* my intent to imply figure 19 is "all it takes" to mathematically represent *Comparison*; my intent is to say this figure seems to me to be a very good network with which to start. Why do I think so?

The **first fundamental property** of *Comparison* is that it must be part of a transformation from representation in sensibility to representation in motor action. This comes immediately from the real explanation above. But a representation in sensibility, regarded in and by itself, carries no immediate implication of any *motor* action. To *make* such an implication requires us to posit that the determination of *Comparison* contains some comparand feature or features that can be related to motor action. Probably the simplest examples of this we find in biology are the basic innate *reflex actions* such as, e.g., the four fundamental reflex arcs empirically found in spinal cord organization [Wells (2003)<sup>41</sup>]. Indeed, this idea was the original starting point in defining the Martian 1 agent system [Wells and MacPherson (2009)<sup>14</sup>]. Such a transformation structure must have at least four properties: (1) a minimum of two comparand representations; (2) a transformation rule for converting the comparand representations to a representation of some motor excitation; (3) one or more motor embedding field networks that convert (2) into actuating signals going out to somatic end effectors; and (4) some function that signals the intersection of the comparands in the same motor network system.

Property (1) is depicted explicitly in figure 19 by the two comparand vertices, *a* and *b*. What I have in mind here is the vector representation  $I_1 = [a \mid b]^T$  where *a* and *b* are vector

<sup>41</sup> Wells, Richard B. (2003), "Spinal sensorimotor system, Part I: Overview," available on the Wells Laboratory web site.

representations of the activity fields depicted earlier by the figure 2 determinables. Property (2) is *implicitly* depicted in figure 19 by the ART resonator network [Wells (2010)<sup>42</sup>, chap. 16] to which  $I_1$  is applied as an input. In Martian 1 we used fixed (non-adaptive) resonators with preset feature patterns<sup>43</sup> to implement both innate sensorimotor reflexes and innate affective preferences [Wells and MacPherson (2009), pp. 15-19]. The resonators used in Martian 1 were designed to exhibit an output property that embedding field theory calls a *fair distribution* [Grossberg (1973)<sup>44</sup>]. Specific examples of how this worked can be found in MacPherson (2009)<sup>45</sup>. This part of the Martian 1 agent system worked extremely well.

Property (3) is generically depicted in figure 19 by the motor networks subsystem. This network subsystem is tasked with converting the resonator output vector to signals that eventually reach end effectors that produce *kinesis* (changes of any kind) in the state of the somatic system of the agent model. I do not go into more detail here in regard to this subsystem for two reasons: (1) its implementation in Martian 1 was fairly complex [Wells and MacPherson (2009), pp. 10-15, 21-37]; and (2) while I deem it likely that most of the Martian 1 networks will find new application in Martian 2 (because these networks also worked extremely well), I also deem it likely that Martian 2 will involve a number of significant *architectural* level changes from Martian 1 and, therefore, it is more or less superfluous to these notes to go into greater detail.

Property (4) is generically depicted in figure 19 by the adaptive ART classifier network, its projections into the motor networks subsystem, and the feedback line signifying co-occurrence of the resonator network projections and the classifier projections at the same functional subsystem(s) within the motor networks subsystem. This part of the system is new in Martian 2. It was not included in the Martian 1 system and I think this omission is part of the reason why Martian 1 was unable to advance from sensorimotor intelligence stage I to sensorimotor intelligence stage II. What I have in mind as a starting point for the ART classifier network is a network such as an ART 2 classifier [Carpenter and Grossberg (1987)<sup>46</sup>]. This network's output projections (from field  $F_2$  of figure 10) would exhibit what is called a 0-1 distribution<sup>47</sup> (also more widely known as a "winner-take-all" response). I think it likely that my reason is non-obvious at this point when I say this network must be adaptive and that an ART 2 structure seems like a good initial starting point. My reason is that there are *secondary properties* that also appear to be necessary in order to satisfy the mathematical definition of equivalence relation and to conform with what the process of TRJ additionally requires from this embedding field network. I discuss these next.

15. Secondary property set characteristics for Comparison. Up to now I have mentioned, or at least hinted, that the *materia ex qua* of the comparands is to be regarded as some collection (sets) of activity fields drawn from a universal set of possible fields. I have passed in silence over the question: If this is so, what determines the pair of comparand fields in figure 19?

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<sup>42</sup> Wells, Richard B. (2010), *Introduction to Biological Signal Processing and Computational Neuroscience*, available on the Wells Laboratory web site.

<sup>43</sup> One of the many advantages in using an artificial agent as a research vehicle is that we are free to define the agent's innate reflexes and preferences. So long as these are in some way homologous with what we know about biological structure in *H. sapiens*, the agent is relevant for theoretical neuroscience research.

<sup>44</sup> Grossberg, Stephen (1973), "Contour enhancement, short term memory, and constancies in reverberating neural networks," *Studies in Applied Mathematics*, vol. 52(1), pp. 213-257.

<sup>45</sup> MacPherson, Quinn (2009), "The potentially useful effects of contrast enhancement for ART resonators," available on the Wells Laboratory web site.

<sup>46</sup> Carpenter, Gail A. and Stephen Grossberg (1987), "ART 2: self-organization of stable category recognition codes for analog input patterns," *Applied Optics*, vol. 26, no. 23, pp. 4919-4930.

<sup>47</sup> *op cit.* Grossberg (1973).

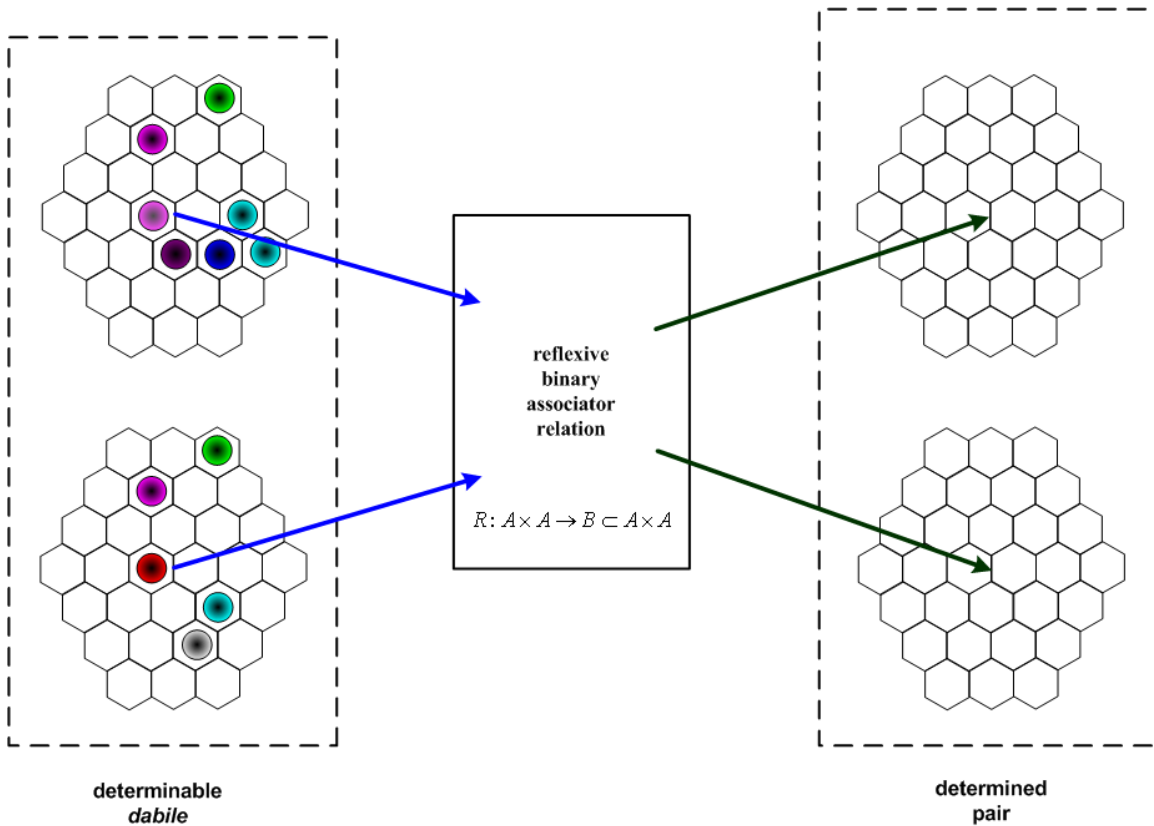


Figure 20: Illustration of the reflexive binary associator (RBA).

That a multiplicity of comparand activity fields is needed comes from the simple fact that there are many sensorimotor modalities, that many of these are initially disjoint in stage I of sensorimotor intelligence, and yet, nonetheless, the synthesis of *Comparison* must have a structure capable of dealing with arbitrary comparand combinations since *Comparison* is involved in *all* stages of the development of intelligence in *H. sapiens*. One way, in principle, all these factors might be reconciled is to simply "hardwire" specific combinations of sensible activity fields to specific subsystems of the sort figure 19 depicts. This is a bad idea on at least two counts. First, our understanding of the neurology of the central nervous system of *H. sapiens* appears to gainsay it. Secondly – and this is a decisive factor all by itself – the combinatorial explosion such a "hard-wired" approach involves is just too computationally impractical. It does not take very many hard-wired pairs – many fewer than we would need by several orders of magnitude – before the number of vertices required in the model *would exceed the total number of biological cells in the CNS of H. sapiens*. In consequence, the hard-wired idea is ludicrous *prima facie*.

A more practical (but not non-trivial) conjecture is illustrated by figure 20, the reflexive binary associator (RBA). The RBA is a mathematical concept (and therefore originates from facet B considerations) and it is not presently known whether or not this construct is a principal quantity because empirical evidence of it in *soma* is currently merely circumstantial. There is a ground for inferring its *Dasein* in *soma* provided by the epistemology of the psychology of functions [Piaget *et al.* (1968)<sup>48</sup>], namely the associative coordinator constitutive function.

<sup>48</sup> Piaget, Jean, Jean-Blaise Grize, Alina Szeminska, and Vinh Bang (1968), *Epistemology and Psychology of functions*, Dordrecht, Holland: D. Reidel Publishing Co., 1977.



There are also anatomical structures in the CNS, such as the claustrum, having the scope of connectivity that would seem to be required by the RBA<sup>49</sup> and which, therefore, hypothetically provide an at least feasible biological substratum for RBA *Existenz*. All of this empiricism, however, is still very speculative at this time.

**Definition 10.** A reflexive binary relation  $R$  on a set  $A$  is a transformation

$$R : A \times A \rightarrow B \subset A \times A$$

such that the ordered pairs  $\langle a_i, a_i \rangle \in B$  for every  $a_i \in A$ .

The set  $A$  is the set of all givable (*dabile*, in Kant's terminology) activity fields that can be comparands for *Comparison*. The defining property, reflexivity, of this relation is a necessary property for the RBA function because reflexivity is a defining property of an equivalence relation. It follows that the RBA and its defining relation, the reflexive binary relation on  $A$ , is a necessary member of the *Comparison* property set. At the present time I have no reason to conclude that the symmetric relation or the transitive relation are required for the RBA because both of these properties are supplied (by definition) in the structures of the network subsystems in figure 19. The RBA function was not part of the Martian 1 system, and my post analysis of Martian 1 leads me to think that this omission was another factor in Martian 1's inability to move past sensorimotor stage I.

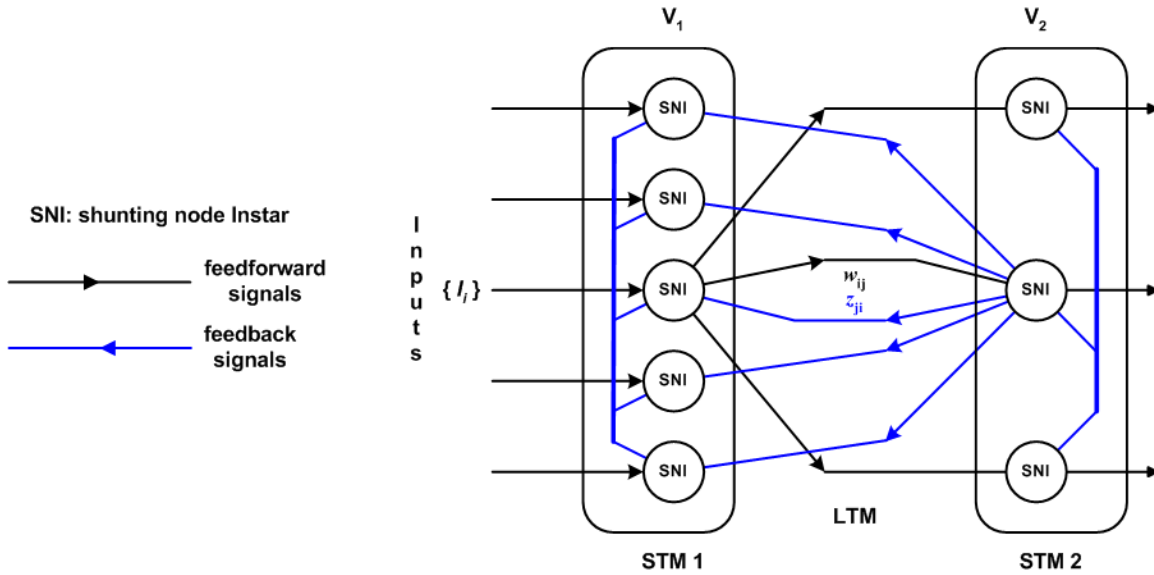
The reason the ART classifier in figure 19 must be adaptive follows from the necessity for the RBA. The classifications encoded by that network must have a starting foundation. Because reflexivity is arguably the most fundamental property of equivalence (because nothing for which the tautology  $a = a$  is not satisfied can be called an "equivalence"), it seems very clear that initial *reflexive* classification,  $I = [a \mid a]^T$ , performed by the ART classifier makes a good starting point. Provided the discriminations made by the classifier (it's "vigilance parameter") are not too tight, these classifications would be compatible with mental physics' requirement that we use set membership theory at the mathematical foundation [Wells (2009), chap. 1 §4].

However, the classifier does not directly classify comparands. It classifies the resonator network output determination. The required symmetry relation property for equivalence therefore places a requirement on either the resonator network or else on the classifier that if a pair  $\langle a_i, a_j \rangle, j \neq i$ , is placed in one particular classification, then the pair  $\langle a_j, a_i \rangle$  must also be placed in that same classification. This is another member of the property set of *Comparison*. It therefore seems likely that some additional network development work will need to be carried out for Martian 2 in regard to either the original MacPherson resonator network system, or in regard to the ART classifier, or both in order to achieve all required properties. I think the resonator network is the likely best subsystem to modify in regard to the symmetry property because if the symmetry property is met by its outputs, the transitivity property will follow automatically from the nature of ART classification mathematics.

There are numerous network schemes capable of enforcing a symmetry property in a fixed resonator anatomy, the simplest of which is to merely encode it into the fixed weights. Figure 21 illustrates an ART resonator network [Wells (2010)<sup>42</sup>, chap. 16]. When used in figure 19, if vertices  $a$  and  $b$  are each dimension- $N$  vectors then field  $V_1$  has  $2N$  vertices. Let the number of outputs from  $V_2$  be  $M$ . If  $a \alpha b$  and  $b \alpha a$  both belong to the equivalence set, then this symmetry condition is enforced in the fixed resonator network by setting  $w_{i+N,j} = w_{ij}$  and  $z_{j,i+N} = z_{ji}$ ,  $i = 1, \dots, N$  and  $j = 1, \dots, M$  in the resonator's feedforward and feedback weights. In this way, the resonator network outputs from  $V_2$  are unaffected by swapping inputs  $a$  and  $b$ .

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<sup>49</sup> Crick, Francis C. and Christof Koch (2005), "What is the function of the claustrum?" *Philosophical Transactions of the Royal Society B*, **360**, 1271-1279.



**Figure 21:** ART resonator with feedforward weights  $w_{ij}$  and feedback weights  $z_{ji}$ . Equations for the vertex nodes are given in Wells (2010), chapter 16. When used in figure 19, field  $V_1$  has  $2N$  vertices and field  $V_2$  has  $M$  vertices. Indices run from  $i = 1, \dots, 2N$  and  $j = 1, \dots, M$ . STM = short term memory, LTM = long term memory (the fixed weights).  $W$  and  $Z$  connectivities are the same at each vertex in the network.

The routing function implemented by the RBA in figure 20 has long been familiar to computer engineers and is called a *crossbar switch* [Bell and Newell (1971)<sup>50</sup>, pp. 66-70, 461]. There are a number of different possible logic networks by which the crossbar switch function can be implemented and it is not known *a priori* what sorts of logic networks (if they actually exist in the CNS) the somatic system of *H. sapiens* might be using. Mathematically they are all equivalent but in terms of connectivity and complexity of implementation these schemes differ widely from one to another. All, however, can be implemented in the embedding field formalism because the switching circuits used in computers are themselves nothing else than McCulloch-Pitts "neural" networks [McCulloch and Pitts (1943)<sup>51</sup>] as reformulated by von Neumann [Neumann (1952)<sup>52</sup>]. The structure is, therefore, biologically plausible although at this time its biological actuality is hypothetical.

16. Other property set characteristics for Comparison. Although *Comparison* takes its context from the *momenta* of teleological reflective judgment, the *Comparison* structure is not itself a judgment of any kind and by itself makes no meaning implications. This is the significance of denoting in figure 19 that the motor networks are non-aroused. What I mean by this is that motor network efferents leading to end-effectors are quiescent even when their afferent inputs are stimulated by the projections from the resonator network. A meaning implication is not *made* before these efferents are aroused. *Comparison* merely provides sets of *possible* meaning implications at the sensorimotor level and has no part in the arousal of the motor network efferents. One way to say this is to say that the depictions of *Comparison* are

<sup>50</sup> Bell, C. Gordon and Allen Newell (1971), *Computer Structures: Readings and Examples*, NY: McGraw-Hill.

<sup>51</sup> McCulloch, Warren S. and Walter Pitts (1943), "A logical calculus of the ideas immanent in nervous activity," *Bulletin of Mathematical Biophysics*, vol. 5, pp. 115-133.

<sup>52</sup> Neumann, John von (1952), "Probabilistic logics and the synthesis of reliable organisms from unreliable components," in *Papers of John von Neumann on Computing and Computing Theory*, William Aspray and Arthur Burks (eds.), Cambridge, MA: The MIT Press, 1987, chap. 13, pp. 553-602.

possible rather than actual percepts or percept-fragments.

Nor is the *Comparison* synthesis under the *immediate* control or direction of reflective judgment. The *synthesis* of the *Verstandes-Actus* (*Comparison*, *Reflexion* and abstraction) as a *process* is under the control of the power of imagination and, specifically, the synthesis of *apprehensive* imagination. Within the power of imagination there are three processing capacities: apprehension, re-cognition and reproduction. Acts of apprehensive imagination are *oriented* by the process of reflective judgment and this orientation pertains to *the synthesis of subjective time*, which is the form of inner sense in organized being [Wells (2009), chap. 3].

Figure 3 depicts the synthesis of the *Verstandes-Actus* and the synthesis of pure intuition (subjective space and subjective time) as separate information transformation pathways. It must, however, always be kept in mind that synthesis requires a synthesizing *process*, and in sensibility this is the process of imagination. The transformation pathways, although separate and distinguishable, are not *operationally* independent since all depend on the synthesis of imaginative apprehension. Imagination "builds an image" and the *Verstandes-Actus* have in this the role of providing the *matter* of the image, whereas the synthesis of pure intuition has the role of providing the *form* of the image. Subjective space is a topological structure and subjective time is a mathematical partial order structure, i.e.,

**Definition 11.** A *partial ordering* is a reflexive, antisymmetric and transitive relation on a set.

**Definition 12.** A relation  $\beta$  on a set  $A$  is said to have the antisymmetric property (to be antisymmetric) if, for  $a$  and  $b$  in  $A$ ,  $a \beta b$  AND  $b \beta a$  implies  $a$  and  $b$  are the same element of  $A$ .

The synthesis of reflective judgment is *not constrained* by subjective time; one way to put this is to say that reflective judgment *stands outside of subjective time*. However, affective perceptions and intuitions *do not*. Affective perceptions in particular constitute the *matter of desire* in reflective judgments while desiration constitutes the form of this judicial manifold (the manifold of Desires)<sup>53</sup>. Speaking specifically of objective representations (intuitions and concepts) in *Critique of Pure Reason*, Kant tells us,

In all subsumptions of an object under a concept the representation of the former must be *homogeneous* with the latter, i.e., the concept must contain what is represented in the object that is to be subsumed under it, for that is just what is meant by the expression: an object is contained under a concept. [Kant (1787)<sup>54</sup>, B176]

Now, the same is true for affective perceptions subsumed under a Desire by acts of reflective judgment with this difference: That which is represented by a Desire is not called an object but, rather, is called *the judicial subject* [*op cit.* Wells (2009)]. A Desire in affectivity is the homologue of a concept in cognition and an affective perception is the homologue of an intuition in sensibility. An affective perception as a perception and a Desire as an affect are not the same thing and so if we are to be able to say the two are *homogenous* (as we do in Kant's transcendental Logic), then we have to have some third thing that stands in homogeneity with the affective perception on the one hand and the Desire on the other. This third thing, which is another representation, is called the *transcendental schema* [*op cit.* Wells (2009), chap. 5, §3]. The transcendental schema is a product of imagination and aims only at unity in the determination of sensibility [*op cit.* Kant (1787), B179].

The *momenta* of transcendental schema are called the *transcendental schemata*. The transcendental schemata with respect to affectivity mirror those with respect to objectivity and

<sup>53</sup> Wells (2009), chapter 7 §2.

<sup>54</sup> Kant, Immanuel (1787), *Kritik der reinen Vernunft*, 2nd ed., in *Kant's gesammelte Schriften, Band III*, Berlin: Druck und Verlag von Georg Reimer, 1911.

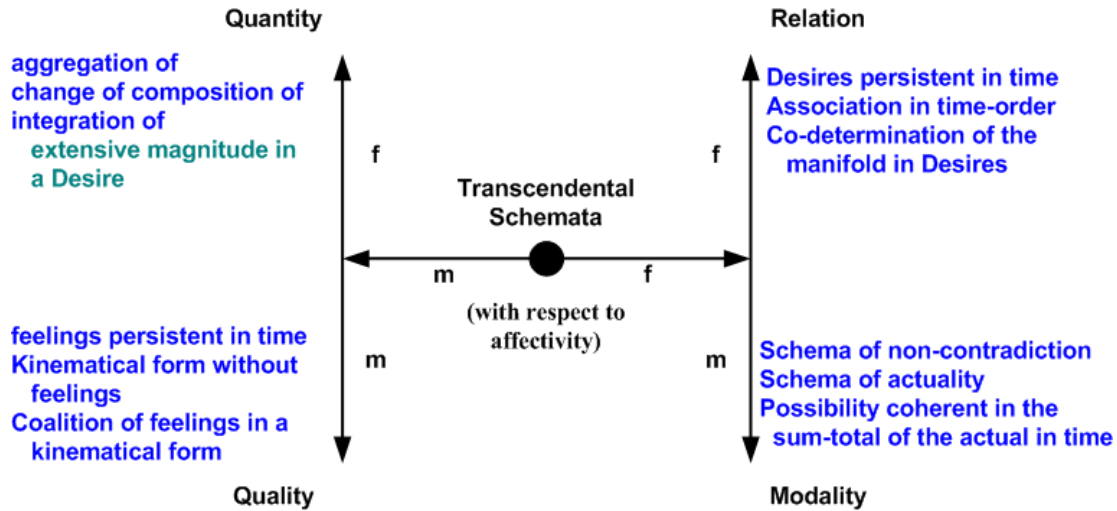


Figure 22: 2LAR of the transcendental schemata of imagination in the context of affectivity.

their deduction and interpretation mirrors the schemata with respect to objectivity that is presented in Wells (2009) chapter 5. The only difference between them is one of context. The schemata with respect to affectivity pertain to Desires and reflective judgment; those with respect to objectivity pertain to intuitions and determining judgment. Figure 22 presents the 2LAR for the transcendental schemata in the context of affectivity. Feelings are the homologues in affective perception of sensations in intuition. A Desire is the combination of desires (judged by aesthetical reflective judgment) and desirations (judged by teleological reflective judgment). Because the mathematical interpretation of the transcendental schemata with respect to affectivity is no different (again, other than in context) from that of the transcendental schemata with respect to objectivity, I refer you to the discussion of the latter in Wells (2009) chapter 5 for those mathematical details when you wish to have them.

What we do need to discuss here is the relationship between the affective schemata in figure 22 and the *momenta* of teleological reflective judgment in figure 17. The Quantity relationship could not be simpler: it is one-to-one. The transcendental schema of aggregation in Quantity pairs with scheme implication in TRJ, the schema of change in composition pairs with contextual implication, etc. When I say TRJ *orients* apprehensive imagination this means that the *momenta* of TRJ in judging desiration co-determines the transcendental schema of desiration in the synthesis of apprehension.

The transcendental schemata pertain to the determination of inner sense, which is to say that they pertain to the *construction* of the ordering structure that *is* the intuition of subjective time. The schemata of Quantity define the determination of **time-series**; those of Quality define the determination of **time-content** ("what is 'in' time"); those of Relation define **time-order**; and those of Modality define **time-embodiment** (the "quintessence" or *Zeitbegriff* of pure subjective time, which means "the way everything in sensibility coheres in the *Existenz* of the Organized Being"). Note that time-series and time-order are *not* the same thing. To compose a form of *aggregation* for what is "in" time is not the same thing as to connect the members of this aggregate in a particular *order*. These are two different acts of synthesis. The first builds a set, the second builds an *operation* on that set.

I think you might be inclined to agree with me that these are rather abstract ideas and we need to "put a bit more specific reality" into the discussion. All meanings are at root practical, so what does all this *practically mean* and, even more particularly, what does it mean in the

context of *Comparison*?

The first thing to note is that *Comparison* gives us a *subcontext* to focus on. *Comparison* pairs with *teleological* reflective judgment and *Reflexion* pairs with *aesthetical* reflective judgment. The first pertains to the *form* of the manifold of Desires, the latter to the *matter* of the manifold of Desires. The subcontext for *Comparison* is **desiration**, the *form* of Desires. The first thing to do, then, is to substitute the word "desiration" for the word "Desire" in figure 22.

The second thing to do is to remember that imagination *builds* something, namely an image. The *practical* context we are dealing with here is the context of *doing* something. The transcendental schemata are *momenta* for imaging *what* we're doing (Quantity and Quality) and *how* we're doing it (Relation and Modality). There is a name for the object of a "what-doing-and-how-doing"; it is called an **algorithm**:

**Definition 13.** An **operation** is a transformation mapping one representation to another representation.

**Definition 14:** An **algorithm** is a sequence of operations, each of which can be executed in a precise manner. The execution of an operation is called a **step**.

The word "algorithm" basically means "procedure." The word itself comes from the name of a celebrated 9th century mathematician, Mohammed Ibn-Musa Al-Khowarizmi.<sup>55</sup> (I bet you probably didn't know that; generally speaking, when you see a modern technical word that begins with "al-" you should probably suspect that the word comes from somewhere among the works of Moslem scholars during Islamic civilization's "golden age" from the 9th to the 12th centuries A.D. For instance, "algebra" comes from the Arabic *al-jabr w'al-muqabala*, which means "restoration and reduction." Al-Khowarizmi invented it, too).

This tells us something important about *Comparison*: **Another of its properties is that the act of Comparison is an algorithm.** The "what-doing" consists of the operations, the "how-doing" is the sequence.

That brings up one of the many puzzling little perplexities that the ontology-centered habit of thinking "time is time" and failing to distinguish between *subjective* time and *objective* time has a tendency to produce. Insofar as it has any temporal context, *the idea of a sequence is an idea of objective time*. Subjective time is a pure form of intuition and we never "sense" subjective time *because subjective time is the form of inner sense*. You can't "sense a sense." It is the peculiar Nature of human understanding that we can only conceptualize appearances after their representations have been presented as intuitions. When, as in mathematics and everywhere else in empirical science, we represent time, *what we are representing is an object* and that object is *never* "subjective-time-itself." Any *theory* is a doctrine of objective concepts and ideas, and so when we say "sequence" we are talking about the *objective* representation of "first this, then that, then this-other-thing." *A sequence is an object of mathematics*. Want to run your sequence "backwards"? Go ahead. As an object, it is a denizen of facet B and you can do with it as you wish. Just don't peg any *ontological* significance to what you do.

Setting up algorithms of *Comparison* is a theoretical (mathematical) exercise of, in a manner of speaking, "figuring out the recipe for cooking up a form of desiration." In doing so, we are

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<sup>55</sup> "How in the world," you might well wonder, "did we get from 'Al-Khowarizmi' to 'algorithm'?" The answer is easier than you might think. When the intellectual treasures of Islam were imported to Europe at the end of the dark ages, the Europeans didn't know how to pronounce Arabic words or names and didn't try very hard to learn how to correctly do so. Mumble the man's name like a Brit and you get "algorithm."

working with ideas of objective time and because of that we can use whatever kind of mathematical recipe seems best. We can "make objective time run backwards" (in more collegiate and eloquent language, use the method of Hamilton-Jacobi-Bellman optimization) and we can employ the methodology of dynamic programming [Werbos (1997)<sup>56</sup>]. We are not even restricted to keeping objective time one-dimensional. We can use *timescapes* rather than a single timeline if we wish [Wells (2011a)<sup>26</sup>]. A computer engineer might be inclined to think of the idea of a timescape in terms of "a multiprocessor system with distributed multirate clocks."<sup>57</sup> The one thing, however, that we have to do is: make sure there is a mathematical pathway to a principal quantity of objective time that can be associated with the objective time representations that characterize appearances of empirical experience (e.g., in the biology lab or in the psychologist's observations). We *must* make sure we have some context within which we can get *from* mathematical facet B secondary quantities *to* empirical facet A. Until we have that, the theory is *ontologically* non-real.

What remains for us to do is to run through what particular kinds of operations and algorithms are required for *Comparison*. To do this, we run down the transcendental schemata of figure 22. When we've finished doing so, we'll have the remaining *a priori* properties that go into the property set for *Comparison*.

17. The first schema of Quantity. In relationship to *psyche* (figure 14), Quantity in teleological reflective judgment pertains to the form of a *nexus* of meanings in *nous-soma* reciprocity [Wells (2009)<sup>58</sup>, chap. 4 §3.1]. Piaget and Garcia found,

[An] object is a set of conjoined predicates and its meaning amounts to "what can be done" with it, and is thus an assimilation to an action scheme (whether the action is overt or mental). As for actions themselves, their meaning is defined by "what they lead to" according to the transformation they produce in the object or in the situations to which they are applied. Whether we are dealing with predicates, objects, or actions, their meanings always implicate the subject's activities, which interact either with an external physical reality, or with elements that were previously generated by the subject, such as logico-mathematical entities.

Furthermore, we may distinguish various degrees in meanings: They may remain "local" in that they relate to limited data and to particular contexts; they may become "systematic" in laying the groundwork for structures; and finally they may become "structural" when they pertain to the internal compositions of already constituted structures. [Piaget and Garcia (1991)<sup>59</sup>, pp. 119-120]

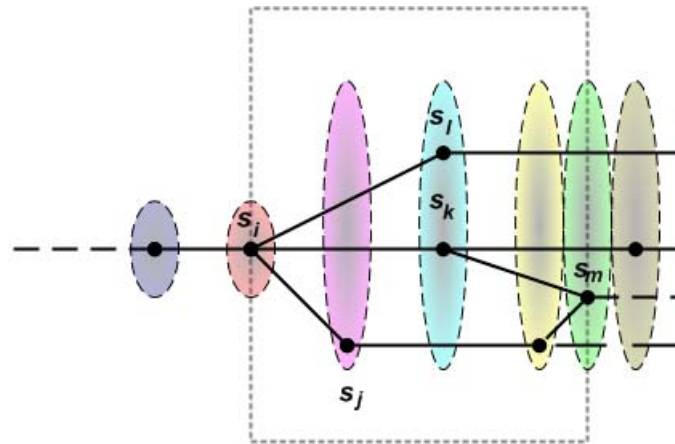
The schema of aggregation of extensive magnitude in a desiration is called the *schema of persistence* in desiration. Here the object of the act of scheme implication in teleological reflective judgment is the composition of the form of a sensorimotor scheme of motoregulatory expression. "Persistence" in this context refers to persistence in subjective time between successive moments in time at which intuitions are marked by judgments.

<sup>56</sup> Werbos, Paul (1997), "A brain-like design to learn optimal decision strategies in complex environments," in *Dealing With Complexity: A Neural Networks Approach*, M. Kárný, K. Warwick and V. Kůrková (eds.), NY: Springer-Verlag.

<sup>57</sup> We use tricks like this all the time in practical simulations. For example, suppose one or two subsystems are characterized by time constants five times faster than the next fastest time constants anywhere else in the system. Then it is common to run the faster blocks for five steps per each step the slower blocks take rather than trying to make the *entire* system run for five times the number of steps than the slower blocks actually require for acceptable numerical accuracy.

<sup>58</sup> Wells, Richard B. (2009), *The Principles of Mental Physics*, available on the Wells Laboratory web site.

<sup>59</sup> Piaget, Jean and Rolando Garcia (1991), *Toward a Logic of Meanings*, Hillsdale, NJ: Lawrence Erlbaum Associates.



**Figure 23:** Illustration of a timescape in subjective time. The black dots denote moments in subjective time,  $s$ , which are defined by the marking of an intuition in sensibility. Colored ovals denote affective perceptions that overlay all the diverse timelines in the timescape between moments in time.

Figure 23 illustrates an example timescape in subjective time [Wells (2009)<sup>58</sup>, chap. 3 §3]. Between any two successive moments in time, let us say  $s_i$  and  $s_k$  in figure 23, there must be something in affectivity that persists (is present) throughout the time between these moments. Furthermore, this persistent in affectivity is common to all the timelines because affectivity is not localized to any one of them in particular. However, affectivity does not itself constitute a structure<sup>60</sup>; affectivity is an energetic<sup>61</sup> (like a "fuel") for the process of structure-building. Thus, this persistent is some depiction of one or more activity field places efficacious (adequate for) producing the expression of some *singular* motor action. To mix the terminology of Kant with that of Piaget and Garcia, this depiction (*parástase*) of activity field place or places serves the role of a *materia circa quam*<sup>62</sup> as a place where a "local" meaning (a *specific* sensorimotor action) nucleates. By "local" I here mean "mathematically localized to some region of the timescape." In topological terms, this locality implies action *generating a specific topological neighborhood* [Wells (2011a)<sup>26</sup>]. In psychic terms (figure 14), this is an idea of *organizing* noetic organization.

The *real context* of this persistent is *persistence of motor action* because we are assimilating sensibility into an action scheme and motoregulatory expression is the assimilation of perception<sup>63</sup>. From this context we finally arrive at the property of *Comparison* implicated by the transcendental schema of persistence in desiration. Figure 20 says that the reflexive binary associator selects determinables (activity fields) to present as comparands. Logically, in order to do so *with specificity* means that the employment of the reflexive binary associator relation requires a *search algorithm*. This is, after all, the practical implication of saying that reflective judgment *orients* the synthesis in sensibility. If we merely piled up every possible pair of

<sup>60</sup> A structure is a system of self-regulating transformations such that (1) no new element engendered by their operation breaks the boundaries of the system (the system is self-conserving) and (2) the transformations of the system do not involve elements outside the system. Affectivity is not self-conserving and for that reason can never be constituted as a structure. The constructed structures in *nous* are the manifold of concepts in determining judgment and the manifold of rules in practical judgment. The manifold of Desires in reflective judgment is not a structure and this is why reflective judgment is said to be *impetuous*. Much (not all) of Freud's idea of the *id* is similar to the power of reflective judgment.

<sup>61</sup> An energetic is that which is efficacious in arousing actions.

<sup>62</sup> "matter around which."

<sup>63</sup> This is an animating principle of *psyche* [Wells (2009) chap. 4 §3.6].

determinables one after another, this would not be an *oriented* synthesis of *Comparison* but rather merely an exhaustive trial of all logical combinations of determinables. Earlier (note 13) I quoted Damasio's observation that "emotion" contributes significantly to reasoning ability. What else did his observations lead him to hypothesize?

[Our] brains can often decide well, in seconds, or in minutes, depending on the time frame we set as appropriate for the goal we want to achieve, and if they can do so, they must do the marvelous job with more than just pure reason. . . Consider again the scenarios I have outlined. The key components unfold in our minds instantly, sketchily, and virtually simultaneously, too fast for the details to be clearly defined. But now imagine that *before* you . . . reason toward the solution of the problem, something quite important happens: When the bad outcome connected with a given response option comes into mind, however fleetingly, you experience an unpleasant gut feeling. Because the feeling is about the body, I gave the phenomenon the technical term *somatic* state ("soma" is Greek for body); and because it "marks" an image, I called it a *marker*. . .

What does the *somatic marker* achieve? It forces attention on the negative outcome to which a given action may lead, and functions as an automated alarm . . . The signal may lead you to reject, *immediately*, the negative course of action and thus make you choose among other alternatives. The automated signal . . . allows you *to choose from fewer alternatives*. . . Somatic markers probably increase the accuracy and efficiency of the decision process. Their absence reduces them. . . When a negative somatic marker is juxtaposed to a particular future outcome the combination functions as an alarm bell. When a positive somatic marker is juxtaposed instead, it becomes a beacon of incentive. [Damasio (1994)<sup>40</sup>, pp. 172-174]

Damasio is, of course, stating his somatic marker hypothesis in terms of the too-vague notion of "emotions." But if we alter this slightly, by replacing the notion of emotion with the idea of affectivity in general, we can begin to make out a linkage between high-level psycho-somatic observations, visible to Damasio by the nature of his work with numerous patients, and much more fundamental *psyche*-somatic mechanisms of mental physics. Combine this as well with Damasio's convergence zone hypothesis (note 8), and a picture of orientation in comparand selection begins to come into focus. We know from fundamental grounds in mental physics that an orientation of comparand search is theoretically necessary. Damasio's hypotheses give us an empirical context in which to place this otherwise mathematically abstract idea. Put them together and we get this: *The search algorithm for the reflexive binary associator limits the selection of comparand determinables to just those activity field places for which the association with the somatic activity field pertains to activities in limbic and motor functions.*

These places are *not* Damasian somatic markers or convergence zones, although it might well be the case that construction of determined embedding field networks could be mathematical images of somatic markers and convergence zones. *Damasio's hypotheses, taken in the proper Critical context, are not-incongruent with the dictates of mental physics.* In my opinion, this necessitates taking them quite seriously.

What is "the limbic system"? Neural science has not altogether settled this question, but there is a kind of cautious working consensus that the model of figure 24, or one similar to this, is at least a reasonably close approximation. The anatomical factors of *Comparison* are: (1) the anatomical sub-regions of those blocks in figure 24 making efferent projections to non-limbic anatomical structures; (2) the direct targets of these efferent projections; (3) the sub-regions receiving afferent projections from non-limbic anatomical structures; and (4) the non-limbic sub-regions making these projections. *These are empirically plausible somatic places where the presence of activity is a qualifying condition for inclusion in the comparand search process* in regard to the transcendental schema of aggregation of extensive magnitude in desiration. *They jointly define a property set for Comparison* in regard to its RBA network.



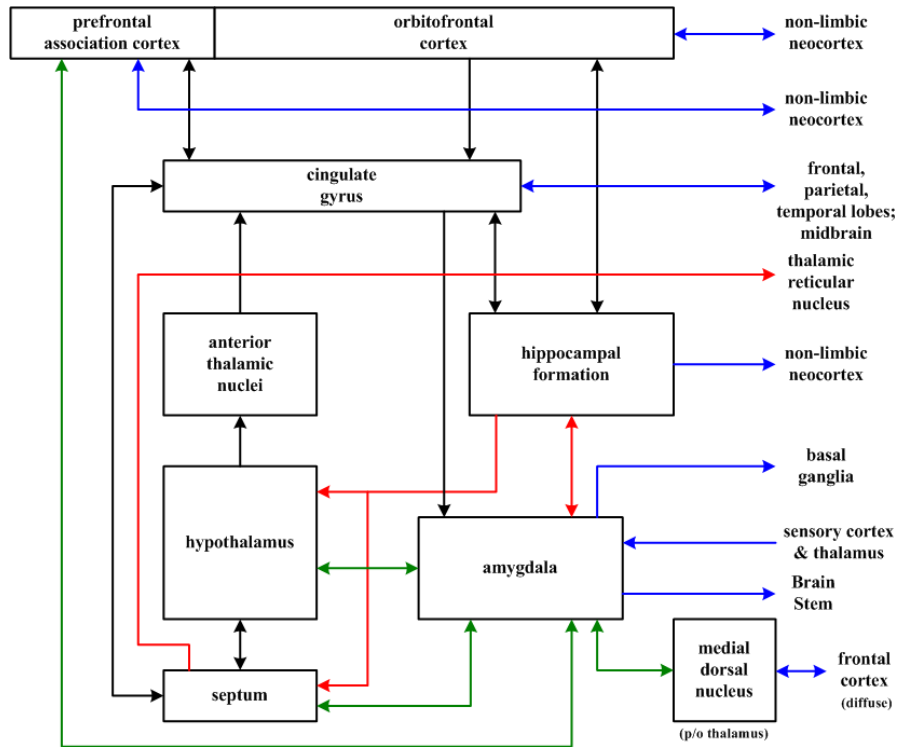


Figure 24: One current and popular working model of the limbic system.

At the present time, it is not yet feasible to apply this theory directly to *H. sapiens*. *H. sapiens* is simply too poorly understood at the present time and we still lack the requisite experience that is needed to develop good heuristic experimental procedures. We also still lack adequate measuring instruments to conduct experiments of the sort required here. However, in a Martian agent, which is a model that does contain a "mock limbic system" *function*, there is no barrier that would prevent us from applying this theory and gaining some experience on at least the mathematical modeling side of the empirical science. Indeed, one goal of Martian research is to *propose* well-posed experiment objectives for practical experiment designs useful to neural science and psychology.

In regard to the mathematical ramifications of all this: Along with the property set just deduced we also need to have a specific understanding of the type of search operation suitable to be applied to this property set. The elements of the property set constitute subsets within a determinable activity field. The compilation process implied by the reflexive binary associator relation is a compilation oriented by activities *contained in* the subsets. In the terminology of computer engineering, a search where the selected items are selected on the basis of content is called a **content addressable search algorithm**. Thus we have not only a property set definition for the activity fields but also a property definition for the operations of the algorithm. Over the years, computer engineering research has yielded up numerous practical algorithms and algorithm logic structures for carrying out content addressable searches. Computers and processors based on such algorithms are called *associative computers* or *content addressable parallel processors* [Foster (1976)<sup>64</sup>, Brennan (1998)<sup>65</sup>, Laverty (1996)<sup>66</sup>,

<sup>64</sup> Foster, Caxton C. (1976), *Content Addressable Parallel Processors*, NY: Van Nostrand Reinhold.

<sup>65</sup> Brennan, Aaron J. (1998), *Binary Connectionist Networks*, M.S. Thesis, the University of Idaho.

<sup>66</sup> Laverty, Mark Wayne (1996), *A VLSI Template Generator and Search Engine*, M.S. Thesis, the University of Idaho.

Crane (1968)<sup>67</sup>, Digby (1973)<sup>68</sup>, Estrin and Fuller (1963)<sup>69</sup>, Falkoff (1962)<sup>70</sup>, Stillman and Defiore (1971)<sup>71</sup>, Unger (1958)<sup>72</sup>, and Weinstein (1963)<sup>73</sup>]. So far as the mathematical issues involved in mind-body modeling are concerned, the task of the model-maker is actually little different from the task of the computer architect who defines the structure of a computing device. This is not to say "the brain is a computer." That is, at best, a strained metaphor. But it is to say that "logic is logic and mathematics is mathematics" and it makes no *mathematical* or *logical* difference if the object of the modeling is H. sapiens, a Martian, or a computer. The technical skills of the computer engineer are relevant to mind-body science.

18. The second schema of Quantity. We need not spend too much time examining the second schema of Quantity because we have already covered most of it. The schema is change of composition of the extensive magnitude of desiration, and with regard to teleological reflective judgment this amounts to the aggregation of multiple contexts. This, however, is precisely what the capacity to form numerous classifications with the ART classifier of figure 19 accomplishes. All that really needs to be added to this is an observation concerning the finite capacity of apprehension to hold multiple contexts as separate timelines in subjective time. It is well known in psychology, not to mention each individual's personal experience, that the ability to attend to multiple things all at once is limited. William James wrote,

The number of *things* we may attend to is altogether indefinite, depending on the power of the individual intellect, on the form of the apprehension, and on what the things are. When apprehended conceptually as a connected system, their number may be very large. But however numerous the things, they can only be known in a single pulse of consciousness for which they form one complex 'object,' so that properly speaking there is before the mind at no time a plurality of *ideas*, properly so called.

The 'unity of the soul' has been supposed by many philosophers, who also believed in the distinct atomic nature of 'ideas,' to preclude the presence to it of more than one objective fact, manifested in one idea, at a time. . . Such glaringly artificial views can only come from fantastic metaphysics or from the ambiguity of the word 'idea,' which, standing sometimes for the mental state and sometimes for the thing known, leads men to ascribe it to the thing, not only the unity which belongs to the mental state, but even the simplicity which is thought to reside in the Soul.

When the things are apprehended by the *senses*, the number of them that can be attended to at once is small, "*Pluribus intentus, minor est ad singula sensus.*"<sup>74</sup> [James (1890)<sup>75</sup>, vol. 1, pp. 405-406]

When Grossberg was developing the theory that lead to classifiers of the form of figure 10 he

<sup>67</sup> Crane, Bently (1968), "Path finding with associative memory," *IEEE Transactions on Computers*, **C17**, no. 7, 691-693.

<sup>68</sup> Digby, David W. (1973), "A search memory for many-to-many comparisons," *IEEE Transactions on Computers*, **C-22**, no. 8, 768-772.

<sup>69</sup> Estrin, G. and R. Fuller (1963), "Algorithms for content-addressable memories," *1963 IEEE Pacific Computer Conference*, pp. 118-130.

<sup>70</sup> Falkoff, A.D. (1962), "Algorithms for parallel-search memories," *JACM* **9**, no. 10, 488-511, Oct.

<sup>71</sup> Stillman, Neil J. and Casper R. Defiore (1971), "Associative processing of line drawings," *Proc. SJCC*, pp. 557-562.

<sup>72</sup> Unger, S.H. (1958), "A computer oriented toward spatial problems," *Proceedings of the IRE*, pp. 1744-1750, Oct.

<sup>73</sup> Weinstein, H. (1963), "Proposals for ordered sequential detection of simultaneous multiple responses," *IEEE Transactions on Elect. Comp.*, **EC-12**, no. 10, pp. 564-567.

<sup>74</sup> "A single experience is extensible at great length yet small in degree."

<sup>75</sup> James, William (1890), *The Principles of Psychology*, NY: Dover Publications, 1950.

was focused on psychological phenomena of learning and long-term cognitive memory. He was, therefore and quite correctly, concerned with the mathematical issue of *recoding* and with the stability-of-learning problem [Grossberg (1976a)<sup>76</sup>]. There is, he showed, a tradeoff inherent in the theory of self-modifying automata as that theory stood at that time. It has since come to be known as "the stability-plasticity dilemma." ART arose from his efforts to resolve this dilemma [Grossberg (1976b)<sup>77</sup>].

However, in the context of the synthesis in *sensibility* long-term coding stability in the ART classifier is not only not a pertinent issue for the object but, in point of fact, such long term stability is *contrary to the function of sensibility*. However many classifications M the  $F_2$  field of the ART classifier can hold, this number is always finite whereas the process of apprehension is unlimited in temporal span. This is to say that there is a real distinction between *apprehension* and *comprehension*, and it is only in regard to the latter where long-term stability of ART classifications is pertinent. Kant wrote,

To take up a quantum intuitively in imagination . . . involves two acts of this capacity: **apprehension** (*apprehensio*) and **concentration** (*comprehensio aesthetica*). Apprehension involves no problem, for it may progress to infinity. But concentration becomes more and more difficult the farther apprehension advances, and it soon reaches its maximum, namely, the aesthetically largest basic measure for the evaluation of magnitude. For when apprehension has gone so far that the partial representations of sensible intuition that were first apprehended are already being extinguished in imagination as it advances to apprehension of further ones, then it loses as much on the one side as it gains on the other, and so there is a maximum in concentration that it cannot exceed. [Kant (1790)<sup>78</sup>, 5: 521-522]

What this means for the ART classifier in *Comparison* is that this function must be *forgetful*. However large its storage capacity M in field  $F_2$  may be, it must still be a finite capacity and will eventually be used up. At this point, some old classifications *must* give way to make room for new ones to form, and this is "forgetful logic." The utility of forgetful logic in artificial neural network engineering has already been demonstrated [Wells *et al.* (2005)<sup>79</sup>]. Now we find the idea reappearing in a much more fundamental context, i.e., as an algorithmic property of the logic and mathematics of apprehension.

Mental physics can tell us that the forgetful logic property is necessary in the modeling of apprehension. It cannot, however, tell us anything about *empirical* forgetfulness. For this we must turn to empirical psychology. Here probably the foremost research issue we face is that of determining the *priority order* in which classifications are to be forgotten to make room for new ones. In algorithmic terms, there are numerous forgetting strategies possible. Computer engineers, again, have long been concerned with this in the design of virtual memory systems in computers, and the algorithm itself is often called a page replacement algorithm [Matick (1975)<sup>80</sup>]. Note, however, that the algorithm *must preserve* the equivalence-defining relations.

<sup>76</sup> Grossberg, Stephen (1976a), "Adaptive pattern classification and universal recoding: I. Parallel development and coding of neural feature detectors," *Biological Cybernetics* 23, 121-134.

<sup>77</sup> Grossberg, Stephen (1976b), "Adaptive pattern classification and universal recoding: II. Feedback, expectation, olfaction, illusions," *Biological Cybernetics* 23, 187-202.

<sup>78</sup> Kant, Immanuel (1790), *Kritik der Urtheilskraft*, in *Kant's gesammelte Schriften, Band V*, Berlin: Druck und Verlag von Georg Reimer, 1913.

<sup>79</sup> Wells, R.B., A. Bhattacharya, B. Sharon, P. Gupta, S. Young, S. Giri, T. Ityavyar, and D. Cox (2005), "Forgetful logic circuits for pulse-mode neural networks," *Proc. Int. Joint Conf. Neural Networks (IJCNN'05)*, Montreal, Quebec, Canada, July 31-August 4, pp. 616-621.

<sup>80</sup> Matick, Richard E. (1975), "Memory and storage," in *Introduction to Computer Architecture*, Harold S. Stone (ed.), Palo Alto, CA: Science Research Associates, Inc., chap. 5, pp. 230-239.

Although a number of different computer strategies for this exist, I note that among them we find systems and algorithms based on associative memories and comment that of the various types of computer approaches extant, the associative approach seems to be the one most consistent with neural system organization. Again, the computer engineering literature provides numerous examples and analyses of different kinds of replacement algorithms, e.g., Shaw (1974)<sup>81</sup> pp. 130-144, Aho *et al.* (1971)<sup>82</sup>, Belady (1966)<sup>83</sup>. One should not expect these examples to directly apply to embedding field graphs, however, because the algorithms were developed independently of embedding field theory. Consequently, there is a design challenge involved in merging these ideas with that of a forgetful ART classifier.

19. The third schema of Quantity. The first two schemata are oriented by the composition of motor actions and the composition of multiple contexts in teleological reflective judgment. The third schema, the schema of integration of extensive magnitude in desiration, is oriented by the syncretism of reflective judgment in presenting compositions leading to the possibility of an objective *Obs.OS* [Wells (2009), chap. 7, pg. 251]. This is still an affective judgment, you should note, but one that is essential for the possibility of *cognition*.

Every timeline in the timescape of subjective time can be regarded as the intuitive form of a particular context *judicially* expedient for *practical* meanings. In transcendental Logic terms, the third schema can be formally looked at as a kind of synthesis of the first two. What this means is that integration of extensive magnitude in sensibility pertains to assembling the matter in subjective time for the different timelines (figure 23). Referring once more to figure 3, the logical outcome of *Comparison* stands as an aliment to *Reflexion* in the synthesis of apprehension. Concurrent timelines in sensible perception implies a functional requirement for distributing comparands in the manifold of timelines in the intuitive timescape. This aspect of transforming the comparands can be called a ***distribution algorithm*** for *Comparison*.

The task at hand, algorithmically, is a task combining the comparand representations with the classifications produced in the *Comparand* operation. This is illustrated by figure 25. Quantity in *Comparison* deals with possible time-series (but not time-order) for desiration. Looking back at figure 23, one obvious character of the imaginative timescape in apprehension is the forking property of diverging timelines emerging from one common moment in time. The divergence is due to difference in context but the particular timelines involve logical commonality in one logical context. We might call this *logical predication conditioning* in sensorimotor meanings.

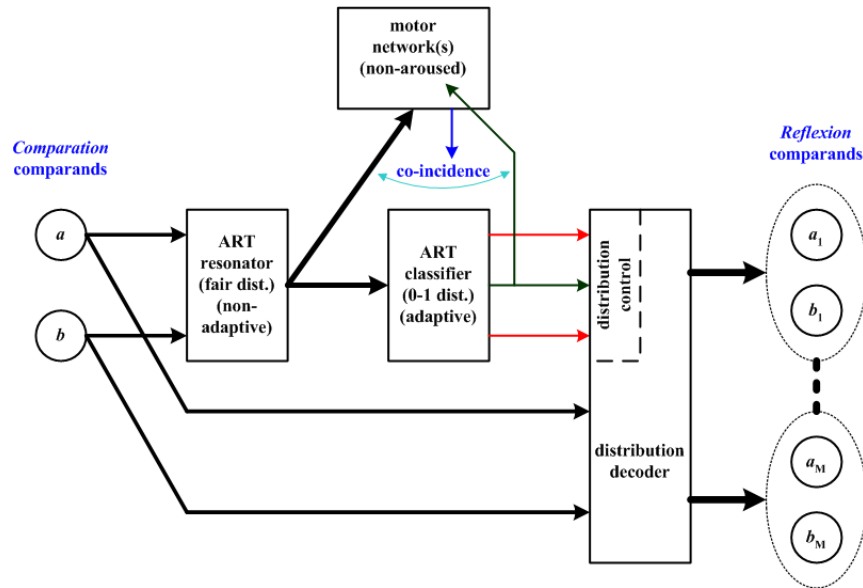
The new "piece" in the *Comparison* minimal anatomy is the block labeled distribution decoder in figure 25. Logically and mathematically, the distribution decoder is a kind of inverse function to the reflexive binary associator function discussed earlier. The RBA can be regarded as a many-to-two mapping-and-selection function. The distribution decoder or ***distributor*** can be regarded as a two-to-many mapping with determination of the mapping being made by the classification operation of the ART classifier.

Now, the adaptability and the forgetfulness of the ART classifier raise two otherwise probably non-obvious issues. As the ART classifier function is adaptable, *so also must be arcs made by its projections to motor networks and to the distributor*. In addition to the property set requirement for a distributor, we also find a property set requirement for a ***distribution algorithm***. Here the property set encounters *systematic constraints* on the Quantity of form.

<sup>81</sup> Shaw, Alan C. (1974), *The Logical Design of Operating Systems*, Englewood Cliffs, NJ: Prentice-Hall.

<sup>82</sup> Aho, A.V., P.J. Denning, and J.D. Ullman (1971), "Principles of optimal page replacement," *JACM*, **18**, no. 1, pp. 80-93.

<sup>83</sup> Belady, L.A. (1966), "A study of replacement algorithms for a virtual storage computer," *IBM System Journal*, **5**, no. 2, pp. 78-101.



**Figure 25:** The logical comparand distribution property of *Comparison* in minimal anatomy representation. The distribution decoder is called the *distributor* operation.

20. The senses. While their explanations have not been trivial, the three functions of Quantity in *Comparison* have been more or less straightforward to deduce. This has been due primarily to the fact that Quantity functions tend to be those that are the most "mathematics-like" functions of representation. *Comparison* is *logical* comparison and it is the nature of formal logic that it makes abstraction of the matter of its constructs and focuses on the forms of composition. However, it is this very character of *Comparison* that makes the other three headings more challenging to place in their proper *real* context.

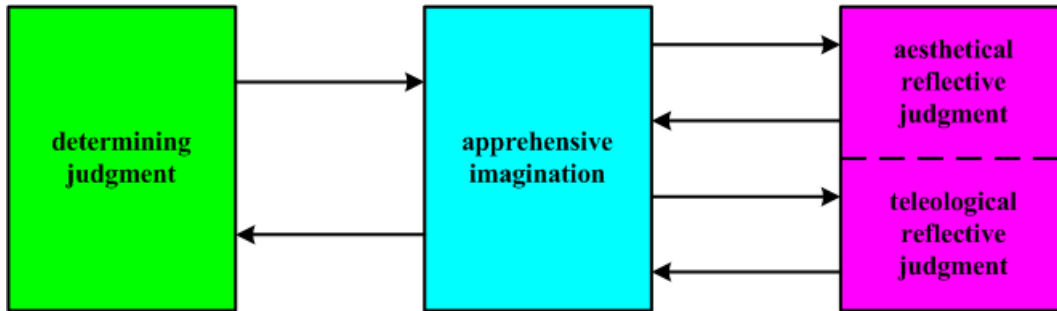
To do so, the first thing we must bear in mind is that *Comparison*, because it is part of sensibility, belongs to that *noumenon* called *sense*. Kant called sense one of the "three original sources which contain the conditions of the possibility of all experience, and cannot themselves be derived from any other capacity of mind" (the other two being the power-of-*imagination* and *apperception*) [Kant (1781)<sup>84</sup>, A94]. When we speak of "the senses" we are referring to some one or another aspect of phenomenal experience whose transcendental place is assigned to the *noumenon* of sense. *Sense-per-se* is supersensible, i.e., we cannot sense "sense"; we can only sense real effects that we credit to a *capacity for sense* in H. sapiens.

*Comparison* is part of this capacity and so to better fix its real context we should take a moment to summarize three *anthropological characteristics* of the senses. Kant tells us,

The inner perfection of the human being subsists in that: he has in his dominion the employment of all his capacities, in order to subject them to his *free choice*. For this, it is required that *understanding* should reign without weakening sensibility (which in itself is plebian because it does not think): because without it [sensibility] there would be given no material that could be processed for the use of legislative understanding. [Kant (1798)<sup>85</sup>, 7:144]

<sup>84</sup> Kant, Immanuel (1781), *Kritik der reinen Vernunft*, 1st ed., in *Kant's gesammelte Schriften, Band IV*, pp. 1-252, Berlin: Druck und Verlag von Georg Reimer, 1911.

<sup>85</sup> Kant, Immanuel (1798), *Anthropologie in pragmatischer Hinsicht*, in *Kant's gesammelte Schriften, Band VII*, pp. 117-333, Berlin: Druck und Verlag von Georg Reimer, 1917.



**Figure 26:** The free play of imagination and judgment, also called the free play of imagination and understanding.

*The senses do not confuse. . . Sense perceptions (empirical representations with consciousness) can only be called inner appearances. . . To be sure, sensuous representations must come before those of understanding and post themselves en masse. But the fruits are all the more plentiful when understanding comes in with its order and intellectual form and brings into conscious, e.g., concise expressions for the concept, emphatic expressions for the feeling, and interesting representations for determination of will. [ibid., 7: 144]*

*The senses do not have control over understanding. Rather, they offer themselves to understanding merely in order to be at its disposal. [ibid., 7: 145]*

*The senses do not deceive. . . not because they always judge correctly, but rather because they do not judge at all. [ibid., 7: 146]*

Understanding is not a distinctly separate process of mind. Note that in figure 12 there is no block labeled "understanding" at all. Rather, understanding is an organic human capacity, by which I mean it is what we call the entire outcome of the processes of judgment, the syntheses of imagination, the expressions of pure Reason, and the synthesis of apperception. Kant described it in *Critique of Pure Reason* as the organized capacity (faculty) for Self-made rules. In regard to sensibility specifically, the capacity for understanding is realized through a system of thorough-going interactions between sensibility and the processes of determining and reflective judgment depicted in figure 26. Kant called this the *free play of imagination and understanding*, although it is more precise to call it the free play of imagination and judgment.

In looking at figure 26, Kant's two remarks quoted above – that the senses do not control understanding (specifically, judgment processes) and do not judge at all – are of especial importance. This is because these characteristics of sense and imagination have to be built into any model of sensibility and its parts. Sense is not a *passive* process, but it is a subservient process, *stimulated* into acts by *reproductive* imagination as much as by receptivity in *psyche* and *oriented* in its actions by the processes of judgment.

21. First schema of Relation in Comparison. The transcendental schematism of Quantity in *Comparison* deals with time-series, a term that means the *form* of aggregation of sensuous *materia ex qua* that are coalesced to compose possible sensations and feelings (the matters of, respectively, objective perceptions – i.e., intuitions – and affective perceptions). This schematism does not, however, deal with the time-order in which these "fill time" in the *nexus* of subjective time. The schematism of time-order is Relation in apprehensive imagination, and the property set of *Comparison* must include within the properties of logical comparison sufficient functions that make the determination of time-order possible. Again, sensibility does not judge, and so this time-order is not *determined by Comparison* (or, likewise, *Reflexion* or

abstraction in the *Verstandes-Actus*) but, rather, is only *depicted* by means of *Comparison* (and the other two acts of understanding).

The form of *nexus* in the timescape of subjective time (figure 23) shows us the real context for this aspect of the *Comparison* property set. Referring to this and the schemata of Relation in figure 22, we can make out three *topological forms* of representation *trajectories* that *Comparison* must support. The first is called the *modus* of persistence-in-time. In the context of *Comparison*, this refers to *everything that "fills" a particular timeline between the moments in time marked by the making of intuitions*. In Critical epistemology, we say that a moment in time "grows out of" the immediately antecedent moment in time to which it is connected by a timeline. One analogy I tend to like is to envision each timeline as a kind of "pipe" through which comparand representations "flow" with the ends of the pipes aligning with the conscious moments in time. All the sensuous *materia* "inside the pipe" is then to be regarded as *materia* persistent in time.

There are a couple of observations in regard to this metaphor that need to be set down. The first is that *a timeline has no fixed duration*. In Critical terminology, duration in the time of a thing is the measure of the magnitude of the *Dasein* (presence) of that thing *insofar as it is a phenomenon* [Kant (1783)<sup>86</sup>, 29: 842]. Representations "between moments in time" are not even conscious representations in relationship to intuition, much less representations of phenomena. Therefore, no idea of "duration" can be attached to the length of a graphical representation of a timeline. Timelines in subjective time are ideas of secondary quantities in the mathematical facet B of Reality-in-general.

Of course, this does raise the issue of how a timeline is re-presented in a somatic embedding field graph, but this is not an especially difficult puzzle to solve. Draw a line underneath the timescape of figure 23 and call this line the *somatic-objective-time timeline*. This construct will stand as a *principal* quantity of facet B determinable, with respect to facet A, by measuring instruments ("clocks"). A moment in time is "where a pulse of consciousness takes place" and, to the extent that one can "clock" the presentation of a perception<sup>87</sup>, those "clock moments" in the objective timeline can be defined as corresponding to moments in subjective time. The subjective-time timeline representations can then be "stretched or shortened" as appropriate to make their moments in time line up with those of the objective timeline. Stated in terms of the *Comparison* property set, **Comparison requires a temporal synchronizing algorithm** as part of its property set definition.

The second observation is a reference to a particularly prescient conjecture Grossberg made in 1969 [Grossberg (1969)<sup>88</sup>; Wells (2011c)<sup>89</sup>]. Grossberg observed that sensory experiences, while seeming to be spatio-temporally continuous, are also adequately describable by relatively discrete processes (e.g., by means of language). He concluded that "the discrete representation of continuous processes must be a *universal representation* of some kind." In our mental physics construct of "time pipes" and their context as a sensuous "persistent-in-subjective-time," *we find both the continuous and the discrete aspects* of which Grossberg wrote. This tells us Grossberg's conjecture is objectively valid and his idea of a "pyramid of

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<sup>86</sup> Kant, Immanuel (1783), *Metaphysik Mrongovius*, in *Kant's gesammelte Schriften, Band XXIX*, pp. 745-940, Berlin: Walter de Gruyter & Co., 1983.

<sup>87</sup> For example, see Obhi, Sukhvinder S. and Patrick Haggard (2004), "Free will and free won't," *American Scientist*, vol. 92, no. 4, July-August, pp. 358-365.

<sup>88</sup> Grossberg, Stephen (1969), "Embedding fields: A theory of learning with physiological implications," *Journal of Mathematical Psychology*, 6, 209-239.

<sup>89</sup> Wells, Richard B. (2011c), "The applied metaphysic of the somatic code," available on the Wells Laboratory web site.

discrete acts" [*op cit.* Grossberg (1969)] is congruent with mental physics.

22. Second schema of Relation in Comparison. The second transcendental schema of Relation is the schema of succession in subjective time. For the most part, this schema would seem to be simple and one might almost say "intuitive." The notion of succession in time is even to a degree inherent in the idea of "timeline pipes." The question raised, however, is this: What determines the order in which comparand pairs are selected for presentation to *Comparison*? The issue comes up because apprehensive imagination is a synthesis process and the set of activity field comparands as thus far explained carries in its representation *no depiction of any particular time-order*. Indeed, this *unattachment* can almost be said to be an identifying trait of the phenomenon of imagination. James wrote,

Sensations, once experienced, modify the nervous organization so that copies of them arise again in the mind after the original outward stimulus is gone. No mental copy, however, can arise in the mind of any kind of sensation which has never been directly excited from without. . . The phenomena ordinarily ascribed to imagination . . . are those mental pictures of possible sensible experiences, to which the ordinary processes of associative thought give rise. When represented with surroundings concrete enough to constitute a *date*, these pictures, when they arrive, form *recollections*. . . When the mental pictures are of data freely combined, and reproducing no past combination exactly, we have acts of imagination properly so called. [James (1890)<sup>90</sup>, vol. 2, pp. 44-45]

What James calls "sensation" [*ibid.*, chap. XVII] corresponds more or less to what mental physics calls the receptivity of *psyche*. What he calls "imagination properly so called" is what mental physics calls the synthesis of reproductive imagination. What James calls "association" [James (1890)<sup>90</sup>, vol. 1, chap. XIV] is more or less the synthesis of apprehensive imagination in mental physics. In James' day psychology was a young science and some of his hypotheses regarding sensation, imagination-properly-so-called, association (and *especially* his "copy" notion) have not withstood the test of time. Others, however, retain currency today and in either their original or a modified form are compatible with modern views in psychology.

It is from the fact that reproductive imagination contributes *materia ex qua* to sensibility that the major issues of time-order spring. Furthermore, empirical psychology finds that there is a great deal of individual diversity in "imaginings." James saw in this diversity implications for brain organization. He wrote,

The commonly-received idea [in James' day] is that [imagination] is only a milder degree of the same process which took place when the thing now imagined was sensibly perceived. . . But the question remains: Do currents run *backward*, so that if the optical centers (for example) are excited by 'association' and a visual object is imagined, a current runs *down to the retina* also, and excites that sympathetically with the higher tracts? In other words, *can peripheral sense-organs be excited from above, or only from without? Are they excited in imagination?* . . . The truth seems to be that the cases where peripheral sense organs are directly excited in consequence of imagination are exceptional rarities, if they exist at all. *In common cases of imagination it would seem more natural to suppose that the seat of the process is purely cerebral, and that the sense organ is left out.* [*ibid.*, vol. 2, pp. 68-70]

The present-day form of this hypothesis is presented by Damasio's convergence zone hypothesis, which we discussed earlier. It also shows up in another of Damasio's hypotheses, which he calls the "as-if body loop" [Damasio (1994)<sup>40</sup>, pp. 155-164].

This, however, does not get us immediately closer to apprehensive imagination (Jamesian

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<sup>90</sup> James, William (1890), *The Principles of Psychology*, in two volumes, NY: Dover Publications, 1950.



"association"). James, who was a positivist but one who held the virtue of recognizing that positivism had its severe limitations, sought a brain-explanation for this while remaining cognizant that brain-theory was only the *material* aspect of the question. He wrote,

I shall try to show . . . that there is no other *elementary* causal law of association than the law of neural habit. All the *materials* of our thought are due to the way in which one elementary process of the cerebral hemispheres tends to excite whatever other elementary processes it may have excited in some former time. . . Let us then assume as the *basis* of all our subsequent reasoning this law: *When two elementary brain-processes have been active together or in immediate succession, one of them, on reoccurring, tends to propagate its excitement into the other.*

But, as a matter of fact, every elementary process has found itself at different times excited in conjunction with *many* other processes, and this by unavoidable outward causes. Which of these others it shall awaken now becomes a problem. Shall *b* or *c* be aroused next by the present *a*? We must make a further postulate, based, however, on the fact of *tension* in nerve-tissue, and on the fact of summation of excitements, each incomplete or latent in itself, into an open resultant. The process *b*, rather than *c*, will awake, if in addition to the vibrating tract *a* some other tract *d* is in a state of sub-excitement, and formerly was excited with *b* alone and not with *c*. In short, we may say:

*The amount of activity at any given point in the brain-cortex is the sum of tendencies of all other points to discharge into it, such tendencies being proportionate (1) to the number of times the excitement of each other point may have accompanied that of the point in question; (2) to the intensity of such excitements; and (3) to the absence of any rival point functionally disconnected with the first point, into which the discharges might be diverted.*  
[James (1890), vol. 1, pp. 566-567]

Slightly less than sixty years later, psychologist Donald Hebb would rediscover one tiny piece of James' theory, known today as Hebb's postulate, and would receive much acclaim for his impoverished conjecturing of a small piece of James' theory.<sup>91</sup> Today this is called "Hebbian learning." The historical facts, however, are: (1) James was there first with much more; and (2) Hebb's theory does not actually work out in practice without extensive modifications to his "rule" for determining associational strengths in embedding field networks (as modern theory puts it).

Part of the problem of determining succession-in-time is solved in embedding field networks simply by the arrangement of the graph's arcs and "time delays" assigned to these arcs. But this begs the broader question: if two comparand pairs are brought forth for comparison via the reflexive binary associator, and as a consequence are subsequently ushered into *the same* "timeline pipe," *what is the correct order in which comparisons should be made if they are not made all at the same mathematical objective time? If they are all made at the same mathematical objective time* (which, you will remember, is a secondary quantity of facet B), *what nonetheless determines the order of their introduction into the timeline pipes?* We can call this ***the enumeration problem of Comparison***. It tells us that one of the algorithms necessary in the *Comparison* property set is ***an enumeration algorithm***.

What is mathematical enumeration?

**Definition 15.** A *consistent enumeration* of a finite weakly-ordered set  $S = \{s_1, s_2, \dots, s_n\}$  is an assignment of integers  $i(s_j)$  to  $s_j$  such that  $s_p \leq s_q$  implies  $i(s_p) < i(s_q)$ .

<sup>91</sup> I don't know the extent of how true it may be, but my observations of the behaviors of the psychology community lead me to think most psychologists today have never read *The Principles of Psychology*. It is a peculiar form of near-sightedness not uncharacteristic of most of the scientific community today. Sometimes my fellow scientists seem to me like twelve year old boys who refuse to wear their eyeglasses.

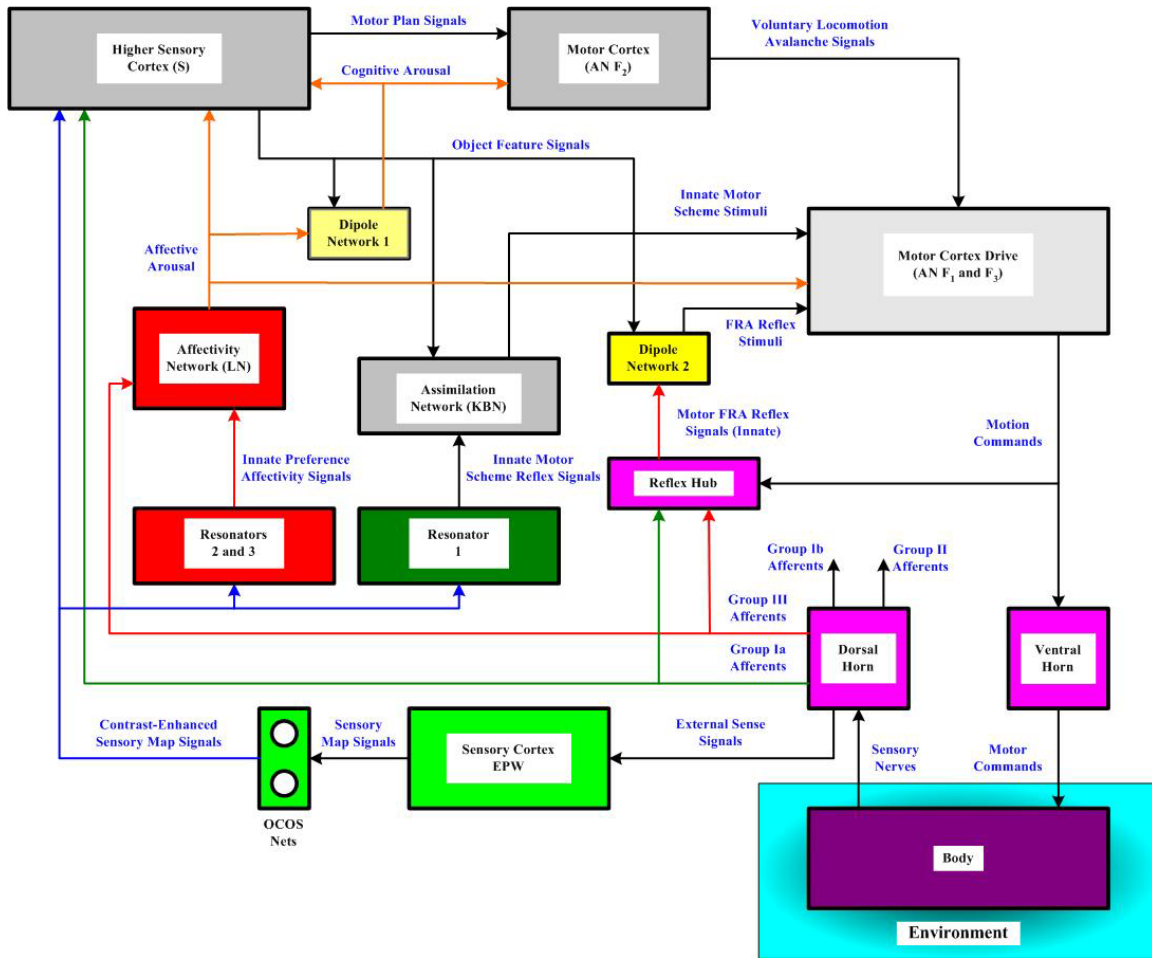


Figure 27: Block diagram of the Martian 1 system.

What can be said *a priori* about enumeration in sensibility is rather limited. We can say that enumeration is necessary. We can also say that mathematical enumeration must be made consistent with facet A observable appearances. In somatic appearances we have *one* object of appearance and, assuming it is measurable, *one* activity field appearance at any point in somatic-objective-time. This activity field appearance therefore has a privileged position, namely as a *gauge standard* for facet A correspondence. This brings up a matter of possible confusion we might unwittingly inherent from Martian 1. Figure 27 is a block diagram illustration of the Martian 1 system, and I call your attention to the constructs called the "external perception window" or EPW and the afferents emanating from the dorsal horn construct. These constructs correspond to sensory nerve signals and it is a not-unnatural temptation to regard them as the "present activity field" itself. **They are not.** The somatic activity field (which is an image of the *semantic* activity field) *involves the entire body including brain activities.* The *semantic* activity field *includes* the state variables assigned to the embedding field networks modeling brain-object functions.

I call this privileged *semantic* activity field **the primary gauge field** because it provides a reference object through which the linkage from facet B to facet A must pass. Three logical deductions follow from this. Firstly, while it is not necessarily true that "timeline pipe" *materia* must include the *somatic* gauge field comparand, it nonetheless is the case that the enumeration of time-order must take its *standard reference* from the primary gauge field.

Secondly, the enumeration issue lays before us in the starkest way possible that embedding field theory is a *field* theory in the full sense of that word in relativistic objective space-time, and it probably requires rather exotic mathematical analogues to some of the "tricks" played in the theory of quantum electrodynamics as these are illustrated in, e.g., Feynman graphs<sup>92</sup>. I do caution and entreat you, however, to not get too wrapped up in the Platonic wonders of mathematical field theory. There is no surer route to undertaking a lot of very hard work that will end up being quite useless for and irrelevant to Martian 2. Martian 2 is simply not an electron, a positron, or an atom. Semantic space-time, somatic space-time and noetic space-time **are not identical**. Semantic space-time does have priority of transcendental place with respect to the other two *because it is semantic space-time that is directly relevant to meanings*, and this must be robustly kept in mind when mappings from semantic space-time to homeomorphic images of somatic and noetic space-time are considered.

Finally, any particular and specific embedding field network model constitutes an *empirical hypothesis* of mind-body organization. This includes any conjectured enumeration algorithm that might be used and, indeed, *the very structure of any proposed embedding field network places a priori constraints on the enumerations possible*. It is important to clearly recognize this. Development of enumeration algorithms for *Comparison* belongs to empirical science, and findings from psychology and neural science must be consulted in the proposing of them.

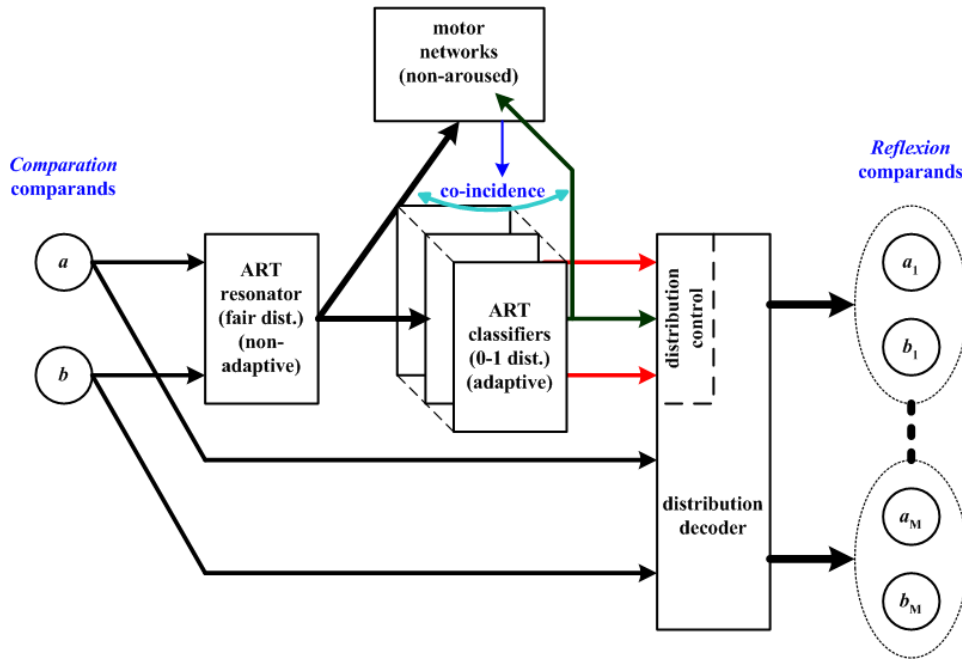
23. The third schema of Relation in Comparison. The third schema of Relation pertains to the *modus* of coexistence in subjective time. *Materia in qua* in sensibility is regarded as coexistent *materia* when it is contained in the same specific perception, whether this be affective or objective. To those who know just enough about Einstein's theory to wield it recklessly, I will here note in passing that *simultaneity* is not "banned" in relativity theory; it is quite valid at any one particular and localized place in space-time.<sup>93</sup> Therefore, there is nothing objectively invalid about the notion of coexistence in time when the *context* is properly understood and delimited.

The Relation of coexistence in the transcendental schematism of time *is not yet covered* in the previous diagrams of the *Comparison* process. We have another modification to our minimal anatomy that must be made in order to bring in this essential schematic of representation. To see this, examine once more figure 23, this time paying particular attention to the moments in time labeled  $s_i$  and  $s_m$ . At  $s_i$  we have a divergence of timelines, and this corresponds to the second schema of Quantity. Such a divergence is sometimes called a "fork" by computer scientists and is sometimes called a "join" by mathematicians (in our context of a timescape). In logic it is often represented by the disjunction symbol  $\vee$  and in algebraic notation it is often represented by the symbol  $\leq$ . Now consider  $s_m$ , where we have a re-convergence of timelines. Computer scientists often refer to such a convergence as a "join" while mathematicians often call it a "meet." In logic, it is often represented by the conjunction symbol  $\wedge$ , and in algebraic notation it is often represented by the symbol  $\geq$ . Hereafter I will call these operations join and meet in conformity to the mathematics convention.

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<sup>92</sup> An accessible qualitative introduction to Feynman graphs can be found in Feynman, Richard P. (1985), *QED: The Strange Theory of Light and Matter*, Princeton, NJ: Princeton University Press. The scholar bold enough to take on the advanced graduate level physics of QED itself – an undertaking not likely to be very applicable to Martian 2 – can consult Feynman, Richard P. (1961), *Quantum Electrodynamics*, A.R. Hibbs (note-taker) Reading, MA: Addison-Wesley Publishing.

<sup>93</sup> As an historical footnote, the word Einstein actually used in his 1905 relativity paper that gets translated into English as "simultaneous" was *gleichzeitig*. The Critical term for "coexistence" is *Zugleichsein*. This warns us that Einstein and Kant are not talking about the same Object where coexistence-in-time is concerned. *Zugleichsein* literally means "conjointly being."



**Figure 28:** Modified minimal anatomy incorporating an ART classifier bank rather than a single classifier.

Our previous minimal anatomy already contains the structure for performing the join operation; it is the combination of the ART classifier and the distribution decoder. However, it does not contain any structure by which the meet operation is possible. This is because the meet operation in logical essence amounts to routing the same pair of comparands into *two* timeline pipes rather than just one. (This is merely a convenient logical method of symbolizing that the two timelines *can* (but do not necessarily *have to*) "meet each other" at their common comparand pair).

In an ART classifier, the fineness or coarseness of its classification is determined by a parameter called *the vigilance parameter*. One classifier has at any given instant one specific vigilance setting, and this setting determines how the solution set the classification represents is determined. To make possible the routing of a single pair of comparands to *more than one* "timeline pipe" it is necessary to have *grades* of vigilance so that two different resonator outputs are distinguishable at more discriminating levels of vigilance and undistinguished at coarser levels of vigilance. This can be accomplished by replacing the single ART classifier by a bank of ART classifiers, each with different vigilance parameter settings, as illustrated in figure 28. Control of the distributor routing to permit multiple timeline pipe routings for the same comparand pair can then be incorporated into the distributor by using the bank of classified 0-1 distribution outputs as the routing information. There are, naturally, a number of more minute details of logic design involved in this distribution *algorithm*, but they are not fundamentally indefinable. (Any definition, however, does amount to another empirical postulate built into the structure of the embedding field network system).

24. Modality in Comparison. *Comparison* is logical comparison. It is the nature of formal logic to make abstraction of the material aspects of premises, retaining only the form of predication. Desiration in teleological reflective judgment is likewise formal in character because it is not concerned with the matter of Desires (desire) but only with the form of the *nexus* of desires. Quantity and Relation in the transcendental schematism are likewise formal schemata and it is for this reason that Quantity and Relation in *Comparison* are more straightforward, albeit not trivial, ideas of representation in comparison with Quality and Modality.

How is the situation not as straightforward in the cases of the matter terms, Quality and Modality? The schemata of Quality pertain to "what 'fills' subjective time" and the schemata of Modality have to do with the "embodiment" of the *nexus* of subjective time. But if *Comparison* is only *logical* comparison, what "matter" is there for its synthesis to pertain to?

Modality functions always pertain to the relationship between representation and the state of *Existenz* of the representing Subject. They add not one jot to the thing represented (whether it is an object or an affect) and only speak to "*how* my representations are *mine*." This type of relationship is called the *metaphysical nexus* of the phenomenon of mind. It is "the matter of the form of combination" in representation. One should not be too naive about the challenge of understanding this. We all come into the discussion bearing a lifetime of habits-of-thinking in regard to the question "what is time?" Most of these habits are ontology-centered and not objectively valid. Among all the scholars of classical times whose works or reports of their works have come down to us, I know of only two who came close to objectively valid notions of "what time is." They are Augustine<sup>94</sup> and, apparently, Epicurus<sup>95</sup>. Explanations of time-embodiment (*Zeitbegriff*) can have no objective validity other than *practical* objective validity, i.e., objective validity in terms of *effects* for which time is held-to-be a *noumenal* condition at the horizon of possible experience. This context is not-dissimilar to the context Santayana used in explaining pain:

That we desire to escape pain is certain; its very definition can hardly go beyond the statement that pain is that element of feeling which we seek to abolish on account of its intrinsic quality. . . The bitterest quintessence of pain is its helplessness and our incapacity to abolish it. The most intolerable torments are those we feel gaining upon us, intensifying and prolonging themselves indefinitely. This baffling quality, so conspicuous in extreme agony, is present in all pain and is perhaps its essence. If we sought to describe by a circumlocution what is of course a primary sensation, we might scarcely do better than to say that pain is consciousness at once intense and empty, fixing attention on what contains no character, and arrests all satisfactions without offering anything in exchange. [Santayana (1905)<sup>96</sup>, pp. 224-225]

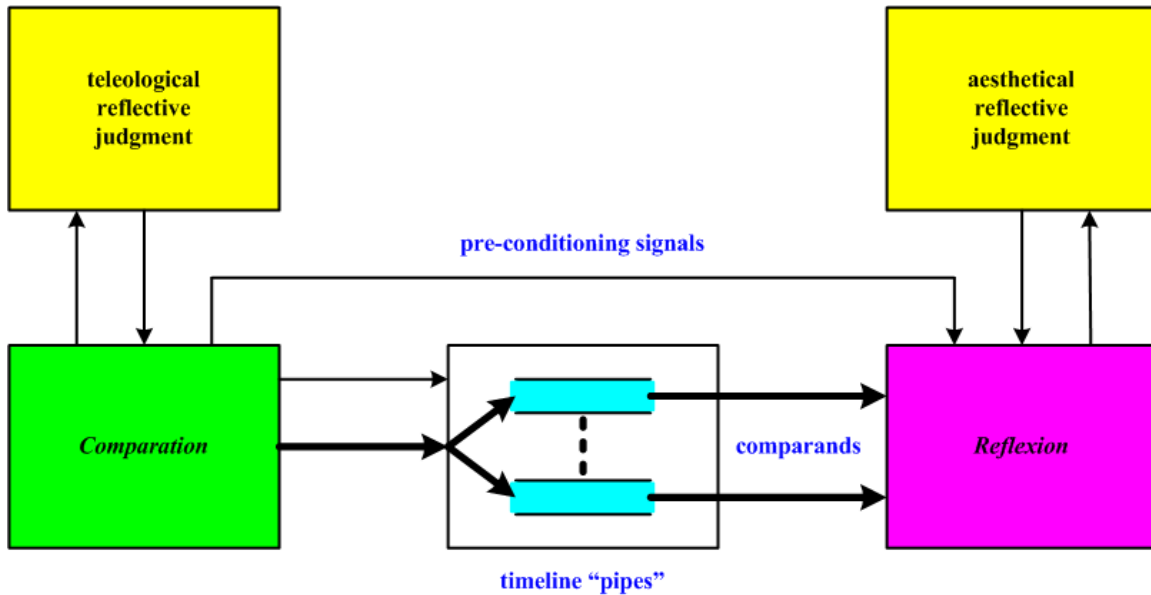
Observe that Santayana has not explained what pain-as-pain *is*. Rather, he is only describing the flavor of experiences a person is *feeling* when he says, "I am in pain." This is what he was getting at with his word circumlocution ("speaking around"). So, too, it is for explaining the notion of time-embodiment. We can only explain the notion by describing effects.

The practical *Realerklärung* ("real explanation") of the transcendental schemata of Modality is provided in chapter 5, §3.4, of *The Principles of Mental Physics* [Wells (2009)]. The context that is most pertinent to our discussion here is this context of an "embodiment" of time. As the form of inner sense, subjective time is *formulated* by apprehensive imagination oriented by reflective judgment. Our "circumlocution" of time-embodiment here revolves around the idea of the processes of imagination and reflective judgment in interaction with each other – what is called the "free play" of the synthesis of imagination with the synthesis of reflective judgment in figure 26 insofar as this "free play" produces a mathematical order structure. The *outcomes* of this interplay are ordering structures, and we say these structures "embody" the representation of subjective time. Sensibility does not control judgment. As we saw Kant say earlier, its processes are "at the disposal" of the judicial processes, which is as much as to say that the *Verstandes-Actus* are tasked with *making it possible* for judgment to fulfill *its* function. For *Comparison* this is as much as to say that its Modality function pertains to *rules*

<sup>94</sup> Augustine, *Confessions*, book XI, chapters X-XXX.

<sup>95</sup> Diogenes Laertius, *Lives of Eminent Philosophers*, book X. 72-73.

<sup>96</sup> Santayana, George (1905), *Reason in Common Sense*, NY: Dover Publications, 1980.



**Figure 29:** The functional relationship of *Comparison* and *Reflexion* with respect to reflective judgment.

of sensible representation that must be enforced. Sensibility is not responsible for the legislation of these rules (because that function belongs to judgment) but only with seeing to it that these rules are carried out. Metaphorically speaking, it is the "enforcement arm" for the judicial "legislature."

*Comparison* logically precedes *Reflexion*, which means that the rulings of reflective judgment having to do with the form of affectivity *take logical precedence* over those having to do with its matter. Modality speaks to what is required of *Comparison* in order for enough information to be presented by sensibility so that Modal rulings of judgment can be enforced. These would amount to structural logical rules that *Reflexion* is bound to follow in the synthesis of the matter of perception. This is as much as to say ***Comparison preconditions Reflexion***. Figure 29 illustrates this *property* of *Comparison*.

The *task-object* for the synthesis of apprehension is *perception*. In this context one can say that *the Verstandes-Actus prepare possible percepts* offered up to the processes of judgment for the synthesis of understanding. The *momenta* of teleological reflective judgment imply what TRJ "needs" from *Comparison* so that TRJ can do *its* job. Teleological Modality pertains to the preferences of judgment (presupposing judgments, demanding judgments, requiring judgments). What is required of perception and the embodiment of time insofar as logical comparison is concerned so that these Modal types of judgment are possible? The first pre-condition – amounting to a *sine qua non* of perception – is *non-contradiction*. For any given pair of timeline pipes, their contents might be either compatible or they might be incompatible with each other if both are brought into conscious representation at the same moment in time. Definition 8 provided the mathematical definition of a compatibility relation. Compatibility relation applied to a set (in this case, the set of comparand pairs coming into *Reflexion* through the timeline pipes) produces what are called *compatibility classes*:

**Definition 16.** Given a compatibility relation  $\gamma$  on a set  $A$ , a *compatibility class* induced by  $\gamma$  is a subset  $C$  of  $A$  such that for any two members  $a_1$  and  $a_2$  of  $C$ ,  $a_1 \gamma a_2$ . Note that  $a_1$  and  $a_2$  can be the same, i.e., that a compatibility class can consist of just one member.

To be more specific, some of the comparand pairs implicate motor network actions that

oppose those implicated by other comparand pairs. Thus, these are incompatibles and may not be combined in the same perception. Any two such comparand pairs will belong to *different* compatibility classes.

Next, it obviously makes a difference in perception whether a comparand arose immediately from receptivity or from the synthesis of reproductive imagination. The first type is said to be actual-in-sense, the other non-actual. An individual expresses action differently in the two cases. How one expresses in the presence of an actual external stimulus is generally not the same as how he expresses in the presence of a fictive product of his own imagination.

Third, howsoever he expresses himself, he is *bound* to complex expressions necessary for *coherence* in his experience. A child might, out of curiosity, try to grasp the dancing flame of a match head the first time he sees one, but he is unlikely to do so a second time. A man once lied to by another man is unlikely to trust that man's word a second time. All this is to say that *experience modifies the formal nexus of desiration in reflective judgment*. This effect of experience implies preconditioning rules for *Comparison* to signal ahead to *Reflexion*.

In these considerations, there is something of relevance in James' long-ignored theory of instincts that is pertinent to our deduction of *Comparison* property sets. James wrote,

A very common way of talking about [instincts to act] is by naming abstractly the purpose they subserve, such as self-preservation, or defense . . . . But this represents the animal as obeying abstractions which not once in a million cases is it possible it can have framed. The strict physiological way of interpreting the facts leads to far clearer results. *The actions we call instinctive all conform to the general reflex type*; they are called forth by determinate sensory stimuli in contact with the animal's body, or at a distance in his environment. The cat runs after the mouse, runs or shows fight before the dog, avoids falling from walls and trees, shuns fire and water, etc., not because he has any notion either of life or death, or of self, or of preservation. He has probably attained to no one of these conceptions in such a way as to act definitely upon it. He acts in each case separately and simply because he cannot help it; being so framed that when that particular running thing called a mouse appears in his field of vision he *must* pursue; that when that particular barking and obstreperous thing called a dog appears there he must retire, if at a distance, and scratch if close by; that he must withdraw his feet from water and his face from the flame, etc. His nervous system is to a great extent a preorganized bundle of such reactions – they are as fatal as sneezing, and as exactly correlated to their special excitants as it [sneezing] is to its own. [James (1890)<sup>90</sup>, vol. 2, pp. 383-384]

This is the phenomenal Nature of motoregulatory expression linked to actual sensory stimuli (of receptivity) *in the absence* of comparands reproduced through the synthesis of reproductive imagination. In effect, James more or less equates innate sensorimotor reflexes and innate instincts. So far as this goes Piaget agrees with James [Piaget (1981)<sup>97</sup>, pp. 16-20]. Where he, and many other psychologists, correctly take issue with James' theory is in the particulars of how one should *empirically* decide what sort of behavior is "instinctive" and what sort of behavior is not. For example, Piaget remarked, "to speak of a play instinct is to say that the child has an instinct to be a child. Again, we have only a tautology." Child's play is a behavior, but it should not be called an *instinctive* behavior.

The situation becomes far more involved and convoluted once comparands of reproductive imagination are involved. James wrote,

Remember that nothing is said yet of the origin of instincts, but only of the constitution of those that exist fully formed. How stands it with the instincts of mankind?

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<sup>97</sup> Piaget, Jean (1981), *Intelligence and Affectivity*, Palo Alto, CA: Annual Reviews, Inc.

Nothing is commoner than the remark that Man differs from lower creatures by the almost total absence of instincts, and the assumption of their work in him by 'reason.' A fruitless discussion might be waged on this point by two theorizers who were careful not to define their terms. 'Reason' might be used, as it often has been since Kant, not as the mere power of 'inferring,' but also as a name for the *tendency to obey impulses* of a certain lofty sort, such as duty or universal ends. And 'instinct' might have its significance so broadened as to cover all impulses whatever . . . Were the word instinct used in this broad way, it would of course be impossible to restrict it, as we began by doing, to actions with no prevision of an end. We must of course avoid a quarrel about words, and the facts of the case are really tolerably plain. Man has a far greater variety of *impulses* than any lower animal; and any one of these impulses, taken in itself, is as 'blind' as the lowest instinct can be; but owing to man's memory, power of reflection, and power of inference, they come each one to be felt by him, after he has yielded to them and experienced their results, in connection with a *foresight* of those results. It is obvious that *every instinctive act, in an animal with memory, must cease to be 'blind' after being once repeated*, and must be accompanied with foresight of its 'end' just so far as that end may have fallen under the animal's cognizance. . .

It is plain, then, that, *no matter how well endowed an animal may originally be in the way of instincts, his resultant actions will be much modified if the instincts combine with experience*, if in addition to impulses he have memories, associations, inferences, and expectations on any considerable scale. [*op cit.*, James (1890), vol. 2, pp. 389-390]

To this, mental physics adds that these notions of "memories, associations, inferences, and expectations" can, for *Comparison*, only be *affective* because reflective judgment knows no objects as such and deals only with the subjective in representation. This is the sort of, if you will forgive the term, "non-cognitive cognizance" some people call *emotional intelligence*. The possibility for imaginative reproductions to alter sensorimotor reaction is grounded in the fact that concepts are schematized by imagination (synthesis of re-cognition in this case), i.e. have transcendental schemata *prescribed when they are conceptualized*, and these schemata come into play again during the synthesis of reproduction as *preconditions on how imaginatively reproduced concept comparands may be employed* in acts of *Comparison*.

Finally, James said,

Wherever the mind is elevated enough to discriminate; wherever several distinct sensory elements must combine to discharge the reflex-arc; wherever, instead of plunging into action instantly at the first rough intimation of what *sort* of a thing is there, the agent waits to see which *one* of its kind it is and what the *circumstances* are of its appearance; wherever different individuals and different circumstances can impel him in different ways; wherever these are the conditions – we have a masking of the elementary constitution of the instinctive life. . . *Nature implants contrary impulses to act on many classes of things*, and leaves to it slight alterations in the conditions of the individual case to decide which impulse shall carry the day. . . [Affects] are all impulses, congenital, blind at first, and productive of motor reactions of a rigorously determinate sort. *Each one of them, then, is an instinct*, as instincts are commonly defined. *But they contradict each other* – 'experience' in each particular opportunity of application usually deciding the issue. *The animal that exhibits them loses the 'instinctive' demeanor* and appears to lead a life of hesitation and choice, an intellectual life; *not, however, because he has no instincts – rather because he has so many that they block each other's path*. [*ibid.*, vol. 2, pp. 392-393]

From all this, we come away (in regard to Modality) with the following: (1) that projections to motor networks set conditions on possible sensorimotor actions; (2) that comparands sourced immediately through reflectivity without going through the reproductive pathway of imagination imply one kind of conditioning, imaginative comparands another; and (3) coherence in experience is a necessitating precondition on all sensuous representation.



25. Quality in Comparison. This brings us to our final heading in the representation of *Comparison*, namely, the role merely *logical* comparison has to play in the overall determination of what *materia in qua* of sensibility "fills time" in apprehension. Once again, the acts of *Comparison* can have nothing to do with *matter of desire* because the judgment of matters of desire is vested in *aesthetical* reflective judgment and its composition belongs to the synthesis of *Reflexion*. What, then, is left for *Comparison* to do?

Here, again, we find the clue in the transcendental schemata (of Quality in this case). For sensuous representation to be possible (1) something has to persist between moments in time as a *material* element; (2) along with this something there must be some raw *kinematical* (action) form; and (3) these two factors must be capable of being combined. For this to all be possible, *Comparison* must be able to precondition the "downstream" operations of the *Verstandes-Actus* in appropriate *logical* forms of combination. For this ***matter of preconditioning*** we find the following:

1. Teleological reflective judgment must be able to *make* imaginative apprehension either dwell upon or quench specific comparands according to the requirements of composing equivalence relations (persistent *materia* between moments in time);
2. Teleological reflective judgment must be able to *make* imaginative apprehension either dwell upon or cut particular arcs to motor networks (kinematical factor between moments in time); and
3. Teleological reflective judgment must be able to *make Comparison* "signal ahead" to *Reflexion* what *pairs* of comparand-pairs *Reflexion* (a) is *allowed* to process because they are *formally* expedient in desiration; (b) is *not allowed* to process because they are *formally* inexpedient in desiration; and (c) is *conditionally allowed* to process because the expedience or inexpedience is conditioned by some other factor such as a third comparand.

In short, the process of *Comparison* not only has the property of setting conditions, but it also ***has the property*** of having conditions of its operations *set for it* by teleological reflective judgment. A computer engineer would say that *Comparison* regarded-as-a-processor needs an instruction set, and that it receives its "operation codes" (op codes) from TRJ.

26. Odds and ends. What has gradually emerged from these many pages is a picture of the process of *Comparison* that can logically and mathematically be regarded as a specialized form of *processor* in a very computer engineering context of that word. While it is perhaps a marvel that so many pages have been spent in detailing the "simple, obvious idea of logical comparison," both the "simple" and the "obvious" characterizations sink out of sight as soon as we pose the transcendental question, "What does 'to logically compare' mean *in general*?" Computer engineers typically do not have to wrestle with this question, but only because they have a special-purpose mathematical definition of "compare" (usually in arithmetic terms) already provided to them and their task is merely to implement the definition. So it has been here, *except* that we had to first seek out what that definition is in a context with *real* objective validity and in *conformity* with the principles of mental physics. It was this *Realerklärung* that took so many pages to dig out.

You will likely not have failed to notice that, despite the length of this paper, there are many *specific* details not covered. What, exactly, goes into the property sets? What sort of algorithms specifically are called for? What sort of "motor functions" are involved and what are the explicit preconditions for *Reflexion* and from teleological determining judgment? It is perhaps obvious to you, though, why these details are passed over in silence here: they are the empirical details of a *specific* Organized Being (H. sapiens vs. Martian). The empirical is that part of scientific knowledge that *by definition* is not *a priori*. Critical metaphysics and mental

physics can deal with epistemological necessities, but neither can pre-ordain empirical *facts*.

For *factual context* we have recourse to the contingent findings and hypotheses of empirical psychology (particularly, empirical psychophysics but behavioral and developmental psychology as well) and of neural science. However, our research must remain cognizant of the difference between interpretation of empirical observations and the observation data themselves. Above all, the semantic context has priority of place in theorizing. At root, all meanings are practical and the facet B hypotheses of mathematical *Comparison* must anchor principal quantities in this fact.

Within this overriding general context there are potentially useful computer engineering metaphors we can employ as aids to theorizing. Teleological reflective judgment sets conditions on *Comparison* operations, and these conditions might be likened to "op codes" (operation codes) in a processor. There are objects, namely the comparands, that are operated on by the processes of *Comparison*, and these can be likened to "operands" in computer terminology. *Comparison* is not a simple combinatorial logic operation; it has to have its own internal laws of transformations ("algorithms"), and these can be likened to so-called "micro sequences" in the central processing unit of a computer. In the timeline pipe constructs and the notion of "dwelling" upon particular comparands and motor connections we see an analogue to the idea of "memory function" in a computer. In the distributions of these things, as well as in the arcs of embedding field graphs, we see a homologue to "bus architectures" in computers. In specific functions, e.g. the ART resonator, the ART classifier, etc., we can see a homologue in the "arithmetic/logic unit" of a computer processor. There are, indeed, many potentially useful analogical ideas we can glean from the now-more-than-half-a-century practices of computer architecting and computer engineering.

Even so, it is Critically necessary that we not let our model-making and theorizing drift into a Platonic fog and thereby lose contact with the real-in-Nature that is the Object of our research in the first place. Mathematics exists to provide us with the important capacity to express ideas with rigorous specificity and clarity; it does not exist, nor can it function, as a substitute for Reality and Nature. That would be Platonism of the worst and most useless kind.

However tempting it may be to interpret the idea of "motor networks" strictly in analogy with the cerebral motor cortex, brainstem and cerebellum motor functions, and, of course, spinal cord motor functions, we must not lose sight of the fact that "motor" (and "locomotion") fundamentally refers to actions (changes in appearances of somatic accidents) *of any kind*. This takes in not only the obvious neural-motor functions of *soma* but the limbic and endocrine actions as well. To whatever extent Damasio's somatic markers and convergence zones stand up over time and in future experience with the physical Nature of *H. sapiens*, the transformations carried out in acts of *Comparison* are relevant for these ideas.

It should be obvious that the "memory functions" called out in this paper (e.g. timeline pipes) are not *cognitive* memory functions, nor even permanent or semi-permanent affective structures (the manifold of Desires is not a *structure*). There *is* an interactive *linkage* between cognition and affectivity. In this context we should bear in mind that *H. sapiens* is not *aware* of transformations or even of possible structures that might form in, e.g., the amygdala or the hippocampal formation. Rather, we become conscious of *end effects* neural science thinks are traced to these organs. To the extent that anatomical analogues are useful as aids to theorizing, we must bear in mind that within the mathematical world (facet B) of *Comparison*, any such analogues we might find useful as context-guiding aids are *mock-anatomies*, not biological ones, as Grossberg so correctly pointed out decades ago.

Above all, remember that to conduct scientific research is to conduct oneself with patient open-mindedness. Be enthusiastic, but do not succumb to enthusiasm. □

**Postscript 1: The Critical principles of the natural schema of judgmentation**Judicial phoronomy (Quantity)

- **composition of a motoregulatory act:** in the synthesis of apprehension, the determination of the representation of every appearance requires the co-determination of an act of moto-regulatory expression by which the *materia circa quam* for synthesis of the pure intuition of subjective space is givable through kinaesthetic feedback;
- **generation of topological neighborhood:** the apprehension of change necessarily requires the on-going construction of an aggregation of points in time that produces topological neighborhoods from the extensive magnitude of change in the representation of kinaesthetic feedback data;
- **presentation of syncretic *Obs.OS*:** representation in intuition cannot distinguish between the representation of object-in-space and sensuous matter of the representation of that space and, thus, every intuition is a syncretic representation of object-and-space called *Obs.OS*.

Judicial dynamics (Quality)

- **well-being:** the matter of the affective perception in apprehension is a feeling of *Lust*<sup>98</sup>;
- **ill-being:** the matter of the affective perception in apprehension is a feeling of *Unlust*<sup>99</sup>;
- **happiness:** the matter of the affective perception in apprehension is a feeling of equilibrium.

Judicial mechanics (Relation)

- **law of inference of ideation:** schematization of representation in sensibility through persistence in motoregulatory expression through a determination of appetitive power in pure practical Reason;
- **law of inference of induction:** schematization of representation in sensibility through actions that work to extinguish the intensive magnitude of *Lust per se*;
- **law of inference of analogy:** schematization according to the reciprocity of motivation (accommodation of perceptions) as the cause of an effect in practical appetite and, at the same time, of appetite as cause of an effect in motivation.

Judicial phenomenology (Modality)

- **law of indifference:** any joining of desiration with the motor faculties that satisfies the principle of formal expedience in reflective judgment is a possible subjective ground for the marking of an intuition in sensibility;
- **law of attentiveness:** the perception of a change in *kinesis* draws the attention of the Organized Being to a particular content of presentation in sensibility and away from other content of the presentation;
- **law of coherence in context:** no presentation of an object of attention can be determined except if this determination also includes a mutual determination of other objects of experience which provide context for the first object and presents this context in sensibility.

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<sup>98</sup> pronounced "loost." This German word, like *Gestalt*, has no English equivalent. The general connotation of the term is captured in the American colloquialism, "I'm up for that!" A feeling of *Lust* denotes a desire to maintain, promote and perfect the OB's current state of *Existenz*.

<sup>99</sup> the opposite of *Lust*. A feeling of *Unlust* denotes a desire to prevent or abolition a current state of *Existenz*.

**Postscript 2: The *momenta* of teleological reflective judgment**Extensive functions of implication (Quantity)

- **scheme implication:** synthesis of a logically singular representation of desiration that presents a local meaning implication through composition of a sensorimotor scheme expression;
- **contextual implication:** synthesis of a logically particular representation of desiration that presents a plurality of meaningful contexts in an intuition by means of generating a topological neighborhood in the pure intuition of subjective space;
- **objective implication:** synthesis of a logically universal representation of desiration that fuses an action scheme with an object of intuition, the appearance of which is called *Obs.OS*, to compose the form of a meaning implication.

Intensive functions of implication (Quality)

- **real tendency:** synthesis of a logically affirmative action expedient for satisfaction of a feeling of *Lust* by means of producing an accommodation to sensibility such that sensuous factors antagonistic to an immediate assimilation of perception in a condition of equilibrium are compensated by the action; the compensation behavior is called type- $\beta$  compensation;
- **real repugnancy:** synthesis of a logically negative action expedient for abolition of a feeling of *Unlust* by means of *negating* (not *accommodating*) the sensuous *Existenz* of factors regarded as disturbances of equilibrium; the compensating behavior in this case is called type- $\alpha$  compensation;
- **implication of real significance:** synthesis of a logically infinite action that makes a meaning implication by structuring the interplay of type- $\alpha$  and type- $\beta$  compensations such that by limiting the scope of the overall sensorimotor action an equilibration state is achieved; the compensating behavior in this case is called type- $\gamma$  compensation, i.e., an *adaptation* behavior.

Persuasions of judgment (Relation)

- **reflective subjection:** logically categorical desiration setting the focus of attention of the Organized Being; this act of judgment symbolizes the intuition as an inference of ideation;
- **reflective expectation:** logically hypothetical desiration setting a practical rule of expectation; this act of judgment symbolizes the intuition as an inference of induction;
- **reflective transferal:** logically disjunctive desiration determining the orientation of the synthesis of imagination and symbolizing the intuition as an inference of analogy<sup>100</sup>.

Preferences of judgment (Modality)

- **presupposing judgment:** logically problematic desiration expressed as simple behavioral repetitions when teleological reflective judgment is in free play with the synthesis of apprehension;
- **demanding judgment:** logically assertoric desiration (demand for expression of a specific action scheme);
- **requiring judgment:** logically apodictic desiration necessitating an action scheme that acts to condition both the form of desiration in reflective judgment and the interacting free play of imagination and understanding in thinking by generalizing the applicability of a sensorimotor action scheme.

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<sup>100</sup> it is worth noting that analogy is describable as "induction applied to predicates" rather than to subjects.

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<sup>i</sup> My purpose in publishing these notes on the Laboratory's web site is twofold. First, these notes are published for the purpose of training people in the Wells Laboratory about the theory and methods of mental physics as these are used in the Martian 2 research program. Second, I make these notes available to the broader public to provide other researchers and new students of the Critical Philosophy and/or mental physics with specific examples of reasoning in Kant's transcendental Logic and of applying the principles of mental physics in research. These notes are my own working research notes and from time to time there may be additional notes added. If and when this happens, the updated posting will be announced in the "What's New on the Wells Lab Website?" bulletin.

<sup>ii</sup> The first three notes in this paper are concerned with some of the deeper metaphysical aspects of Critical metaphysics. To grasp their significance fully, one must be knowledgeable in both the metaphysics and with mental physics, and this knowledge is not expected to already be in possession of the average reader. These first three notes, therefore, are aimed more at the metaphysician than anyone else. However, they do serve a general purpose in pointing out some of the propaedeutic background one eventually requires for full mastery of the topic. This background is contained in *The Critical Philosophy and the Phenomenon of Mind* [Wells (2006)] and *The Principles of Mental Physics* [Wells (2009)]. Both books are available on the Wells Laboratory web site free of charge. *CPPM* is a kind of "voyage of discovery" in the connotation that its purpose was to "peel back layer by layer" what we learn from experience to discover the fundamental acroams and laws that govern the phenomenon of mind. Along the way, it must fend off hostiles (namely, the failed ontology-centered systems of metaphysics and pseudo-metaphysics that lead only to error and paradox), clearing the ground (so to speak) for resettlement by epistemology-centered metaphysics. *CPPM* is not written as a textbook. The newcomer is advised to first try to grasp the "final destination" where *CPPM* ends up. This is the purpose of *PMP*, which is written in the form of a textbook and is intended to serve teaching purposes. If *CPPM* were likened to Lewis and Clark, then *PMP* would be likened to the trail guides who conducted the early pioneers moving westward.

<sup>iii</sup> Unless noted otherwise, all Wells citations are available free of charge on the Wells Laboratory web site, <http://www.mrc.uidaho.edu/~rwells/techdocs>.