On the Synthesis of Polysyllogisms in Critical Logic

I. Introduction

The synthesis of polysyllogisms in Critical Logic is an important topic within the larger topic of Critical mathematics. My previous treatises (Wells, 2006; Wells, 2009) have touched upon the logical organization of the manifold of concepts and the manifold of rules in *nous*, but the levels and scopes of these works are too high and broad to have permitted a detailed exposition of the Logic of structuring series of combined representations in an organized manifold. The aim of this paper is to make a beginning in filling in this omission as one of the early steps in the development of an organon of Critical mathematics and a doctrine of transcendental Logic. A second aim of this paper is to provide a specific illustration of the methodology of doing Critical Logic. Such an illustration is a necessary propaedeutic for teaching other scholars how the Critical theory is applied in general Logic. The Kantian system is far different from and far richer than the contentless impoverished organon of traditional logics, whether these be one of the so-called Aristotelian versions or the more modern *mathēma* called symbolic logic.

The traditional doctrines of mathematics and logic are static inasmuch as the mathematician or the logician lays out his arguments in a particular sequence of his own choosing and, assuming he has made no errors, that layout is fixed and unchanging in the progression from premises to conclusions. Complete abstraction is made of all processes of thinking and reasoning that go into his constructions, the presupposition being that mathematical truth ought not to depend upon the order or progression of the mathematician's thoughts or the details of the process of his reasoning. In Critical Logic the situation is quite different. This Logic deals with the *laws* of human thinking and reasoning, these laws determine the cognition that results, and these results depend very much on the *dynamical* process of judgmentation in *nous*. Some will object to injecting human subjectivity into mathematics but this objection is misdirected. The practice of mathematics, and of formal logic of whatever kind, *is* foundationally subjective because there can be no mathematics without the mathematician, no logic without the logician, and the doer cannot remove himself from his doings. He is bound to use theorized or posited axioms that are themselves foundationally subjective, and as these are the foundations of his mathematics or his logic, the doctrine itself and all its results are likewise subjectively founded. Critical Logic and Critical mathematics are and can only be *dynamic*, not static, doctrines. This means that the doctrine of elements employed depends as much – even, in fact, more – on the human being's *processes* of knowledge-construction as upon the formal Objects of the discipline.

This inherent subjectivity was recognized long ago by mathematician Henri Poincaré, who wrote,

> Can mathematics be reduced to logic without having to appeal to principles peculiar to itself? There is a whole school full of ardor and faith who make it their business to establish this possibility. They have their own special language, in which words are used no longer, but only signs. This language can be understood only by the few initiated, so that the vulgar are inclined to bow before the decisive affirmations of the adepts. It will, perhaps, be useful to examine these affirmations somewhat more closely, in order to see whether they justify the peremptory tone in which they are made. . .
>
> Many mathematicians have followed in [Cantor's] tracks . . . They have become so familiar with transfinite numbers that they have reached the point of making the theory of finite numbers depend on that of Cantor's cardinal numbers. In their opinion, if we wish to teach arithmetic in a truly logical way, we ought to begin by establishing the general properties of transfinite cardinal numbers, and then distinguish from among them a quite small class, that of the ordinary whole numbers. Thanks to this roundabout proceeding, we might succeed in proving all the propositions relating to this small class (that is to say, our

whole arithmetic and algebra) without making use of a single principle foreign to logic.

> This method is evidently contrary to all healthy psychology. It is certainly not in this manner that the human mind proceeded to construct mathematics, and I imagine, too, its authors do not dream of introducing it into secondary education. But is it at least logical, or, more properly speaking, is it accurate? We may well doubt it. . . It is time these exaggerations were treated as they deserve. [Poincaré (1914), pp. 143-145]

Unfortunately, the new mathematics' proponents, particularly the Bourbaki mathematicians of the 1940s and 1950s, *did* "dream of introducing it into secondary" and even into primary education. The result in the United States has been a disastrously misguided system of mathematical pedagogy that has produced at least two entire generations of people who can "do math" if a mathematics problem is set before them *as* a mathematics problem, but who have no idea either how to *use* existing or *create* new mathematics. Today the United States is as mathematically illiterate as a band of Kalahari Bushmen. This paper is not for the formally-well-trained ignorant.

In Critical Logic there is an important *epistemological* distinction between a mere formal layout of coordinated representations in a manifold of representations and the organized *structure* of a combined *series* of representations. It is with the construction of a series of organized representations of knowledge *as* series that Critical polysyllogisms are concerned. For purposes of brevity in exposition, this paper will confine its treatment of the topic to polysyllogisms in the manifold of concepts in *nous* because there is little formal difference between this treatment and that of the manifold of rules in practical Reason. The main difference between them is Critical Standpoint. The dominant Standpoint of this paper is the theoretical Standpoint of Critical metaphysics, but this does not exempt the other two Standpoints – judicial and practical – from consideration. This is because our topic-at-hand is the synthesis of representations-in-a-series through the process of judgmentation, and this synthesis (as shall be demonstrated) involves considerations from all three Standpoints.

Figure 1 illustrates the Object of this paper, namely, the structure of series representations in the manifold of concepts. The node denoted by "subject" in this figure is the initial point in the represented manifold from which the processes of synthesis originates. From this concept one either ascends to higher concepts (regressive synthesis) or descends to lower ones (progressive synthesis) in constructing a series by polysyllogism.

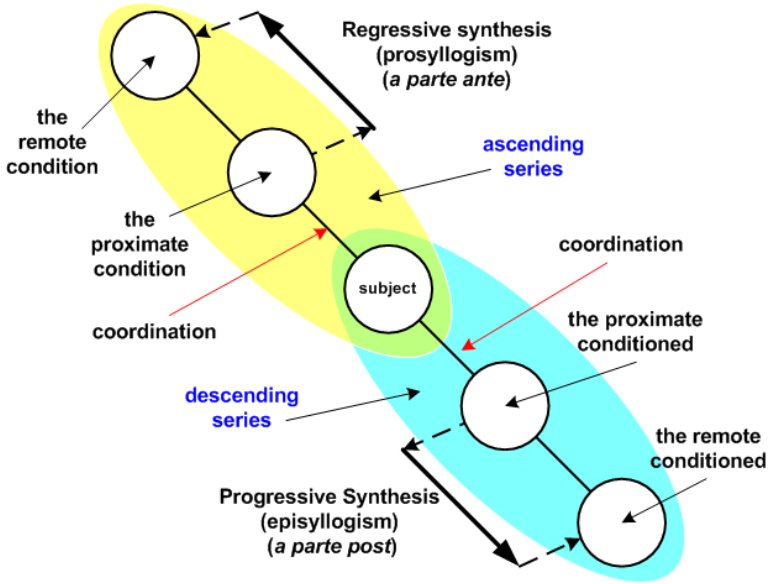

Figure 1: Illustration of the representation of series by regressive and progressive synthesis.

II. The Polysyllogism and the Organized Being

A polysyllogism is an act of judgmentation that produces what Kant called a composite inference (*zusammengesetzer Schluß*). It is this composite representation *as a whole* that constitutes a Critical *series*. Kant tells us,

> A composite inference in which many inferences of reason are combined with one another, not through mere coordination but through *subordination*, i.e., as grounds and consequences, would be called a chain of inferences of reason (*ratiocinatio polysyllogistica*). [Kant (1800), 9: 133-134]

It is from here that the term polysyllogism is obtained. It is important to stress that the Object of a polysyllogism is not the sequence of inferences as such but, rather, the combined whole of the outcome of a synthesis (the yellow and light blue ovals in figure 1). Let us suppose that we begin with a substructure within the manifold of concepts, illustrated in figure 2, in which the concepts in the manifold are combined merely by coordination (i.e., each higher concept is a mark of the one immediately below it and is regarded as a coordinate of that lower concept). S is the subject concept (the subject of the composite inference action to follow), HC denotes higher concept, LC denotes lower concept relative to S.

The Organized Being's understanding of this structure of concepts is quite limited to *pairwise* understandings between an S and its immediate HC and LC. At this point it has no *cognizance* whatsoever that concept HC_2 and concept LC_2 stand in a relationship of any sort. That there is a potential relationship, e.g. that HC_2 is a higher ground for the cognition of LC_2, is quite evident to us as *observers* of the figure, but the Organized Being himself is not yet aware of this *even potentially*.[1] Its understanding at this point is a mere collection of pairwise cognitions (HC_2-HC_1), (HC_1-S), (S-LC_1), and (LC_1-LC_2). This structure is not yet constituted as a series in which the concepts are subordinated one under another. A higher concept is a mark of its immediately lower concept but not yet a ground for recognition of that lower concept, nor is that lower concept regarded as a consequence of its immediately higher coordinate.

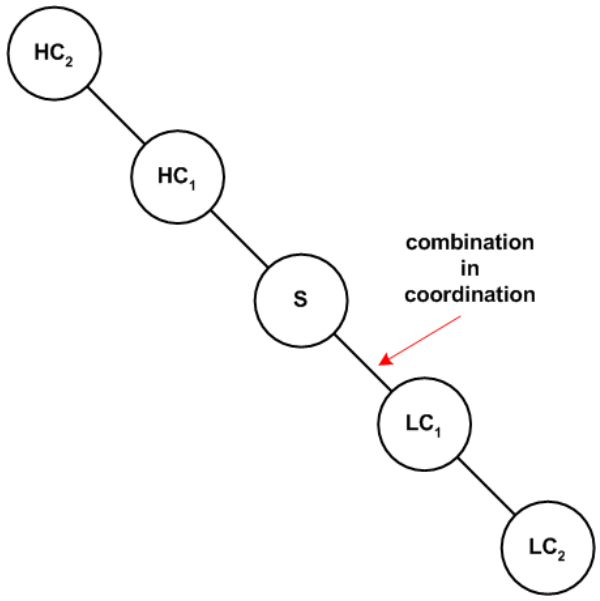

Figure 2: Illustration of a chain of merely coordinated pairs of concepts in the manifold of concepts. The connecting lines in the figure denote mere coordination, and not subordination, in the structure.

[1] Such an awareness would require an homunculus, an idea Critical epistemology utterly rejects as invalid.

It is pertinent to remark here that this epistemological state-of-affairs carries a fairly obvious and testable implication for human cognitive psychology. This implication has been tested with the finding that the limited pair-wise understanding explained above *is found in studies of young children*. Piaget *et al*. tell us,

> We have observed for some time that when children are asked to order a series of objects in increasing sizes $A < B < C$, the younger subjects simply proceed in pairs, such as *CF*, *AD*, *EH*, etc., maintaining the order 'a little one, a big one', but they are not subsequently able to coordinate them into a whole series. This also occurs with the first classifications made from 'figural collections', with the youngest subjects doing well in constructing a composite figure, such as a row, but in general placing an object in relation to the one just before it (e.g., a square after a square, or a sheep following a shepherd), without, however, taking into account the other preceding terms, in other words, once again without coordinating the pairs in spite of the linear form of the whole. In 1947, Wallon . . . focused on this idea of pairs seeing in it the most elementary form of cognitive structuring. This idea was also contained in Hoeffding's comparison . . . of the workings of thought to the successive positions of a compass, where one of the legs, although placed on a given point, provides no information until the other leg is placed on a second point or on another object.
>
> If we define functions mathematically . . . as ordered pairs, then the pairs just referred to show, however elementary they may be, already constituted functions. But if in addition functions are considered, from a psychological standpoint, as the expressions of schemes of assimilation of actions, then functions are already present in the conceptualization of any action that modifies an object x into x' or y, thereby also constituting as an ordered pair (x,x') or (x,y). Since we hypothesize that functions constitute the common source of operations and causal systems, it must therefore be possible, even in the most elementary cases, to find such (x,y) pairs in general as a substitution of y for x. [Piaget, *et al*. (1968), pg. 4]

Piaget's characterization of a "function" in terms of an ordered pair is mathematically quite correct. As this paper demonstrates, his further remark regarding functions as schemes of assimilation from a psychological standpoint was remarkably prescient in terms of cognitive structuring. As is explained here, a polysyllogism is not a mere isolated act of *nous* but, rather, a cyclic process of judgmentation in general in which an accommodation of perception leads to assimilation of a new Object (the series) and produces an equilibrium in the overall state of the Organized Being. This balancing of assimilation and accommodation producing an equilibrium is the Critical definition of *adaptation*. This is so in Piagetian psychology as well. Figure 3 is a graphical illustration of adaptation as Piaget presented it in Piaget (1952).

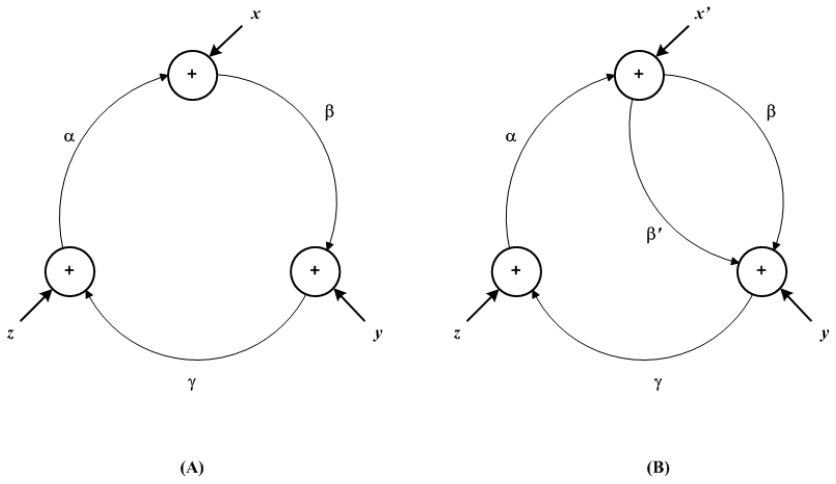

Figure 3: Cycles of assimilation (A) and adaptation (B). α, β, γ: internal elements; *x*, *y*, *z*: external aliments

The *Existenz* of an Organized Being, regarded in its dynamics, exists in cycles of organized processes consisting of internal functioning structures and external aliments of the environment of these structures. Figure 3(A) depicts a cycle of *assimilation* in which the organized elements of its structure (α, β, γ) are transformed by external aliments (x, y, z) such that the functioning of the Organized Being does not destroy the cycle of organization but, rather, coordinates the given environmental aliments in such a way as to incorporate their effects into that cycle. This process of assimilation is said to be in *equilibrium* because the cycle of activities and transformations is stable and repetitive. The processes in equilibrium can be symbolized using the mathematical notation

$$\alpha + x \to \beta;$$
$$\beta + y \to \gamma;$$
$$\gamma + z \to \alpha; \quad etc.$$

Figure 3(B) illustrates the process of *accommodation* in response to a transformation of an environmental factor x into some new factor x'. One of two outcomes is possible. Either the Organized Being does not adapt, in which case the cycle is ruptured, or else *adaptation* takes place, e.g. by transformation of β into a higher *structure* $\{\beta, \beta'\}$, such that the cycle closes up on itself. It is in this context that *an adaptation is an equilibrium between assimilation and accommodation*. We denote this mathematically by

$$\alpha + x' \to \beta';$$
$$\beta' + y \to \gamma;$$
$$\gamma + z \to \alpha; \quad etc.$$

The nature of accommodation is *conservative*. This means the new structure $B = \{\beta, \beta'\}$ does not lose its former ability to assimilate factor x as it gains the ability to assimilate x' [Piaget (1952), pp. 5-6]. The system is said to *preserve stability* as it *exhibits plasticity*. The overall organized structures depicted in figure 3 are what mathematicians call stable limit cycles.

This natural character of living organisms has long been noted by biology. Empirically we find it exhibited down to the cell level. Neurologist Antonio Damasio wrote,

> One key to understanding living organisms, from those that are made up of one cell to those that are made up of billions of cells, is the definition of their boundary, the separation between what is *in* and what is *out*. The structure of the organism is inside the boundary and the life of the organism is defined by the maintenance of internal states within the boundary. . .
>
> Through thick and thin, even when large variations occur in the environment that surrounds an organism, there is a dispositional arrangement available in the organism's structure that modifies the inner workings of the organism. The dispositional arrangement ensures that the environmental variations do not cause a correspondingly large and excessive variation of activity within. When variations that trespass into a dangerous range are about to occur, they can be averted by some preemptive action; and when dangerous variations have already occurred, they can still be corrected by some appropriate action.
>
> The specifications for survival that I am describing here include: a boundary; an internal structure; a dispositional arrangement for the regulation of internal states that subsumes a mandate to maintain life; a narrow range of variability of internal states so that those states are relatively stable. [Damasio (1999), pp. 135-136]

Damasio likewise conjectures that this same situation is exhibited in the phenomenon of mind. Piaget and many other psychologists concur with this conjecture, as do many neuroscientists. For

the theoretician this equilibrium between stability and plasticity has long been a vexing issue. Stephen Grossberg wrote,

> The design problem on which I will base my article is called the stability-plasticity dilemma. The design problem is easy to state because it is so basic. . . How can a system's adaptive mechanisms be stable enough to resist environmental fluctuations that do not alter its behavioral success, but plastic enough to change rapidly in response to environmental demands that do alter its behavioral success? How are stability without rigidity and adaptability without chaos achieved? [Grossberg (1984)].

Considerations such as these – cycles, assimilation, accommodation, stability, plasticity – must underlie all mathematical and logical treatment of mental phenomena if the mathematical model is to speak of the Nature of the Organized Being with objective validity. Thus this pertains to the making of polysyllogisms as well. By what *objectively valid* process does the Organized Being go from a simple condition of pair-wise *local* understandings (figure 2) to the integrated *totality* that is knowledge of a series (figure 1)? The Critical answer to this dictated by mental physics is discussed next.

III. Episyllogisms and Prosyllogisms

In this section I describe the step-by-step sequence of synthetic steps leading from the localized structure of understandings illustrated in figure 2 to the unified structure illustrated in figure 1. Then in the following section I briefly review some of the principal systemic Critical considerations grounding this sequence. In the next to last section I discuss the *process* of synthesis involved in making polysyllogisms. The explanation uses a key mathematical idea first developed by Palmquist – the first-level synthetic relation or 1LSR construct.

Even a superficial glance at figure 1 shows that there are two polysyllogism syntheses that must be considered. The first, the episyllogism, begins with the subject concept (S) and descends to lower concepts in synthesizing a series. This synthesis is said to occur *a parte post* and is called the *progressive* synthesis. The second, the prosyllogism, likewise begins with S but ascends to higher concepts. This synthesis is said to occur *a parte ante* and is called a *regressive* synthesis. As will be shown, both are special cases of a single *general* process of synthesis.

The term *synthesis* has a specific technical meaning and Kant's use of this term was consistent across the span of his career. He tells us,

> In *synthesis* we create and beget a concept, as it were, which simply was not there before, completely new both as to matter and also as to form, and make it distinct at the same time.
>
> All ideas of mathematics are of this kind, e.g., the ideas of triangle, square, circle, etc.
>
> All concepts feigned by reason[2] are at the same time distinct, but only by synthesis. If a concept is made distinct by analysis then it must already be given; thus we are occupied with making clear and distinct what was confused and obscure in this given concept, to develop it, to set it with another [concept], and thus to illuminate it. This includes, e.g., each and every idea of metaphysics. [Kant (*c*. 1770), "*Logik Blomberg*," 24: 130-131]

There is an important distinction between the idea of a concept as a *co*ordinate mark and this same concept as a *sub*ordinate mark of a lower concept connected with it in series. Kant explains,

> Marks are *coordinate* so far as each of them is represented as an *immediate* mark of the subject-matter, and *subordinate* so far as one mark is represented in a thing only by means

[2] "*durch die Vernunft fingirte*". The word "feign" (Latin root: *fingere*, to touch, handle, shape) originally meant "to form or shape." This is not the usual connotation today but Kant was an 18th century author.

> of another. The combination of coordinated marks for the whole of the concept is called an *aggregate*, the combination of subordinated marks a *series* . . .
>
> The series of subordinated marks pushes: *a parte ante*, or on the side of the grounds, on to indissoluble concepts, which of their simplicity do not permit them to be further dissected; *a parte post*, on the other hand or with respect to consequences, it is *endless because we have a highest genus* but no lowest *species*. [Kant (1800), *Logik*, 9: 59]

The directions *a parte ante* and *a parte post* are indicated in figure 1. When Kant says we have a highest genus, he means at some point the synthesized higher concept reaches the horizon of possible experience, where it is a notion only of pure form. This is the ultimate level for a thing-as-we-can-know-it, and if synthesis is carried past this point we have only illusory speculations concerning a thing-as-we-cannot-know-it (*Ding an sich selbst*). The object of such a concept is, ontologically, *simple*. When he says we have no lowest species he means that in consideration of the empirical matter of the object of a lower concept, we have no *a priori* knowledge that our knowledge of this matter is complete and that we will never find a matter-of-the-matter and form-of-the-matter for which a currently-lowest concept is a *mark*. For example, it was once thought that neutrons were "elementary particles"; it is now known that neutrons can "decay" into a proton, an electron, and an antineutrino. At present we have no evidence suggesting that an electron *can* "decay" into "more elementary" constituents, but we do not know this is so *for a fact*.[3]

The significance of Kant's statements above is this: the *concept* of a series is a different concept than the concept of an aggregation. In figure 1 the two colored ovals represent concepts distinct from the concepts contained inside them. More specifically, the concept of a series is a concept regarding the manner of *combination* of higher-to-lower or lower-to-higher concepts.

Figure 4 illustrates the step-by-step synthesis of the series connection, by episyllogism, resulting in a concept (S $\rightarrow$ PC $\rightarrow$ RC). We assume for the initial state of the manifold of concepts that concepts S, PC, and RC are represented. We further assume the combination S > PC is represented (as a coordination, "S understands PC") and that, likewise, the combination PC > RC is also represented as a coordination. The Organized Being, however, is not conscious of any connection between S and RC. This is to say the Organized Being has never thought of S > PC as a series, nor has he ever thought of PC > RC as a series. In terms of the elementary notions of understanding in Critical metaphysics, the categories of understanding, the object of combination-in-a-coordination is understood by a determinant judgment of the form

{totality, a notion of Quality, substance & accident, a notion of Modality},

whereas the object of combination-in-series is understood by a determinant judgment of the form

{unity, a notion of Quality, causality & dependency, a notion of Modality}.[4]

In mental physics, a combination of concepts combined by determining judgment is also a concept. Concepts are rules for the reproduction of an intuition, and a concept in which one finds other combined concepts is a different rule for intuition than the individual concepts present alone. The fundamental difference above pertains to *forms*, that is, to Quantity and Relation.

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[3] Physics currently *hypothesizes* that the electron is an "elementary particle" that cannot be divided because no one has ever observed electron decay nor do we have any empirical reason for expecting electrons to decay. But once upon a time the same thing was true of the proton and the neutron. Then sound empirical reasons for physicists to think protons were not "elementary" were discovered and present theory makes the hypothesis that protons are composed of "elementary" things called quarks. Quarks have not yet been directly observed experimentally, but the theory says there has to be six different kinds of them.

[4] See (Wells, 2009) for the explanation of these categories and the form of determinant judgment.

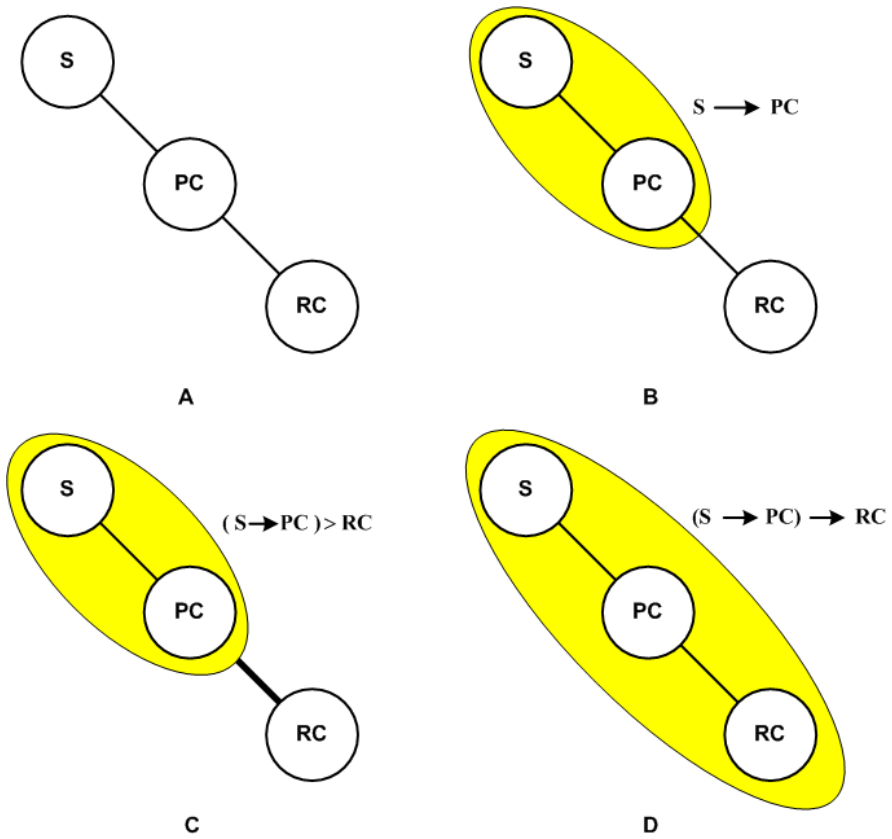

Figure 4: Illustration of synthesis by episyllogism of a series S → PC → RC. S is the subject concept, PC is its proximate conditioned, and RC is its remote conditioned. (A) Manifold of concepts prior to the episyllogism. Here S > PC is merely a coordinated combination and is not understood to be the first link in a connected chain leading from S to RC. (B) Synthesis of the concept that S and PC are series-connected. (C) Synthesis of the concept that *concept* (S → PC) is *coordinated* with RC. (D) Completion of the episyllogism in the concept of the series (S → PC → RC). PC and RC are not understood to be "conditioned" until after the concept of their series connection has been synthesized.

Figure 4(B) illustrates this first step, by which the Organized Being understands, for the first time, a concept of a causal connection between S and PC as condition-to-conditioned. Concept PC is at this point regarded as "conditioned by" S for the first time. *This* concept is what the oval represents in the figure. Symbolically, we can denote this as (S → PC) and PC is now a *proximate* conditioned for S. Note, however, that the reverse is not yet true; S is *not* regarded as a proximate *condition for* PC. *That* concept requires a *pro*syllogism (PC ← S). Reciprocal relationship is not automatic merely because two concepts have been combined. This is a theorem of mental physics and has implications for empirical psychology. It is worth noting that psychological studies have indeed found that the empirical evidence accords with this theorem. This evidence shows up in many ways, e.g. childish notions of causality, the child's ability to form classifications, or the child's ability to do seriations. To take one early example,

> A third process in the evolution of causality is the progressive establishment of reversible series. . . If we examine a mechanism of any complexity that has been correctly understood by a child of 8-10 [years of age], we shall always find that it is a reversible mechanism. When the stone has been understood to be composed of little particles of earth, the child admits that the stone can be decomposed into earth. . .
>
> Now if we really look into the matter, we shall find that all the more advanced forms of explanation in the child are reversible. . . The primitive forms of causality, on the contrary,

> are all irreversible. . . Of all the types of causality that come before that of mechanical causality, explanation by reaction of the surrounding medium is the one that points most clearly to a beginning of reversibility. But this is precisely the type that clears the way for the higher forms of explanation.
>
> The progress from irreversibility to reversibility is thus continuous. . . For the primitive universe is strewn with subjective adherences and very near to immediate perception. Now, in so far as it is tinged with the child's subjectivity, the universe is irreversible . . . Similarly, in as much as it is near to immediate perception, the child's universe is irreversible . . . It is the mind that builds up reversible sequences underneath perception. To the extent that the child's universe is removed from constructions and close to the immediately given, it is irreversible. Thus the advance towards reversibility shown by the development of child causality follows exactly the same lines as those underlying the processes defined in connection with the idea of reality. [Piaget (1930), pp. 269-271]

Note the reference to subjective adherences in this character of the development of intelligence in children. As Piaget's observations illustrated, the earliest examples of childish reasoning in regard to causality are strongly bound up to notions of purposiveness in nature. The following two sections will show that this is not a strange quirk but, rather, a basic property of the cycle of judgmentation. Years later, and based upon a still broader set of empirical findings, Inhelder and Piaget reported,

> [We] would remind the reader that nearly all the structures whose development we have been studying in the course of this work are in fact completely formed at the level of concrete operations[5]. This means that they correspond to elementary "groupings" of classes and relations, and do not cover the whole of the logic of classes and relations. In particular, these structures do not include classificatory structures which are isomorphic with respect to propositional structures . . . We have in fact touched on the way in which children later discover this sort of transformation, one which implies a fuller structure than these "groupings." The question was: when can a child understand the transformation
>
> $$(A < B) \rightarrow [(\text{not-}A) > (\text{not-}B)] \ ?$$
>
> Not surprisingly, we found that the transformation is not understood until the level of formal operations, because it implies the combined application of negation (or complementarity) and reciprocity . . .
>
> What we have seen once again is that logical operations (i.e. in particular, classification and cross-classification, and seriation and multiple seriation) are closely linked with certain actions which are quite elementary: putting things in piles, separating piles into lots, making alignments, and so on. The development is astonishingly continuous: after the actions we have various adjustments to these actions and these in turn become increasingly complex so that in time the entire process in interiorized and generalized. . . First of all, there is a beginning of co-ordination between segments of overt behavior; next the adjustments become sufficiently far-reaching to make the actions increasingly internal; finally they take the form of mobile and reversible operational structures. This is essentially the same order of development as that which we have found in our previous investigations of development involving the logico-mathematical processes, and the definition of these stages is if anything more precise than in any of those others. [Inhelder & Piaget (1964), pp. 290-292]

Here something of importance to note is the reference to practical actions and their interiorization into schemes of reasoning. This involves the development of practical judgments

[5] In the technical terminology, the development of intelligence and reasoning in children progresses through four successive stages: sensorimotor intelligence, preoperational thought, concrete operations, and formal operations. The stage of concrete operations becomes observable around average ages 7 to 8 years.

of actions that mental physics likewise tells us are necessary *a priori* from general metaphysical laws. It is to be noted especially that formal logic as taught to students is not innate to childish reasoning but, quite to the contrary, is a *developed* ability.

Finally, in rather more precisely stated terms, Piaget tells us,

> [There] is a very primitive ordering structure in children's thinking, just as primitive as the classification structure. A very simple example is the structure of seriation. We have given children the following problem. First we present them will a collection of sticks of different lengths. The differences in length are small enough so that it takes a careful comparison to detect them; this is not an easy perceptual task. . . Then we ask the children to put them in order from the smallest to the biggest. Preoperational children approach this problem without any structured framework . . . That is, they take a big one and a little one, and then another big one and a little one, and then another big one and a little one, but they make no coordinations among these pairs of sticks; or they may take three at a time – a little one, a middle-sized one, and a big one – and make several trios. But they will not manage to coordinate all the sticks together in a single series. Slightly older children at the end of the preoperational stage succeed in putting all the sticks together in a series, but only by trial and error; they do not have any systematic approach. By contrast, children from about the age of 7 years have a totally different way of going about this problem. It is a very exhaustive systematic approach. They first of all find the very smallest stick, then they look through the remaining sticks for smallest ones left, then they look for the smallest one that is left again, and so on until the whole structure, the whole series, has been built. The reversibility implied here is one of reciprocity. When the child looks for the smallest stick of all those that remain, he understands at one and the same time that this stick is bigger than all the ones he has taken so far and smaller than all the ones that he will take later. He is coordinating here at the same time the relationship "bigger than" and the relationship "smaller than."
>
> There is even more convincing evidence of the operatory nature of this structure, and that is the fact that at the same time children become capable of reasoning on the basis of transitivity. Let us say that we present two sticks to a child, stick A being smaller than stick B. Then we hide stick A and show him stick B together with a larger stick C. Then we ask him how A and C compare. Preoperational children will say that they do not know because they have not seen them together – they have not been able to compare them. On the other hand, operational children, the children who proceed systematically in the seriation of the sticks, for instance, will say right away that C is bigger than A. According to logicians, seriation is a collection of asymmetrical, transitive relationships. Here we see quite clearly that the asymmetrical relationships and the transitivity do indeed develop hand in hand in the thinking of small children. It is very obvious, moreover, that the structure here is one whose reversibility is reciprocity and not negation. The reversibility is of the following sort: A is smaller than B implies B is larger than A, and this is not negation but simply a reciprocal relationship. [Piaget (1970), pp. 28-30]

In this specific summary we see precisely the manifestation of what was said above, namely that cognition of a descending series by episyllogism does *not* automatically bring with it cognition of the ascending series, and that cognition of the latter requires a second synthesis, namely a prosyllogism.

Figure 4(C) illustrates the next step in the synthesis, namely the *coordination* of the new concept ($S \rightarrow PC$) with RC. Note in the Inhelder quote above as well as the Piaget (1970) quote that coordinations intermingled with ordering are observable behaviors in the children they studied. Despite the fact that coordination $PC > RC$ already exists in the manifold, this is not the same thing as the coordination $(S \rightarrow PC) > RC$. A distinct synthesis of coordination is required. Finally, figure 4(D), the episyllogism is completed by the recognition of the coordinated structure just formed into a series structure $(S \rightarrow PC) \rightarrow RC$.

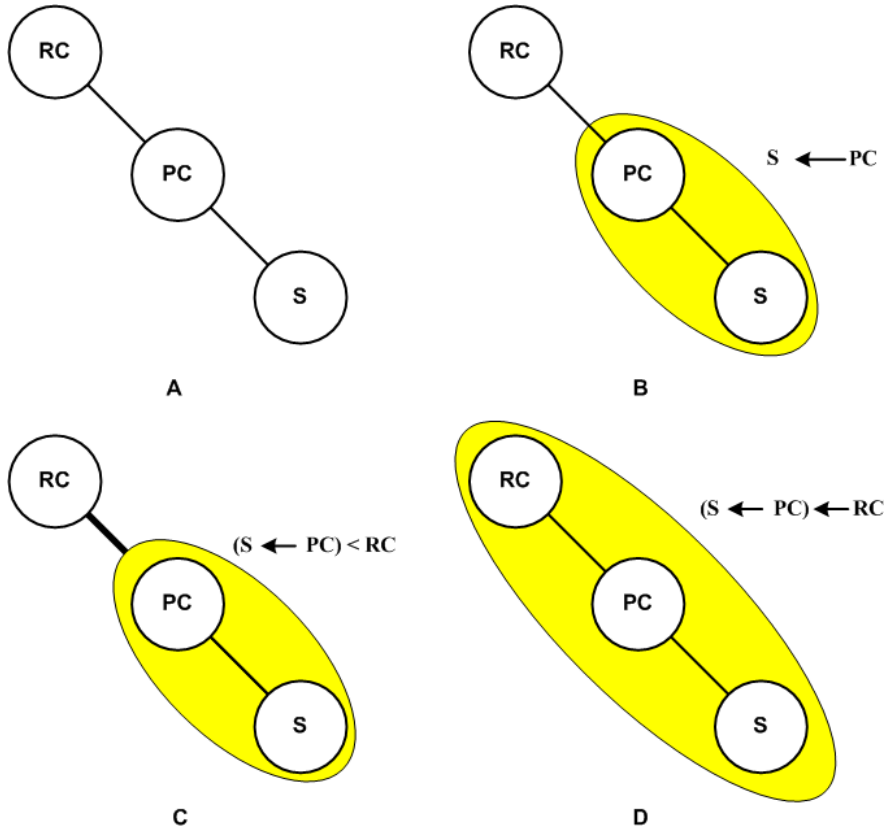

Figure 5: Illustration of synthesis by prosyllogism of a series (S ← PC) ← RC. S is the subject concept, PC is its proximate condition, and RC is its remote condition. The prosyllogism is a mirror image of figure 4, beginning again with S but this time ascending the structure toward the remote condition RC.

Figure 5 illustrates the steps in the synthesis of a prosyllogism. Here the Organized Being forms a new concept, that of the series (S ← PC) ← RC, by ascending the manifold of concepts from S, the subject term, to its remote condition RC. The synthesis process is a mirror image of figure 4 with the key difference being the reciprocal relationships, e.g. S ← PC vs. S → PC. The relationships in the prosyllogism are understood as the reciprocal relationships to that of the episyllogism, differing in the transcendental schema[6] of the intuition of (S ← PC) ← RC vs. that of (S → PC) → RC and the material difference and placement of the constituent concept terms within the structures. Immediate reciprocity of relationship occurs for the case of an episyllogism (S → A) → B that is either preceded by or later followed by a prosyllogism (B ← A) ← S, i.e. when the two syntheses involve the same constituent concepts in the manifold of concepts. Kant described this in logical terms in the following way:

> In a series of composite inferences one can close in a double manner, either down from the ground to the consequence or upwards from the consequence to the ground. The first is done through episyllogism, the other through prosyllogism.
>
> The episyllogism is namely that inference in the series of inferences whose premise becomes the conclusion of a prosyllogism, thus of an inference which has the premise of the former [the episyllogism] in the conclusion. [Kant (1800), *Logik*, 9: 134]

One can see that the material contexts and meanings of concepts is essential in the

[6] see Wells (2009), chapter 5, for an explanation of the transcendental schemata

understanding of what is represented in these structures. Kant's Logic is not, as many scholars have thought, a mere rehashing of so-called Aristotelian logic[7], i.e. a mere logic of form. It is instead a logic of *meanings*. As Kant himself said, it is not a dry and lifeless organon but, rather, an expression of the laws of thinking and reasoning *as human beings think and reason*. Polysyllogisms must be understood in this context. For example, Kant tells us

> All concepts as to matter are either given (*conceptus dati*[8]) or made concepts (*conceptus factitii*[9]). The former are given either *a priori* or *a posteriori*.
>
> All empirical, or given *a posteriori*, concepts are called *concepts of experience*, [all] given *a priori* [are called] *notions*. [Kant (1800), *Logik*, 9: 93]

In matter as much as in form, the representations contained in concepts that go into the making of other concepts affect the form and structure, as well as the root meanings, of the concept that is synthesized. This brings us to the next Critical consideration.

IV. Judgmentation and the Motivational Dynamic

A proper real understanding of polysyllogisms requires us to call upon the mental physics of the processes of judgmentation, reasoning, thinking, and conceptualization in *nous*. Figure 6 shows the organized structure and processes involved in this (Wells, 2009).

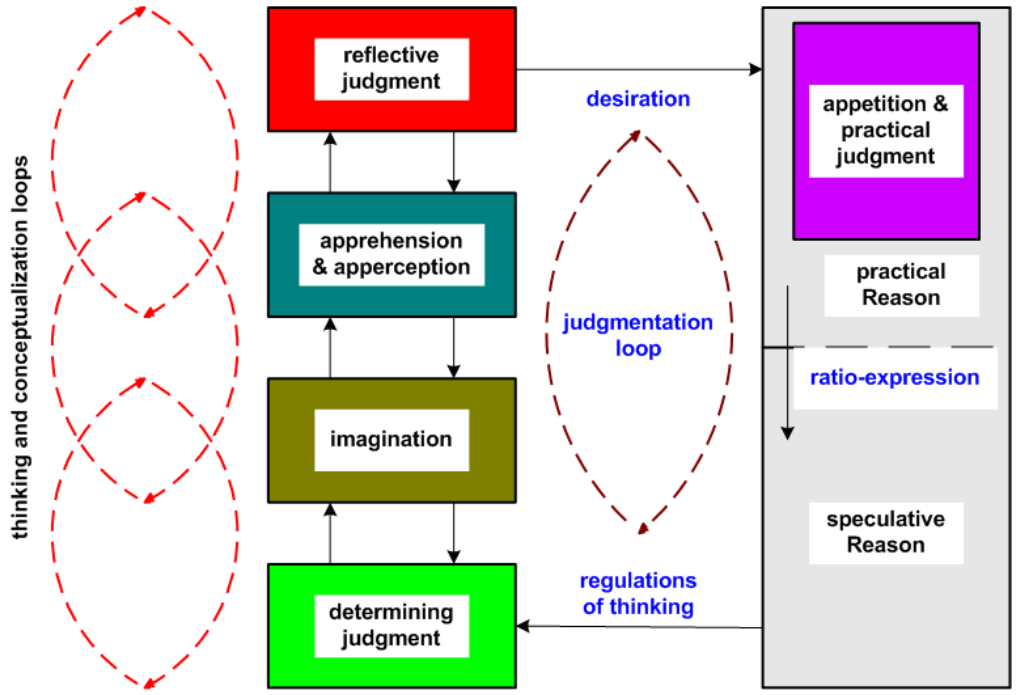

Figure 6: Organized structure of reasoning, thinking, and judgmentation cycles in *nous*.

[7] Aristotle's "science of demonstration" and its companion "dialectics" is also not "Aristotelian logic" as that term came to be interpreted by the Neo-Platonists and the medieval Scholastics. Like Kantian Logic, Aristotle's "logic" is heavily laced with metaphysics – ontology-centered in Aristotle's case, epistemology-centered in Kant's case. The numerous famous issues scholars debate in regard to the interpretation of Aristotelian logic (see, e.g., Smith, 1995) can really only be resolved by taking his metaphysics seriously. Kant's Logic cannot be correctly interpreted at all once it is divorced from Critical epistemology.

[8] "acts giving a concept"

[9] "the concept that may have been habitually made"

It is not impertinent to note that a number of signal processing anatomies having strong *prima facie* resemblances to the structure of figure 6 were proposed and qualitatively described over three decades ago by Grossberg in an epic if opaque paper (Grossberg, 1978, pp. 318+). These signal processing anatomies were proposed on the basis of psycho-mathematical arguments and are outgrowths of Grossberg's theory of adaptive resonance (Grossberg, 1976a, b). As of this date, Grossberg's models have not been subjected to any Critical analysis, and the clear ontology-centeredness of his basic arguments does make it necessary to apply Kant's Copernican turn in perspective to an epistemology-centered applied metaphysic for their proper assessment in order to bring them into alignment with proper Critical doctrine of method (Wells, 2011). Nonetheless, there is enticingly strong reason to speculate at this time that Grossberg's adaptive resonance theory and its functional structures may prove to be of great importance in the Critical development of brain-theory. As the structure of figure 6 derives directly from mental physics, one reason for interest in Grossberg's anatomies is a suggestive inference of analogy presenting as a possibility that fundamental structures and capacities in the Critical mathematics of brain-theory might possess the features of scaling and self-similarity to at least some significant degree. Certainly there is a significant degree of congruence between Kantian ideas, such as harmonization and the free play of imagination and understanding, and Grossberg's idea of adaptive resonance. Furthermore, the layer-by-layer approach to elucidating the consequences of mental physics, such as is represented in this paper, is a method with close kinship to Grossberg's method of minimal anatomies from embedding field theory (Grossberg, 1972).

Two characteristics of the making of a polysyllogism are clear. The first is that the synthesis is a mental action under the regulation and control of the process of pure Reason (figure 6; Reason overall is the combination of practical Reason and speculative Reason). This action is what we commonly call *reasoning*, which mental physics defines as the process of Self-regulation of the general process of judgmentation. *Judgmentation* is the overall process of exercising reasoning, determining judgment, imagination, the synthesis in sensibility, reflective judgment, and the regulation of motoregulatory expression. Judgment in general is the act of subsuming a particular representation under a general rule; as figures 4 and 5 illustrate, each step in the synthesis of a polysyllogism involves the making of a judgment. The synthesis in sensibility is a complex process involving apperception (the synthesis of unity in the Organized Being's state of empirical consciousness) and apprehension (synthesis of the data of the senses to produce representation of intuitions and affective perceptions). Motoregulatory expression is co-determination of the capacities of *soma* and *nous* for realizing the agency of the Organized Being. It belongs to the logical division of *psyche* (which is why it is not shown in figure 6) and is the logical complement of receptivity in sensory impression. Noetic changes in representation have corresponding changes in the appearances of *soma*, and these correspondents include changes in such observable physical phenomena as neural signaling in the brain, hormonal actions of the endocrine system, etc. Motoregulatory expression is controlled and regulated by the joint actions of reflective judgment and practical Reason.

Although the actual synthesis of concepts that occurs during the synthesis of a polysyllogism is carried out by means of the interactions taking place between determining judgment, the process of imagination, the process of apprehension and apperception and reflective judgment in figure 6 (the interaction cycles or "loops" labeled thinking and conceptualization), the *overall* unity in the action is regulated and controlled by the cycle labeled the judgmentation loop. This is the mental Nature of reasoning as an action. Determining judgment does not determine its own employment. Rather, it is a process under the employ and direction of pure Reason (the power to Self-regulate all non-autonomic acts of the Organized Being). This regulation and orientation of acts of determining judgment is denoted "regulations of thinking" in figure 6, and Critical meta-physics calls these regulations the *transcendental Ideas* of pure speculative Reason. In the specific case of a polysyllogism, the particular regulation involved is called the *cosmological*

Idea, the general acroam of which is: *absolute completion in the series of conditions*.

Now, pure Reason is an affectively cold and cognitively dark process. Reason knows no objects of sensuous experience and feels no feelings of affectivity. Its role is that of the master regulator of *nous* and some (but not all) of the properties Freud used to describe his idea of the superego likewise describe to some degree the properties of the process of pure Reason[10]. The affectively cold and cognitively dark mental Nature of pure Reason has consequences for how one must epistemologically regard the idea of "absolute completion" in the cosmological acroam. Initiation of the synthesis of a polysyllogism *does not mean* that the Organized Being will undertake to run through its entire chain of coordinate concepts from top-to-bottom (or vice versa) in the manifold of concepts. Such an interpretation is absurd and is contrary to common human experience of what each of us do when making an inference of this kind. The reasoning process is carried out only until the practical appetite that initiated the action is satisfied by the mental state that is eventually produced. Simply put, the action is maintained until either equilibrium is achieved or the cycle of activities is ruptured (figure 3). This is called *satisficing behavior*.

This brings us to the second characteristic of the making of a polysyllogism. The synthesis of a polysyllogism brings about a change in the manifold of concepts. It is, therefore, a synthesis of accommodation in this manifold and results in new assimilations if the activity cycle is not ruptured (that is, unless the Organized Being "cannot figure it out" and "gives up" the effort). The satisfaction of the appetite of Reason, in other words, is realized when a new equilibrium is achieved. Thus, completion of the synthesis of the polysyllogism is an adaptation in the context in which that term was earlier explained. The notion of absolute completion in the cosmological acroam must be understood as a *practical* (not speculative) completion, i.e., what Critical epistemology calls *practical perfecting*. Practical judgment judges that such an achievement is realized, and this judgment is based upon the successful accommodation of perceptions such that practical conditions mandated by the master regulative law of pure Reason are all met. The chief condition is that the intensive magnitude of feelings of *Lust* ("loost") and *Unlust* ("un-loost") in affective perception are judged to be negated by the process of reflective judgment. This state is the judicial mark of judgment that equilibrium is achieved. The capacity for achieving this is what mental physics calls *the motivational dynamic* (Wells, 2009, chapter 10).

The *Existenz* of the motivational dynamic is represented in 2LAR form by figure 7. Its four headings of Quantity, Quality, etc. are here labeled *want*, *drive*, *drive state*, and *type-of-motive*. The synthesis of a polysyllogism occurs when the motivational dynamic is determined by the four specific *momenta* under these headings of

{expression of interest, affirmation of reevaluation, enforcement of law, groping for equilibrium}.

The detailed explanation of these *momenta* is provided in (Wells, 2009, chap. 10). A summary of this must suffice for the purposes of this paper.

Want is representation *in concreto* of a condition for adjusting accommodation of perception through behavior grounded in psychological causality according to particular practical *a priori* standards. That the power of pure Reason necessarily contains a standard gauge[11] for perfecting the state of the Organized Being was proved by Kant in *Critique of Pure Reason* (B597-599). He called the representation of such a standard a *transcendental Ideal*.

[10] Freud's tripartite model of the id, ego, and superego has some rough alignment with the processes of reflective, determining, and practical judgment in figure 6. However, Freud's theory is not objectively valid overall and I am in no way suggesting that an understanding of this theory is equivalent to an understanding of the mental physics of *nous*. Mental physics can be used to correct errors in Freud's theory; Freud's theory can not be used as an explanation of mental physics.

[11] *Richtmaß*

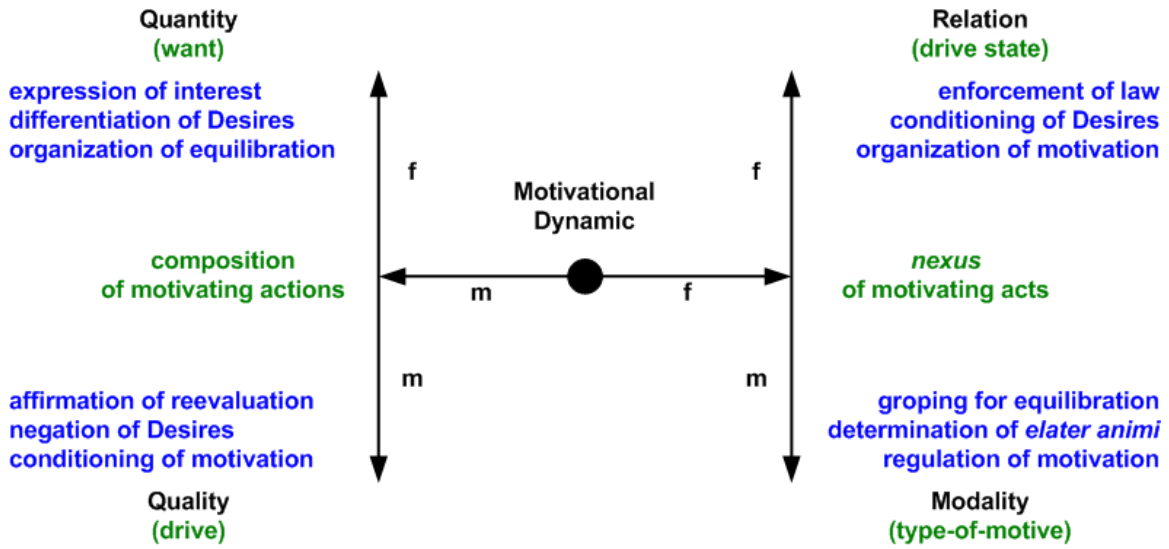

Figure 7: 2LAR structure of the motivational dynamic.

Expression of interest is ratio-expression[12] by pure Reason. It is an act of re-evaluation, i.e. acting to change perception so as to bring practical appetition into alignment with the regulatory directives of equilibrium. The act is initiated by a disturbance brought on by an act of aesthetical reflective judgment in marking an affective perception as a mark of judicial incompleteness. The perception serves as an energetic for taking rational action. The aesthetical notion of Quality for such a judgment is called the *momentum* of sublimity (Wells, 2009, chapter 8). Reason's interest is grounded in the *formal* expedience in removing or negating this disturbance. An act of reflective judgment triggers the process of ratio-expression and synthesis.

Affirmation of reevaluation is Quality in the motivational dynamic specifying, in combination with Quantity as expression of interest, that the act of the Organized Being is the act of ratio-expression. The regulation of practical Reason is directed at the process of determining judgment and the manifold of concepts because of this composition.

The action of Reason is directed at the Organized Being's internal organization of its manifold of concepts, and this is what is meant by the function of enforcement of law. However, because pure Reason knows no objects of sense and feels no feelings, the action it takes can only be a seeking or *groping* for a means to restore judicial balance and equilibrium. Hence the *momentum* of Modality here in the motivational dynamic is called groping for equilibration.

All of this, however, *practically* accomplishes nothing more than to arouse the Organize Being by consciousness of a disturbance and to set up *a posteriori* (after the event) an indication of the direction in which to orient determining judgment through ratio-expression. In the next section we will call this first step in the sequence of synthesis *the construction of consciousness*, and this kind of act of synthesis is adjudicated by reflective judgment in the making of a new *general* concept (in particular, the series S → PC in an episyllogism or S ← PC in a prosyllogism). This one act, by itself, does not complete the synthesis because it does not yet satisfy Reason's demand for equilibrium under the regulation of the cosmological Idea. This is because the new concept, e.g. S → PC, is not yet *coordinated*. It has no context established yet and therefore still lacks any *real meaning*. This is a cognitive lack, constituting a different type of disturbance, and its resolution requires the synthesis of what in the next section is called *the construction of*

[12] i.e., the determination of the employment and direction of determining judgment through the transcendental Ideas. Speculative Reason is the capacity for ratio-expression.

reasoning.

A glance back at figure 6 shows that neither determining judgment nor sensibility has an *immediate* connection with Reason. Rather, this feedback must pass through the process of reflective judgment (by means of the manifold of Desires in representation produced by this process). The cognitive lack in the manifold of concepts resulting from the first act is formally inexpedient under the practical laws that regulate non-autonomic actions. Judgment of a resolution of this condition falls to practical judgment in Reason, but the nature of the action required has now been changed as a consequence of the previous act. The next act is an act of *reconciliation* and the motivational dynamic for this act is (Wells, 2009, chapter 10)

{differentiation of Desires, negation of Desires, conditioning of Desires, determination of *elater animi*}.

To reconcile is to bring into harmony, i.e., to make compatible. Critical harmonization is making diverse representations be compatible and homogeneous with each other such that they can be combined. The reconciliation being effected here is one between the manifold of Desires in reflective judgment and the practical manifold of rules in pure practical Reason. The harmonization in the particular case being considered here pertains to the form of coherence in representation. The reason for this is as follows.

If the recognition of a new concept (S $\rightarrow$ PC), an act which subsumes concepts S and PC under a general rule (S $\rightarrow$ PC), had fully satisfied the practical conditions of equilibrium, then the act of recognition would have been sufficient to bring the synthesis process to closure. In this case, however, the Organized Being would not have been effecting an episyllogism but merely an inference of judgment. To effect the synthesis of a polysyllogism means that some inexpedience with the practical purpose of pure Reason is still present after the recognition of the new concept. Because the act of reflective judgment that produced the recognition (S $\rightarrow$ PC) is *formally* expedient for practical Reason, any remaining inexpedience must be *materially* inexpedient, which means that although the form of affectivity (desiration) in the manifold of Desires coheres with the requirements of the practical manifold of rules, some matter of affectivity (desire) is *unsuited* for assimilation under the Organized Being's rule structure.[13]

What specifically is materially unsuitable is not important for our purposes here. It is enough that the unsuitability still exists in perception and the unsuitable parts of the manifold of Desires are *disvalued* by practical Reason. Recall that the first act described above carried the Modality of groping for equilibration in the motivational dynamic. Metaphorically, pure Reason must "know what to do" the next time whatever stimulus that triggered the cycle is encountered again, and it has not yet found this knowledge. Furthermore, although the connection PC > RC already exists in the manifold of concepts (and therefore this representation is formally expedient), failure to assimilate perception in the existing manifold of rules means that something is still lacking and further accommodations are required. Desires must be differentiated into not-unsuitable and unsuitable ones, the unsuitable ones must be negated (in Freud's terminology, a "censorship function"), and affectivity conditioned in the external Relation between the manifold of Desires and the manifold of rules. The metaphysical *nexus* in the motivational dynamic subsists in asserting the *Dasein* of a cause of the disturbance to the Organized Being's equilibrium, and this is called the determination of an *elater animi* or "driver of mind." The act of synthesis in response to this motivation is called *the construction of reasoning*. It amounts to the formation of a new

[13] Connections of Desires are never judged, by practical judgment, as *suited* for the practical manifold of rules. Such a judgment is impossible for a faculty of judgment that knows no objects and feels no feelings. Rather, the constitution of the manifold of Desires is differentiated into those Desires that are unsuitable and those that are not-unsuitable. The latter are those that can be transformed into practical appetites without coming into conflict with the manifold of rules.

practical maxim in the Organized Being's manifold of rules, for which the determinant judgment of the combination (S → PC) > RC is merely an expedient accommodation in perception.

A simple judgment of coordination requires nothing more than the free play of imagination and understanding, and so the combination (S → PC) > RC is a local act of understanding. Yet this is not the final act of the overall synthesis because by itself it does not allow the validation by practical Reason of its accommodation of the manifold of rules. One more action is needed to lead to equilibrium and bring the cycle to a close. The motivational dynamic for this act is

{organization of equilibrium, conditioning of motivation, organization of motivation, regulation of motivation}

and the process of judgmentation closes with a *synthesis of experience*. The last act of judgment here belongs to the process of determining judgment in the construction of the concept of the series (S → PC → RC). Note that this final act of judgmentation bears in the motivational dynamic the Quantity of integration, Quality of subcontrarity, transitive Relation, and the determining factor in Modality by which judgmentation is held-to-be completed.

It is quite apparent that these epistemological considerations just discussed provide a context and content to the process of synthesizing a polysyllogism far beyond anything that mere formal logic (whether symbolic or classical) provides. This is an illustration that Kant's transcendental Logic is a Logic of meanings and not another dreary instrument for a mere logico-mathematical organon. This was something Kant himself noted in *Critique of Pure Reason*. Still, the multiple considerations that can only be summarized here are rather abstract and need placing in a closer personal relationship with human experience. A consideration first enounced by Santayana can help us along towards accomplishing that:

> Reflection is pregnant from the beginning with all the principles of synthesis and valuation needed in the most comprehensive criticism. So soon as man ceases to be wholly immersed in sense, he looks before and after, he regrets and desires; and the moments in which prospect or retrospect takes place constitute the reflective or representative part of his life, in contrast to the unmitigated flux of sensations in which nothing ulterior is regarded. Representation, however, can hardly remain idle and merely speculative. To the ideal function of envisioning the absent, memory and reflection will add (since they exist and constitute a new complication in being) the practical function of modifying the future. Vital impulse, however, when it is modified by reflection and veers in sympathy with judgments pronounced in the past, is properly called reason. Man's rational life consists in those moments in which reflection not only occurs but proves efficacious. What is absent then works in the present, and values are imputed where they cannot be felt. Such representation is so far from being merely speculative that its presence alone can raise bodily change to the dignity of action. Reflection gathers experiences together and perceives their relative worth; which is as much as to say that it expresses a new attitude of will in the presence of a world better understood and turned to some purpose. The limits of reflection mark those of concerted and rational action; they circumscribe the field of cumulative experience [Santayana (1905), pp. 2-3].

The synthesis of a polysyllogism is merely one exemplar among uncounted ones of what Santayana calls "reflection," and its acts of synthesis are Critical representations of what he called "vital impulse." This leaves us, though, with the question: what determines if the synthesis is to be an episyllogism or a prosyllogism? Formally, as noted earlier, the process is the same except for the terms plugged into the judgments and the direction of the synthesis (regressive rather than progressive). Perhaps it is evident that the distinguishing characteristic between the act of making an episyllogism vs. the act of making a prosyllogism cannot belong to objective understanding (because the synthesis is itself a means of achieving objective understanding) nor can it belong to pure practical Reason (which knows no objects of experience). This leaves only the capacity for

affectivity as the factor that motivates the distinction in the beginning of the cycle of judgmentation between S > PC ("S understands PC") in the episyllogism versus S < PC ("S stands under PC") in the prosyllogism. It depends upon the particular inexpedience that triggered the synthesis, and we must seek its source in mental continuity, specifically as *continuity in perception*, between *nous* (which knows feelings of *Lust* and *Unlust*) and *psyche* (which is the faculty of the organization of *Lust per se*). In mental physics this function of continuity is called the aesthetic Idea, the 2LAR of which is shown in figure 8 (Wells, 2009, chapter 7).

The number of possible situations and circumstances that come into play at this point is so vast that pursuing the issue further in the abstract is rather pointless in the context of this essay. For our current purposes, let it suffice to say: (1) that the aesthetic Idea is active in the synthesis of reproduction in imagination by which concepts in the manifold of concepts enter into the synthesis of apprehension (figure 6); (2) that it is active in the orientation of aesthetical reflective judgment in establishing the affective energetics of motivation; and (3) that its continuity function between *nous* and *soma* is the function which binds the reflective Quality of judgment to the Organized Being's noetic capacity to produce or suffer effects.

The aesthetic Idea can be regarded as a regulating function for local perception in the thinking and conceptualization loops in figure 6 (but not for the judgmentation process overall because this regulation belongs to pure Reason). It is an affective value function such as Piaget described in the following words:

> [We] have argued that values play a distinct role in primary actions and are evident from the moment the subject begins to relate to the external world. In learning to walk, for example, previous success or failure can be seen to influence the child's interest and endeavor. This clearly indicates that some sort of self-estimation is taking place. As early as the sensorimotor level, then, the child draws not only practical knowledge but also confidence about himself from previous experience. These feelings are, in a way, analogous to feelings of superiority or inferiority . . . They will play a large part in determining the finality of action proper and soon will be extended to all of the interpersonal relationships that appear with imitatory behaviors. . . One sees, therefore, that values, although difficult to discern at first, are organized bit by bit into a system that is broader, more stable, and distinct from the system of energetic regulations. [Piaget (1981), pg. 32]

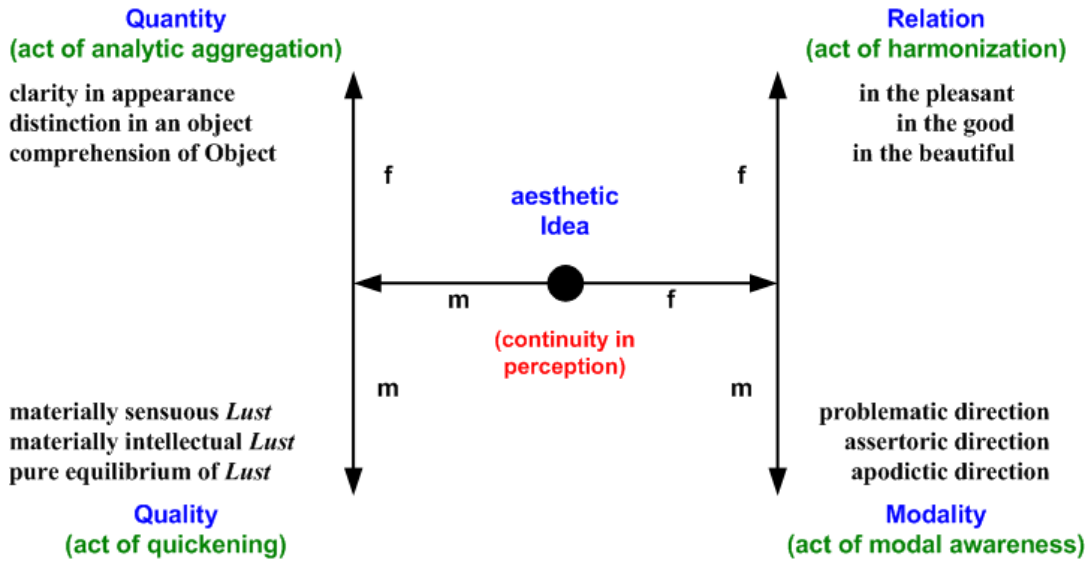

Figure 8: 2LAR organization of the aesthetic Idea in sense.

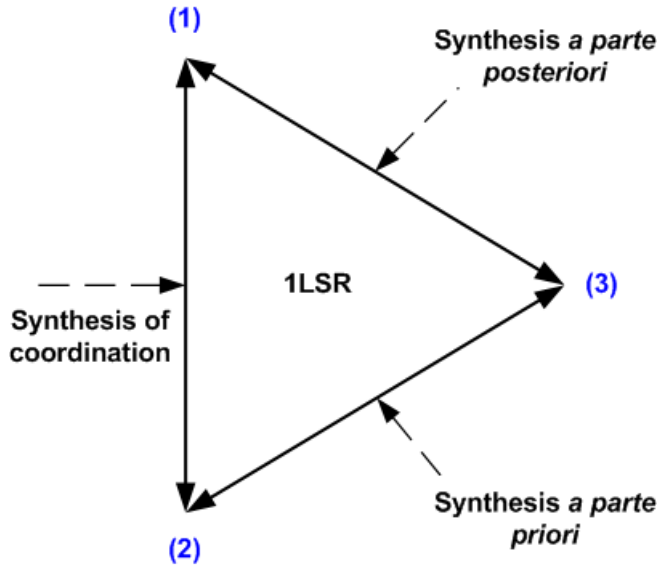

Figure 9: The 1LSR organization of acts of the three basic acts of synthesis.

V. The 1LSR and the Synthetic Cycle

Kant's transcendental Logic, as noted, is not an organon of logic. However, putting its theory to use in mental physics' investigations is greatly aided by having the instruments and tools of an organon. A great contribution to this sphere of knowledge was made by Palmquist in 1993 with his introduction of the *first level synthetic relation* or 1LSR. Figure 9 illustrates the 1LSR representation. Palmquist introduced it as a theoretical tool for analyzing acts of synthetic integration in Kant's Critical system. He wrote,

> Synthetic operations differ from analytic operations in two important respects. First, the relationships between the components of an analytic operation are static . . . whereas those between the components of a synthetic operation are dynamic (i.e., each component can be understood only if it is viewed as part of the development process in which an integrated whole is being constructed). To reflect this difference I will refer to synthetic operations . . . as synthetic *integration*. . . . As Wolff[14] rightly says in describing Kant's theory: 'Knowledge is an activity, not a state of mind. Judgment can be understood only if we first analyze judging' . . . thus a simple (first-level) synthetic integration consists of three one-term components . . . An appropriate geometrical model for this first-level synthetic relation (1LSR) is the triangle [Palmquist (1993), pp. 83-85]

In an act of synthetic integration, two terms are opposed (e.g. a ground and a grounded, or a determinable and a determination, or a condition and a conditioned) and a third term is produced which is a synthetic unification of these. Philosophers will recognize in this the basis for Hegel's famous synthesis triad of "thesis-antithesis-synthesis." There are, as figure 9 illustrates, three possible types of synthetic integration operations (one for each Standpoint in Critical epistemology) and the *process* is primarily adjudicated by one of the three processes of judgment. We can represent these using the "formula convention" employed in chemistry:

(1) + (2) → (3); synthesis of coordination (practical Standpoint, practical judgment)
(1) + (3) → (2); synthesis *a parte posteriori* (judicial Standpoint, reflective judgment)

[14] Wolff, Robert Paul (1963), *Kant's Theory of Mental Activity*, Cambridge, MA: Harvard University Press, pg. 323.

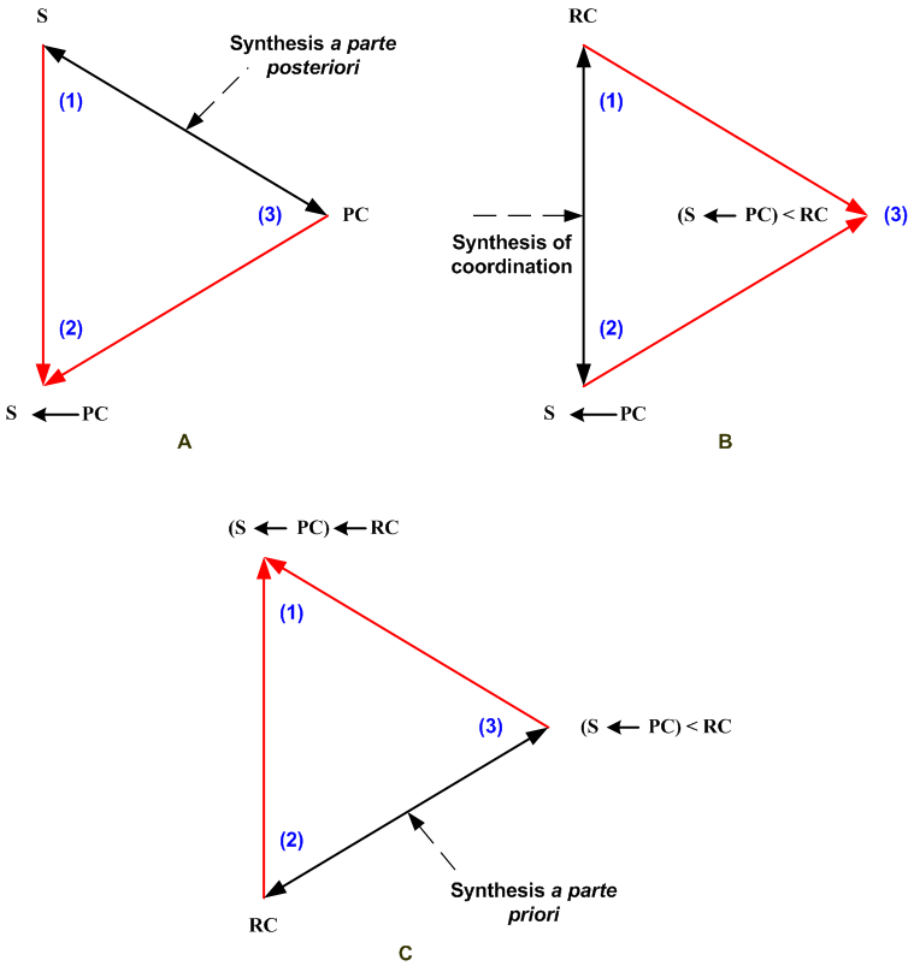

Figure 10: 1LSR representation of the three acts of synthesizing a polysyllogism. This figure depicts, by means of the concept terms indicated at the corners of the triangle, the synthesis of a prosyllogism. The 1LSR sequence is the same for an episyllogism except for these material terms. The red arrows denote the movement of the synthetic integration, e.g., (2) + (3) → (1) resulting in S ← PC ← RC in figure 10 (C). The arrow points from condition to conditioned. S ← PC ← RC reads "RC conditions PC conditions S."

(2) + (3) → (1); synthesis *a parte priori* (theoretical Standpoint, determining judgment).

We use the 1LSR to obtain a more precise statement of the syntheses discussed earlier. Figure 10 illustrates the cycle of judgmentation as the making of the prosyllogism advances from its initial synthesis *a parte posteriori* (***the construction of empirical consciousness***) to its final act of synthesis *a parte priori* (***the construction of experience***). Use of the 1LSR makes clear not only the principal adjudicating process at each step but also the Critical Standpoint from which the act of judgment must be viewed.

The prosyllogism synthesis begins with figure 10 (A) and some may find it strange that this is called synthesis *a parte posteriori* for two reasons. First, mere empirical consciousness (the representation that another representation is present and is to be attended to) does not yet qualify as an experience. Second, the adjudicating process here is reflective judgment, which is a merely subjective capacity of judgment and judges no objects *as* such. In what way, then, is it proper to say this is a synthesis on the side *following* experience (*a parte posteriori*)? Two things are required to properly understand this. First, one must take the viewpoint of the Organized Being himself and not the viewpoint of an outside observer. Whether the synthesis is to be called *a parte*

posteriori or *a parte priori* (on the side prior to experience) is "in the mind of the thinking Subject." Second, the synthesis is not a synthesis of mere cognition (objective perception) but, rather, a synthesis of knowledge[15], and so we must look at the question in that context. There are different kinds of knowledge delineated by Critical Standpoint and "cognition" (as a technical term) has two contexts: (1) objective perception as representation in intuition; and (2) objective perception as representation in a concept. The first belongs to aesthetics in sensibility and its proper Standpoint is the judicial Standpoint (the Standpoint adjudicated principally by the process of reflective judgment). The second belongs to empirical knowledge from the theoretical Standpoint (the Standpoint adjudicated principally by the process of determining judgment).

One must, therefore, approach the question beginning with Kant's distinction between knowledge *a priori* and knowledge *a posteriori*. In the first edition of *Critique of Pure Reason* he wrote,

> Experience is without doubt the first product that our understanding brings forth as it works up the raw stuff of sensuous sensations[16]. It is for this very reason the first information, and in its progress it is so inexhaustible in new instruction that the melting vat of life[17] in all future generations will never have any lack of new cognizance that can be gathered in this soil. Nevertheless it is by far not the only field to which our understanding can be confined. It tells us, to be sure, what is, but never that it must necessarily be thus and not otherwise. For that very reason it gives us no true universality, and reason, which is so hungry for this kind of knowledge, is more nettled than pacified by it. Now such universal knowledge, which at the same time has the character of inner necessity, must be clear and certain for ourselves independently of experience; hence one calls it knowledge *a priori*: while on the contrary that which is merely borrowed from experience is, as it is put, known only *a posteriori*. [Kant (1781), A:1-2]

Kant phrased this differently in the second edition of the *Critique* but without substantive change in the idea:

> There is no doubt whatever that all our knowledge begins with experience; for how else should the faculty of knowledge be awakened into exercise if not through objects that move our senses and in part themselves bring about representations, in part bring the activity of our understanding into movement to compare these, to connect or to separate them, and thus to work up the raw stuff of sensuous impressions into a cognition of objects that is called experience? *As far as time goes*, then, no knowledge in us precedes experience, and with this[18] is captured by all. [Kant (1787), B: 1]

Here we must understand that by "knowledge" Kant means *empirical* knowledge, i.e., knowledge of experience represented by concepts and pertaining to phenomenal objects of experience. His meaning would probably have been clearer if in the first sentence he had said "all our knowledge begins with experienc*ing*."

As for cognition, a cognition is an objective perception represented either as an intuition in

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[15] It is a nettlesome issue for the translator that the German word *Erkenntnis* translates into English both as "cognition" and as "knowledge" depending on the context of its usage. This is one of many issues that makes rendering Kant's work into English a time-consuming and difficult task.

[16] The phrase "sensuous sensations" (*sinnlicher Empfindungen*) is not redundant. Sensation is the matter of representation in a perception and sensational matter can come either from receptivity in *psyche* (in which case it is sensuous) or from the synthesis of reproduction in imagination (in which case it is spontaneous).

[17] This phrase translates Kant's obscure metaphor *zusammengefettete Leben*. There seems to be no exact American-English equivalent for this phrase, but its "flavor" seems similar to the American colloquialism of a "melting pot," as in, "America is the great melting pot".

[18] i.e., with experience

sensibility or as a concept in the manifold of concepts. The latter are nothing more and nothing less than *rules* by which the synthesis of reproductive imagination reproduces an intuition. The process of reflective judgment is tasked with, among other things, making what Kant elsewhere called *judgments of perception*[19], and such a judgment marks a representation as a knowledge representation but not as a representation of *phenomenal* knowledge. He calls a judgment that does the latter a *judgment of experience*. To make a judgment of this second type, the cognition must contain contributions from the manifold of concepts, these concepts originate from previous experiences, and it is in this way that the synthesis (1) + (3) → (2) in the first step of the synthesis is regarded as synthesis *a parte posteriori*. Palmquist was the first to recognize this subtle but crucial aspect of Kant's theory. He wrote,

> In the third *Critique*[20], by contrast, Kant's use of the phrase 'reflective judgment' is . . . equivalent to his former use of the phrase 'empirical employment of pure reason' . . . thus implying that the perspective which determines Kant's *standpoint* for examining such judgment in [the judicial Standpoint] is the empirical. . .
>
> A person who adopts an empirical perspective reflects upon particular objects of experience without attempting to 'go beyond' their nature as given in immediate experience. In empirical reflection as such there is no need to discriminate the respective roles of the knowing subject and the known object because the two are fused in experience. This continuity between immediate experience and knowledge resulting from empirical reflection is, no doubt, what leads Kant to make the (potentially misleading) claim that 'empirical knowledge is experience' . . . Strictly speaking, 'empirical knowledge' should denote only that synthetic a posteriori knowledge which arises out of empirical *reflection on* the objects of one's experience. [Palmquist (1993), pg. 122]

This first act of judgmentation is not sufficient to bring on equilibrium because the new concept (S ← PC) placed in the manifold of concepts by the synthesis of re-cognition in imagination is a "floating" concept, by which I mean a concept still lacking connections in the manifold that provide it with context and meaning. The object represented by the concept is still, at this point, a *non-real* object from the perspective of the Organized Being. This lack of completion constitutes a lack of equilibrium for the Organized Being, and so the synthesis must proceed at once to a second step *for the purpose* of remedying this lack. The process of determining judgment does not automatically "link up" the new general concept in the manifold because, strange as it may sound, determining judgment is not actually concerned with objects as phenomena but, rather, with rules for reproducing intuitions. Determining judgment does not determine its own employment but, instead, is oriented and directed by pure speculative Reason during the cycle of judgmentation. Thus the next step, the synthesis of coordination, is taken from the practical Standpoint and adjudicated principally by the process of practical judgment in pure practical Reason. This is the Standpoint for the synthesis (1) + (2) → (3) in figure 10 (B). This *construction of reasoning* provides the first connection of the new concept in the manifold. By doing so, the synthesis in judgmentation also prepares for the first raw meaning of this concept and *this meaning is a practical meaning*. *At root*, all meanings are *practical*.[21]

Concept (S ← PC) now has a context, under concept RC, from this step in the synthesis, but the coordination (S ← PC) < RC is not sufficient to provide *objective* meaning under regulation by the cosmological Idea. The transcendental schemata and categories of understanding (Wells, 2009, chapter 5) differ between concepts (S ← PC) < RC and (S ← PC) ← RC. In particular, the first concept is understood in Relation by the category of substance & accident while the second

[19] Kant (1783), 4: 297-302
[20] *Critique of the Power of Judgment* (1790)
[21] Piaget and Garcia (1987)

is understood under the category of causality & dependency. It is the latter *a priori* notion of Relation that establishes the relationship of condition to conditioned, and the cosmological Idea is the regulation of pure Reason that seeks practical completion in a series of conditions. As this is the regulation in play during a polysyllogism, we must conclude that the condition of equilibrium is not achieved, and therefore the noetic adaptation is not completed, by the simple coordination of RC and (S ← PC). One more accommodation, synthesis (2) + (3) → (1), is needed to close the cycle and bring the motivational dynamic to rest. With this the polysyllogism is completed and the manifold of concepts contains the new concept (S ← PC ← RC).

VI. Concluding Remarks

The transcendental Logic illustrated in this paper by the making of polysyllogisms points out the crucial need for, and nature of the development of, Critical mathematics that was first noted in (Wells, 2006, chapter 23). This paper has demonstrated how consideration of the rational *a priori* acroams of the transcendental Ideas of pure Reason, the mental physics of judgmentation, and the motivational dynamic interact to introduce into the static formalism of conventional logic a dynamical element, the flavor of which Aristotle captured in his idea of *enérgeia* in Nature. That Aristotle's idea was teleological in character is quite relevant in mental physics because the ratio-expression of pure Reason and the form of reflective judgment are both teleological in their fundamental natural character (Wells, 2006, chapter 16). This is the case for all of psychology as well as for all objectively valid social-*natural* sciences. Teleology has no place in physics, biology or chemistry, but *is* a key principle in all social-natural sciences. It is for this reason that psychology, economics, political science, and the other so-called social sciences *differ in kind* from physics, biology, and chemistry. Biology, chemistry and engineering can, and no doubt will, continue to make important advances without the immediate aid of Critical mathematics – the case is much more in doubt today for physics – but the social sciences will not take one single step towards becoming *proper* social-natural sciences without it. Santayana could hardly have been more prescient than when he wrote,

> At the opposite pole from immediacy lies intelligibility. To reduce phenomena to constant elements, as similar and simple as possible, and to conceive their union and separation to obey constant laws, is what a natural philosopher will inevitably do so soon as his interest is not merely to utter experience but to understand it. Democritus brought this scientific ideal to its ultimate expression. By including psychic existence in his atomic system, he indicated a problem which natural science has since practically abandoned but which it may some day be compelled to take up. The atoms of Democritus seem to us gross, even for chemistry, and their quality would have to undergo great transformation if they were to support intelligibly psychic being as well; but that very grossness and false simplicity had its merits, and science must be forever grateful to the man who at its inception could so clearly formulate its mechanical ideal. That the world is not so intelligible as we could wish is not to be wondered at. In other respects also it fails to respond to our ideals; yet our hope must be to find it more propitious to the intellect as well as to all the arts in proportion as we learn better how to live in it. [Santayana (1905), pp. 16-17]

Physicist Henry Margenau likewise recognized paradigm-imposed limitations of the so-called "natural sciences" (physics, biology and chemistry) and was insightful enough to perceive an unfilled lack in general science. He wrote,

> This last section has dealt with attempts at augmenting physical reality and with their prospects, but attention was confined to problems of factual apperception and knowledge. All these attempts share with the philosophy developed in this book the quality of being insufficient for the representation of human experience in at least two major fields, the areas of feeling and of value judgment. It is of course not for the philosopher to say that

> these defects are necessary and are traceable to *fundamental* limitations of the scientific method; nor is the scientist free to deny it. The proper attitude on the part of the philosopher is open-mindedness, while it is clearly the job of the scientist to recognize the insufficiencies and then to attempt their removal. . .
>
> Indifference of physical reality with respect to the affective qualities of experience is generally recognized and needs no emphasis here. . . But here again it is not certain that the processes of scientific understanding, and with them the ideas of reality, cannot be enlarged sufficiently to express that added richness of experience. There was a time in the history of the young science of colorimetry when representation was bound to the two dimensions of hue and brightness, and men had reason to suspect that this science would never render adequate account of the living vividness of actual color sensations. Today, with the introduction of saturation as a third variable in the description of color, we are very much nearer that goal. And so it may happen in the representation of emotions.
>
> As to values, however, the story is somewhat different. In my view, which is not the only one consistent with the epistemological theory presented in this book and therefore cannot be argued without an appeal to extra-scientific convictions, natural science contains no *normative* principles dealing with ultimate goals; physical reality is the quintessence of cognitive experience and not of values. . . To know physical reality is to know where to look when something is wanted or needed to be seen; it is to be able to cure when a cure is desired, to kill when killing is intended. But natural science will never tell whether it is good or bad to look, to cure, or to kill. It simply lacks the premise of an "ought." [Margenau (1977), pp. 463-465]

Nor, and even especially, are mathematics and logic free to remain aloof from those necessary developments which the advancement of human science calls upon them to provide. W.V. Quine wrote,

> We must thus distinguish two senses of logic, a broader and a narrower; logic in the narrow sense comprises those truths which contain only the so-called logical vocabulary essentially, while logic in the broader sense includes both logic in the narrower sense and discourse about it. . .
>
> Mathematical logic has long been applied, but the most important applications are surely still to come. The usefulness of a theory is not to be measured solely in terms of the application of prefabricated techniques to preformulated problems; we must allow the applicational needs themselves, rather, to play their part in motivating further elaborations of the theory. The history of mathematics has consisted to an important degree in such give and take between theory and application. Much of the promise of mathematical logic for science lies in its potentialities as a basis from which to construct subsidiary techniques of unforeseen kinds in response to special needs. [Quine (1981), pp. 3, 8]

Mathematicians and logicians, whose professional activities receive patronage from the public at large, are not at liberty to hold themselves aloof from the needs of scientific advancement or hide behind a self-serving veil of professed contempt for the applications of their arts like determined spinsters and old bachelors lining the walls at a public dance. At least, they are not at liberty to do so and then to protest if the public they decline to serve subsequently decides to no longer support them. The axiom systems of modern mathematics and of mathematical logic lack real objective validity. Mathematics and mathematical logic are organons of science, and science therefore calls upon them to supply instruments that can be applied with real objective validity to Nature. Mathematical axioms are not, as the ancient Greeks believed, self-evident truths about nature. Modern day mathematics already recognizes this. But a Critical system of mathematical axioms *is deducible* with objective validity from the acroams of Critical metaphysics. It is and must be the task of these sciences, and of philosophers, to repair their foundations, deduce those Critical axioms, and develop the tools and methodologies of a Critical organon of mathematics.

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