Some Rules of Root Locus Construction

We assume the system has n poles and $m < n$ zeroes. We let K be the gain term of the compensator.

RULE 1 (Loci branches): Continuous curves, which comprise the branches of the locus, start at each of the n poles, for which $K = 0$. As $K \to \infty$ the branches of the locus approach the m zeroes. Zeroes do not move. Locus branches for excess poles (i.e. pole $m + 1$ to n) approach infinity.

RULE 2 (Real axis segments): The locus includes all points along the real axis to the left of an odd number of poles plus zeroes.

RULE 3 (Asymptotic angles): As $K \to \infty$ the branches of the locus become asymptotic to straight lines with angles

$$\theta = \frac{180° + i \cdot 360°}{n - m}$$

for $i = 0, \pm 1, \pm 2, \ldots$ until all $(n - m)$ angles not differing by 360° are obtained.

RULE 4 (Centroid of asymptotes): The starting point on the real axis from which the asymptotic lines radiate is given by

$$\sigma = \frac{\sum open\ loop\ poles - \sum open\ loop\ zeroes}{n - m} \ .$$

This point is called the centroid of the asymptotes.

RULE 5 (Breakaway points): Roots leave the real axis at a gain K which is the maximum possible value of K in that region of the real axis. Roots enter the real axis at a gain K which is the minimum possible value of K in that region of the real axis. Two roots leave or strike the real axis at the breakaway points at angles of ± 90°.

Actually applying rule 5 by hand is generally somewhat difficult and requires a trial-and-error process. However, the root locus tools in the MATLAB control system tool box make this very easy to do. For initial design purposes (e.g. deciding upon what kind of compensator to use), rules 1 through 4 are usually sufficient to “rough out” the initial design. The MATLAB root locus tool kit can then be used to finish off the design.

There are a few additional root locus rules not discussed here. These are not described because nowadays it is generally easier to use MATLAB than to apply these additional rules by hand, and they are not particularly useful for roughing out an initial design.

TABLE 5-1
Some root locus plots.