On the Synthesis of Disjunctive Inferences of Reason in Transcendental Logic

1. Kant's Transcendental Logic

The transcendental Logic of disjunctive synthesis presents one of the more subtle fine points in Kant's Critical system. Kant's own lectures on the subject appearing in Logik can be difficult to follow owing to Kant's annoying habit of not identifying, much less defining, many of his key technical terms. Compounding the problem for the English-speaking world is the fact that English translations of Kant's gesammelte Schriften are frequently misleading. This occurs innocently enough. In order to properly translate, a translator must translate the author's ideas, not merely his words. Because Kant's own words present numerous ambiguities of interpretation, and because translators of his Logic works tend to be logicians themselves and habitually accustomed to interpreting logic terminology in traditional ways, Kant's very nontraditional system of Logic has a tendency to not "travel" well into English. Further compounding the problem is the fact that other key translations that translators frequently call upon to interpret Kant's meanings, such as Critique of Pure Reason, are also sometimes extremely poor, misleading, or even technically wrong. Kemp Smith's "standard" translation of Critique of Pure Reason is probably the foremost example of a translation that is so bad one is better off not reading it at all.

Transcendental Logic is not a redressed version of the Port Royal logic of Kant's day. Nor is it "logic" in the traditions of the European Schoolmen or modern mathematics. Kant tells us,

[We] divide principles of human knowledge a priori . . . into principles of sensibility a priori . . . [and] into principles of intellectual human knowledge a priori, and this is transcendental Logic. These principles of human knowledge a priori are categories of understanding . . . and these exhaust all of that which understanding holds in itself a priori, but from which hereafter yet other concepts can be derived. [Kant (c. 1790-91), 28: 576].

Elsewhere he describes the difference between transcendental Logic and mere formal logics such as the Port Royal logic. He tells us,

This transcendental Logic is distinguished from common [logic] by the following: [the latter] regards knowledge without troubling over whether the objects are a priori or a posteriori1; [the former] regards knowledge of understanding, insofar as [this knowledge] cannot be a posteriori, or the possibility of pure a priori knowledge of understanding [Kant (1783), 29: 802].

It can hardly be put more plainly than to say conventional logics each present a morality of thinking (how one ought to think) but transcendental Logic pertains to how human beings are able to represent knowledge and how they do so. A conventional logic is a kind of mathematical, as Russell liked to say [Russell (1919), pp. 194-5]. Transcendental Logic, while it is described and explained using mathematics, is not itself a mathematic. Indeed, it is because human beings are in possession of transcendental Logic in their homo noumenal aspect of being-a-human-being that mathematics itself is possible in the first place. Transcendental Logic belongs to the metaphysics of Critical epistemology. No matter how closely descriptions and explanations of its methods resemble mathematics or conventional logic, it is an epistemology of: objects-in-Nature; Nature per se; and Reality per se. Unlike mathematics and logic, which have a static sort of character as conveyances of outcomes of thinking and reasoning, the fundamental character of transcendental Logic is dynamical and practical. Its scope is the phenomenon of mind in action.

1 in Critical terminology, a priori implies "prior to experience" and a posteriori implies "after experience."
2. Disjunctive Inferences of Reason

There are two primary ways in which one can look at the mental physics of representing the synthesis of manifolds of knowledge by *H. sapiens*. One of these is functional, i.e., looking at it in terms of the mental processes that are involved. The other of these is schematic, i.e., looking at it in terms of the step-by-step procedures that are carried out in acts of mental representing. To this latter viewpoint belong the synthesis of polysyllogisms and disjunctive synthesis. Both are cases of inferences of Reason, as distinct from inferences of understanding and inferences of judgment. I have previously treated the synthesis of polysyllogisms [Wells (2011)]. This paper is concerned with the synthesis of disjunctive inferences of Reason.

One must take care to distinguish between disjunctive inferences of Reason in transcendental Logic and disjunction in mere mathematical logic. In the contexts of understanding and cognition, an inference of Reason (*Vernunftschluß*) is an inference in which the judgment's conclusion is derived by first subsuming a cognition (the minor premise as ground) under the condition of a rule (the major premise as the argument) in accordance with active regulation by an *a priori* principle of the process of pure speculative Reason. Figure 1 illustrates the effect of making a disjunctive inference on the structure of a manifold. It illustrates accommodation of the manifold by the creation of additional manifold Objects. For specificity let us assume that the manifold is the manifold of concepts. The disjunctive inference of figure 1 then depicts the making of three new concepts, denoted D, Member 1, and Member 2. Member 1 and Member 2 are concepts of subspheres under higher concept C, the concept under which the Classification is made. What a transcendental disjunction has in common with disjunction in formal logic is that the members of the disjunction are held-to-be opposites in some context. However, the manner of this holding-to-be-opposite fundamentally differs between transcendental Logic and conventional logic.

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**Figure 1**: Effect of making a disjunctive inference of Reason on a manifold. Making a disjunctive inference is an act of classifying, i.e. making a Classification. The disjunction is performed on manifold Object C and results in the partitioning of the sphere of this Object into two (or more) subspheres. The judgment action produces manifold Object D as the *parástase* of the judgment. Object C is regarded as being contained in D and D is regarded as being contained under C. The manifold is otherwise unaltered.
In formal logic, disjunction is merely logical opposition, i.e., if \( C = A \lor B \) then if \( A \) is true \( B \) is false and vice versa. In a transcendental disjunction the opposition set up is a real opposition in Relation (Entgegensetzung), i.e. Member 1 and Member 2 cannot both be asserted in the same intuition at the same moment in time. For example, if \( C \) is the concept "human-being" one might have for Member 1 the concept "male-human-being" and for Member 2 the concept "female-human-being." If one later predicates "Joe-is-a-male-human-being" then the disjunction precludes simultaneously predicating "Joe-is-a-female-human-being." The disjunctive inference of Reason places real restrictions on the concept matter that can be reintroduced by the process of reproductive imagination into the synthesis of an intuition in sensibility. By the phrase "real restrictions" I mean restrictions on what Objects can be thought in congruence with the thinker's constructed ideas of the structure of Nature and what his concept structure requires to satisfy the transcendental principle of coherence in Meaning. As Kant put it,

The peculiar character of disjunctive judgments, which determines their specific difference from the rest as to the momenta of Relation, particularly categorical judgments, subsists in this: The members of the disjunction are altogether problematic judgments of which nothing else is thought but that they, as parts of the sphere of a cognition, combined with every other complement of the whole . . . equal the sphere of the first [concept \( C \)]. And from this follows: In one of these problematic judgments truth is contained or – what is the same – one of them must be assertorically valid because outside of these the sphere of cognition under the given conditions possesses nothing more, and one is opposed to the other; consequently, there can be true neither anything else outside them nor more than one among them.

In a categorical judgment, the thing whose representation is regarded as part of the sphere of another subordinated representation is regarded as contained under the higher concept; hence here, in the subordination of the spheres, the part of the part is compared with the whole. But in disjunctive judgments I go from the whole to all parts combined. What is contained under the sphere of a concept is also contained under a part of that sphere. Accordingly, the sphere must first be partitioned. When, e.g., I set down the disjunctive judgment: A scholar is either an historian or a rational learner, I determine that these concepts, as to sphere, are parts of the sphere of scholar, but not all parts of each other, and that all combined are complete.

The following schema of comparing categorical with disjunctive judgments may illustrate that in disjunctive judgments the sphere of the classified concept is not regarded as contained in the sphere of the classifications, but that what is contained under the classified concept is regarded as contained under one of the members of the Classification. [Kant (1800), 9: 107-108]

With appropriate terminological changes, this explanation applies also to the manifold of rules in practical Reason and to the manifold of Desires in reflective judgment (although the manifold of Desires is not conserved as a structure, whereas the other two are structures).

In the making of a disjunctive inference of Reason in the manifold of concepts, the disjunction makes a schematism for thinking, i.e., makes a meaning implication regarding formal expedience in the synthesis of apprehension. The making of an inference of Reason in the manifold of rules similarly makes a schematism for acting, i.e. establishes a rule for constituting practical action schemes. The making of an inference of Reason in the manifold of Desires makes what can justly be called a schematism of preference for determining the affective state of the judging Subject. In terms of epistemological principles in the phenomenon of mind, an inference of Reason can be regarded as a synthesis of the ability to make a categorical inference of Reason and the ability to make a hypothetical inference of Reason. It is, however, the sphere of the manifold that is the direct Object of a disjunctive inference of Reason and not the Object of any parástase that the manifold presents. As Kant put it,
The several given judgments of which the disjunctive judgment is composed make up its matter and are called the *members of the disjunction or opposition*. In the *disjunction* itself subsists the *form* of these judgments, that is, in the determination of the relationship of the various judgments as reciprocally exclusive of each other and complementary members of the whole sphere of the classified cognition.

All disjunctive judgments therefore present various judgments *as in the community of one sphere* and bring about each judgment only by a limitation of the other [judgments] in respect of the whole sphere; they thus determine the relationship of each judgment to the whole sphere and thereby conjointly the relationship which these partitioned members (*membra disjuncta*) have to one another. Thus one member here determines any other only so far as they stand collectively in community as parts of a whole sphere of cognition, *outside which nothing can be thought in fixed interrelation*. [ibid., 9: 106-107]

One thing that is very important to emphasize is that one and the same manifold Object $C$ can be subjected to multiple acts of Classification. This means that one and the same Object can contain under it different and independently constructed partitions of its sphere. Figure 2 provides an illustration of this. For example, an Object concept "human-being" can contain under it the aforementioned Classification "male-human-being/female-human-being," but this does not preclude other Classifications such as "adult-human-being/child-human-being" or "living-human-being/deceased-human-being." These divers disjunctive inferences of Reason are synthesized in separate acts of judgmentation. This capability endows human reasoning with the enormous flexibility and creative capacity that human beings are so clearly known to intellectually exhibit.

All inferences, whether inferences of Reason, inferences of judgment, or inferences of understanding, are dynamical acts of judgmentation in the mental physics of human intellect. It is this dynamical aspect that most fundamentally distinguishes transcendental Logic from mathematical logic and displays the relative poverty of the latter in comparison to transcendental Logic.
3. Classification (Eintheilung) and Division (Theilung)

This brings me to another subtle yet extremely important technical distinction Kant drew. This is the technical distinction between an act of division (Theilung) and an act of Classification (Eintheilung). Both ideas stand under a general Object Kant called Division (Division) [ibid., 9: 108]. The effect of an act of division on the manifold is very different from a Classification. Figure 3 illustrates the effect on the structure of a manifold accommodated by an act of division. In a Classification (figure 1), Object C understands the disjunction D and D stands under C. In a division, Objects M1 and M2 understand the original Object C and C stands under them. Kant explained this, in the context of the manifold of concepts, by saying,

Every concept contains a manifold under it so far as the manifold is congruent, but also insofar as it is different. The determination of a concept in respect of everything possible contained under it, so far as the elements are opposed to one another, i.e. differ from one another, is called the logical Classification of the concept. The higher concept is called the classified concept (divisum), and the lower concepts are called the members of the Classification (membra dividentia).

To divide a concept and to classify it are two very different things. In division of a concept I see what is contained in it (through analysis); in Classification I consider what is contained under it. Here I divide the sphere of the concept, not the concept itself. Thus it is greatly mistaken that Classification is a division of the concept; rather, the members of the Classification contain more in themselves than does the classified concept.

We move up from lower to higher concepts [through division], and afterwards can move down again from these [higher concepts] to lower ones – by Classification. [ibid., 9: 146]
As I stated a moment ago, transcendental Logic is an innate capacity of the *homo noumenal* aspect of being-a-human-being. It is, however, a *practical* capacity and, as such, the individual is not born with any a priori *cognizance* of this ability. There is a great real difference between being able to do something and being cognizant of *how* you do it. Can you, for example, describe to another person in minute detail how you catch a ball? Most people cannot although twelve year old boys routinely catch balls and many of them are very skilled at doing so. There is an anecdote told about Hall of Fame baseball player Yogi Berra. Berra, who was a great hitter in his playing days, was later hired as a batting coach. However, he had great difficulty explaining the proper techniques of hitting to young players. It is reported that on one frustrating occasion he said to his player, "Ah, just watch me do it!" He then stepped into the batter's box and proceeded to hit the ball out of the park. He could *do* it, but he couldn't explain theoretically *how* he did it.

Studies in psychology have empirically demonstrated this aspect of being-a-human-being many times and quite conclusively. Acquiring cognizance of this is nothing more and nothing less than conceptualizing an understanding of it in the manifold of concepts. Like any other skill, including logic and mathematics, gaining this cognizance is greatly assisted by education and training. To understand the *theory* of transcendental Logic we call upon Kant's Critical doctrine of method. From this doctrine we have firstly:

In every Classification of a concept it is to be heeded that
1. the members of the Classification exclude or are opposed to one another; further
2. they belong under a higher concept (*conceptum communem*) and, lastly
3. they all together make up the sphere of the classified concept or are equal to it.

Note: The members of the Classification must be separated from one another by *contradiction*, not by mere contrariness. [*ibid.*, 9: 146-147]

Differently classified matters are regarded as different matters. If *x* is a cat then it is-not a wolf, a baboon, or a ferret. The contradiction of which Kant here speaks is a *real* contradiction, not a merely logical one. It sets up not only a real opposition in Relation (*Entgegensetzung*) but also a real opposition in Quality (*Widerstreit*; conflict) inasmuch as "being *x*" also necessarily means "not-being *y*," i.e. "being a *y*" will contradict "being an *x*." The affirmation in a proposition of a member *x* of a disjunction is conjointly the denial of the other members *y* of the disjunction, and these affirmations and denials are *transcendental* affirmations and denials. In the context of cognitions a transcendental affirmation means the signifying (by the category of reality) of being-in-time for *materia in quae* of intuition. A transcendental denial means the signifying (by the category of negation) of non-being-in-time for *materia in quae* of intuition. Affirmation and denial in the other contexts (affective perception in the manifold of Desires and practical rulemaking in the manifold of rules) is similar to this except, of course, for the specific *momenta* of reflective or practical judgment that functionally correspond to the function of the category of reality in determining judgment.

The conjoint making of transcendental denials accompanying a transcendental affirmation has an interesting if somewhat subtle real implication. The making of a transcendental affirmation is equivalent to making a transcendental denial of the opposite of the predication being affirmed. For example, if I have a cognition that "the-pack-of-cards is on-the-table," this conjointly denies the proposition that "the-table is-not under-the-pack-of-cards." This, however, is equivalent to the affirmation that "the-table is under-the-pack-of-cards." In other words, the effect of affirming one member of a disjunction reciprocally determines judgments concerning relationships with *other* Objects. Because disjunctive inferences of Reason act on spheres in the manifold rather than on the Objects that contain these spheres under them, making disjunctive inferences of Reason performs the function of coordinating knowledge "regionally" and, eventually, "globally" in the
manifold and not merely "locally." It is a topological consequence. This coordination of spheres is brought about by co-Division (Codivision; Nebeneintheilung = comparative-Classification) and sub-Division (Subdivision; Untereintheilung = lower-Classification), two more of Kant's hair-splitting technical terms [Kant (c. 1780), 24: 927-928]. Of these he tells us:

Different Classifications of a concept made in different respects are called comparative-Classifications; and the Classification of the members of a Classification is called sub-Division (subdivisio).

Sub-Division may be continued ad infinitum; comparatively, however, it may be finite. Co-Division, particularly with concepts of experience, is also ad infinitum, for who can exhaust all Relations of concepts?

As one may call co-Division also a Classification according to differences of concepts of the same object (viewpoints), so one may call sub-Division a Classification of the viewpoints themselves. [Kant (1800), 9: 147]

As an act of Classification, sub-Division establishes what one might call taxonomies in knowledge representation. For example, the phylogenetic tree biologists employ is a concrete example of the use of sub-Division in understanding. The series from genus to multiple species to multiple inferior species (varieties) has such fecund usages in biology that further elaboration seems unnecessary here. Co-Division, on the other hand, classifies subspheres according to the context of usage one has for it. Thus, one can speak of a "citizen of the United States," a "citizen of Iowa," or a "citizen of Des Moines." Co-Division is the production of members of a disjunction. Sub-Division classifies in depth and is the production of members of a member of a disjunction. It makes subspheres more distinct as to species-of-Object contained in a member of a disjunction. This brings us to the ideas of dichotomy, polytomy, and trichotomy:

A Classification into two members is called dichotomy; if, however, it has more than two members it is called polytomy.

All polytomy is empirical; dichotomy is the only Classification out of principles a priori – thus the only primitive Classification. For the members of the Classification shall be opposed to one another, and the opposite of every A is indeed nothing more than a non A.

Polytomy cannot be learned in logic because cognition of the object belongs to it. Dichotomy needs only the proposition of contradiction, without knowing the concept one wants to separate as regards contents. Polytomy needs intuition, either a priori as in mathematics (e.g. the Classification of conic sections) or empirical intuition in nature-description. Yet the Classification out of the principle of synthesis a priori has trichotomy, namely (1) the concept as the condition, (2) the conditioned, and (3) the derivation of the latter from the former. [ibid., 9: 147-148]

Nestled in this last quote is Kant's reemphasis that Classification is empirical, i.e., is part of the process of making one's own experience. It is essentially synthesis through analysis, i.e., anasynthesis. The partitioning process is analytic in character because it does contain making a Division (but not a division of the subject-Object) and it is also synthetic because before the members of a Classification can be members of a Classification they must be conceptualized as belonging together – and this is what concepts D1 and D2 in figure 2 are doing and what is depicted by the ovals denoting compound concepts (concepts made up of connected concepts) in the manifold. To make a disjunctive inference of Reason is: to carry out a schematism of reasoning; its possibility subsists in the overall process of judgmentation; and it involves at one step or another in the process the participation of all three processes of judgment (figure 4).

This is obviously something far more extensive in scope than the exercises encountered in the
Figure 4: The mental structure of thinking and judgmentation in *nous*.

Lessons of formal logic and, indeed, makes formal logic seem a comparatively trivial undertaking. One thing that formal logic theory derives from transcendental logic is brought out by Kant's remark that dichotomy is the sole primitive classification and obeys a transcendental principle of representation, namely, the principle of real contradiction. If two conclusions of two inferences: (1) are in real opposition (*Widerstreit*) to each other; and (2) must both apply to the same appearance at the same moment in time, *this constitutes a real contradiction*. This means that it is mentally impossible to make a depiction of representation (*parástase*) in any context for which that depiction must contain a real contradiction. Such an impossible *parástase* might indeed be merely contrary rather than contradictory in a different context. For example, I can *write* "four-sided triangle" but I cannot *draw* one nor can I imagine one. I can at best draw a four-sided solid figure such as a pyramid and nominally *call* that a "four-sided triangle," but the naming convention then does not *mean* the same thing as the idea of a "triangle" and my colleagues in the mathematics department would make fun of me.

Kant's bald statement that dichotomy is the only primitive classification makes nothing less than an epistemological prediction. The prediction states that very young children should only be able to exhibit classification actions that proceed pairwise taking two objects at a time. This prediction can be subjected to experimental testing. This testing has in fact been carried out in detail by developmental psychologists, and their findings appear to very solidly confirm the prediction [Inhelder & Piaget (1964), Piaget *et al.* (1968)]. Kant provides us with a description of what we might call the "mental mechanics" of making a disjunctive inference of reason:

> In disjunctive inferences the *major* is a disjunctive proposition and as such must have members of the classification or disjunction.

Here it will be either (1) inferred from the truth of one member of the disjunction to the falsity of the others, or (2) from the falsity of all members but one to the truth of this one.

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2 Every thing is real in some contexts, unreal in others, and non-real in yet others.

3 This means there must be a sphere before one can be partitioned and partitioning is required.
The former takes place through *modus ponens*⁴ . . . the latter through *modus tollens*⁵.

All members of the disjunction taken together bar one make up the contradictory of that one. Thus a dichotomy takes place according to which if either is true the other must be false and *vice versa*.

All disjunctive inferences of more than two members of the disjunction are thus actually polysyllogistic [refer to Wells (2011)]. For all true disjunction can only be *bimembri*⁶, and the logical Division is also *bimembri*; but the *membra subdividentia*⁷ are, for the sake of brevity [of terminology] counted as *membra dividentia*⁸.

The principle of disjunctive inferences is the *fundamental principle of the excluded middle*: From the negation of one contradictory opposite to the affirmation of the other, and from the positing of one to the negation of the other inference is valid. *[ibid., 9: 129-130]*

### 4. Final Remarks

Categorical inferences of Reason (divisions), polysyllogisms and disjunctive inferences of Reason comprise the three classes of inferences of Reason. Of these three, the disjunctive inference of Reason is the most complex in terms of its effects on a manifold. It also offers the greatest scope of possibilities for what a human being can learn and understand. Disjunctive inferences of Reason provide the capability for a single concept with its sphere to be indefinitely refined through the production of any number of disjunctive concepts (the D concepts in figure 2) setting up a *context-organized* manifold. Context (Zusammenhang) is one of the most crucial ideas in Critical epistemology. That which we name Reality-in-general is the *matter* of organized context because every object is *real* in some contexts, *unreal* in others, and *non-real* in still others. The ghost of Hamlet's father is *real* as a character in Shakespeare's play, *unreal* as an actual specter haunting any living inhabitant of Denmark, and *non-real* as a type of farm produce. The disjunctive inference of Reason is the most powerful act in judgmentation for the production of context for concepts in the manifold of concepts and practical rules of action in the manifold of rules (figure 4).

I think it is likely that this brief paper serves to illustrate the numerous subtleties that attend the theory of transcendental Logic. It has been a grievous error of presumption to suppose that Kant's transcendental Logic is nothing more than a re-hashing of the Port Royal or other so-called "Aristotelian" logics of the eighteenth century, just as it has been a grievous error of presumption to assume Kant's applied metaphysics of physics is an apology or justification for Newton's theory. In point of fact, Kant's epistemology-centered system of metaphysics overturned both classical logic and classical physics at their roots. Until translators of the Kantian corpus come to understand this, and to be more thorough in recognizing its highly technical terminology, misleading translations of Kant's works will no doubt continue to be the norm.

It is rather more difficult to understand why Kant's system is so poorly understood in German-speaking countries where, at least, his ideas do not have to pass through the filter of language. However, no doubt one major contributing factor is the relative paucity of scientific background

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⁴ the norm for positing the series of actual conditions in understanding. The principle of *modus ponens* states that if all the consequences of a cognition are true then the cognition is also true. *Modus ponens* is the positive norm in the principle of sufficient reason.

⁵ the norm for rescinding connections of judgments. The principle of *modus tollens* states that if one false consequence follows from a cognition then the cognition is false. *Modus tollens* is the negative norm in the principle of sufficient reason.

⁶ having two parts.

⁷ members of subordinated *membra dividentia*. See next footnote.

⁸ members of a Classification mentally pictured as separate objects.
that characterizes most philosophers. I would venture to say that a metaphysician in particular is poorly equipped if he does not possess at least an adequate technical grounding in at least one natural science and an adequate background beyond basic calculus in mathematics. This is not because metaphysics is based on any special science. It is not. But experience in the practice of empirical science and applied mathematics is a valuable source of context in reasoning through the historically difficult questions philosophers are regularly called upon to face. This background is not too much to ask of one who professes a love of wisdom. After all, this sort of background every engineer is required to acquire as part of his technical education.

Since the early part of the nineteenth century there has been a popular but dangerously false idea that science does not need metaphysics. The attitude was popularized by the positivists and for many years the attitude seemed to be justified by the successes enjoyed by classical physics. However, this overlooks two things. First, physics is the easiest of the natural sciences in terms of subject-matter. Its objects are the most directly accessible to empirical observation and the most easily manipulated by experiment. It is far more difficult to observe, much less manipulate experimentally, any of the objects of psychology, politics, economics, or any other social science (and this is why these sciences lag so far behind physics in their states of development).

Secondly, the one benefit of the era of positivism was the enforcement of epistemological discipline in the practice of science. Platonic speculations counted for nothing unless there was some observable, testable and unique effect predicted by theory. The discovery of quantum phenomena at the beginning of the twentieth century sounded the death knell for positivism and today positivism is dead. It has been replaced by nothing and today the discipline it once enforced is waning. It is worth remembering that science in classical times was, as historians have recently been discovering, far more advanced than the hubris of the nineteenth century had presumed. It is also worth remembering that Platonic speculation, as, e.g., in the mystic excesses of the Neo-Platonists, brought on the utter breakdown of science in Europe and helped to usher in that long intellectual night we call the European Dark Ages. It is simply not true that the special sciences can do without metaphysics – specifically, epistemology-centered and scientifically disciplined metaphysics. What the sciences can do without is mysticism masquerading as metaphysics.

5. References


