

# **Mathematics and Empirical Science**

Richard B. Wells

© 2020 All rights reserved

## Table of Contents

### **Chapter 1 Describing Empirical Knowledge**

§1. Mathematics, Metaphysics, and the Empirical World .....	1
§2. The Craft of Mathematics .....	5
§3. Measuring and Counting .....	7
§4. Geometry and Its Abstract Objects .....	13
§5. Mathematics and Measurement .....	19
References .....	23

### **Chapter 2 Slepian's Principle**

§1. The Bandwidth Paradox .....	25
§2. Slepian Dimensioning .....	28
§3. Polysylllogisms .....	33
§4. Disjunctive Inferences and Classifications .....	37
§5. The Condition of Epistemological Significance for Secondary Quantities .....	39
References .....	42

### **Chapter 3 Principal Quantities and Solution Sets**

§1. Theories and Measurements .....	44
§2. New Math .....	49
§3. Solution Sets and the Set Membership Paradigm .....	54
§4. Exploring the World of Facet B .....	59
References .....	61

### **Chapter 4 Constructing Structures**

§1. Structures .....	63
§2. The Principle of Permanence .....	64
§3. Algebraic Structures .....	69
A. Mathematical Bookkeeping, Ordered Pairs, Operations, Functions, and Binary Relations ..	70
B. Groupoids .....	71
C. Semigroups .....	72
D. Identity Elements .....	72
E. Monoids and Groups .....	72
F. Fields and Rings .....	75
§4. Universal Algebras and Empirical Science .....	76
§5. Mathematical Infinity .....	78
§6. The Mathematically Infinitesimal and the Idea of Limits .....	81
§7. Binary Relations and Transformations .....	84
§8. Statistics and Probability .....	86
§9. Other Structures .....	93
References .....	96

### **Chapter 5 Mathematics and Aesthetical Judgment**

§1. Mathematics and Aesthetic .....	98
§2. Intuition and Objectivity .....	100
§3. The Approvals of Taste .....	104
3.1 Patterning .....	104
3.2 Coalescing .....	106
3.3 Conceptualizing .....	108

3.4 Precisioning .....	110
§4. Summary .....	112
References .....	112

## Figures

### **Chapter 1**

Figure 1: Rosary beads .....	8
Figure 2: The Nippur cubit bar from ancient Sumer c. 2650 BC. ....	9
Figure 3: Illustration of the Method of Exhaustion .....	10
Figure 4: A predynastic Egyptian vessel from the period 4000 to 3000 BC. ....	13
Figure 5: Photograph of the night sky and the Milky Way .....	14
Figure 6: Photograph of the full moon .....	15
Figure 7: How a baseball is made .....	17
Figure 8: Diagram used by Euclid to prove the Pythagorean Theorem .....	19

### **Chapter 2**

Figure 1: Coordination of concepts in the manifold of concepts .....	30
Figure 2: Illustration of the horizon of possible experience .....	31
Figure 3: Slepian dimensioning of concepts .....	32
Figure 4: Illustration of a simple chain of coordinated concepts .....	34
Figure 5: Illustration of the concept structures for prosyllogisms and episyllogisms .....	34
Figure 6: Multiple polysyllogism constructions .....	35
Figure 7: Manifold structure of a Classification .....	38
Figure 8: Schematic structure of scientific ontology .....	41

### **Chapter 3**

Figure 1: DC voltmeter .....	46
Figure 2: Conceptual illustration of how a PMMC display is constructed .....	47
Figure 3: Margenau's "construct theory" of nature and reality .....	51
Figure 4: Empirically indistinguishable solutions of parameter estimates .....	57
Figure 5: Reduction of the solution set through repeated data collection trials .....	58

### **Chapter 4**

Figure 1: Example of a permutation group .....	74
Figure 2: Operation table for the permutation group example .....	75
Figure 3: The scheme for calculating integrals .....	82
Figure 4: A slide rule .....	85
Figure 5: A histogram .....	87
Figure 6: The difference between a bar graph and a histogram .....	87
Figure 7: Illustration of a multimodal distribution of experimental data .....	88
Figure 8: Two example histograms overlaid by probability distribution functions .....	90

### **Chapter 5**

Figure 1: 2nd level analytic representation of the process of aesthetical reflective judgment .....	102
Figure 2: Base 10 addition table with carries indicated .....	105