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On Critical Representation in Brain Theory, Part II: General Schema of Knowledge Representation

I. The Representation Problem

This is the concluding paper in my series [Wells (2011c, d)] on the metaphysical problem of knowledge representation. Its objective is the statement of the general problem to be solved in order to develop mathematical solutions for mind-body knowledge representation. With such a problem definition, the transition from metaphysics to empirical science proper is made. The task of formulating specific solutions then belongs to a special natural science I call *Critical psychophysics*. The metaphysical solution provides a general *solution schema* as a template or roadmap for solving the knowledge representation problem. As experienced system engineers know quite well, coming up with a precise mathematical statement of *what* one wants to do is usually a far more challenging task than coming up with *how* to do it. This is the case for the knowledge representation problem.

Knowledge (*Erkenntnis*) is any conscious representation *or capacity for making such a representation* by or through which meanings are determined. This critical *Realerklärung* of what "knowledge" means tells us at the outset that the representation problem and the knowledge representation problem are in actuality one and the same problem. In contrast, the main paradigm pursued by science and by engineering as exemplified by work in "artificial intelligence" [Woods (1986], so-called "mind design" [Haugeland (1997)] or so-called "android epistemology" [Ford *et al*. (1995)] have tended to first make a real division between "knowledge" and "knowledge representation." Having made that improper division, most of these efforts more or less treat the "knowledge" division much as Plato did – which is to say they reify knowledge as a primitive *thing* about which nothing more need or *can* be said. About the only practical difference is that Plato proceeded to place knowledge *per se* (the Platonic Ideas) in that peculiarly un-Greek heaven he called "the world of true and full being" [Plato, *Phaedrus*]. Modern scientists and engineers, in contrast, either choose to stay silent about the nature of this mysterious *ὄν* or to pronounce *ex cathedra* that "knowledge is rules." Kosko wrote,

> Rules associate ideas. They relate one thing or event or process to another thing or event or process. In natural and computer languages rules have the form of if-then statements. If it rains, you get wet. If you get wet, you can't play golf. It will rain on Saturday. So you can't play golf on Saturday. It won't rain on Sunday. If you can't play golf on Saturday and if it won't rain on Sunday, you can play golf on Sunday. So you play golf on Sunday. [Kosko (1993), pp. 158-159]

Kosko is a bit fuzzy on the question of why "if-then rules" are knowledge but the decision (the outcome of applying the rules), which is not itself an if-then rule, is *not* knowledge (unless, of course, one argues by *fiat* that "decision = rule $\wedge$ knowledge = rule $\therefore$ decision = knowledge")[1]. Fuzzy logicians, like Kosko, and artificial intelligence researchers are not wrong to introduce the idea of rules into the context. Critically, a *rule* is *an assertion made under a general condition*. As such, a rule (or, rather, its mechanization) is a capacity for making a representation and therefore is knowledge of that type provided that its asserted outcome has *conscious* representation[2]. What does this often-controversial adjective (conscious) imply? The Critical answer to this is far easier than one might predict. ***Empirical consciousness*** is *a representation that another representation is in me and is to be attended to*. A computer scientist or an engineer should feel comfortable

[1] In the language of mathematics, the symbol $\therefore$ is pronounced "therefore" and the symbol $\wedge$ is pronounced "and." The "decision = rule" premise is an *arbitrarily* imposed minor term in this syllogism.
[2] A conscious representation is called a perception.

thinking about this in terms of: (1) a pointer of some sort (the representation *of* 'consciousness,' which belongs to the human capacity for formulating species); and (2) something pointed to (which could be either empirical knowledge or could be another member of the 'capacity for making' species of knowledge). The key thing to note is that this *Realerklärung* is a practical explanation in terms of what the representation leads to (*viz*. attending to the indicated representation, which in its turn also leads to something). *A* representation (*parástase*; "depiction") *means* something and *all meanings are at root* ***practical***. The study of meaning in any and all its manifestations is called ***semantics***.

One could say these are hair-splitting distinctions[3], but if we are unaware of them we fall into the fallacies that arise from homonymous usages of key terms. In particular, practices in AI (artificial intelligence) and fuzzy logic engineering often drift into thinking that knowledge (the object of the *parástase*) is the *parástase* itself, i.e. mistake *data* for knowledge *per se* and lose track of meanings as the necessary *context* of knowledge. Even Plato didn't make that mistake:

> Now, as we have said, every human soul has, by reason of her nature, had contemplation of true being; else would she never have entered into this human creature; but to be put in mind thereof by things here is not easy for every soul . . . Few indeed are left that can still remember much, but when these discern some likeness of the things yonder, they are amazed, and no longer masters of themselves, and know not what is come upon them by reason of their perception being dim.
>
> Now in the earthly likeness of justice and temperance and all other prized possessions of the soul there dwells no luster; nay, so dull are the organs wherewith men approach their images that hardly can a few behold that which is imaged . . . [;] pure was the light that shone around us, and pure were we, without taint of that prison house which now we are encompassed withal, and call a body, fast bound therein as an oyster in its shell. [Plato (*Phaedrus*), 249d-250e]

To behave like an inattentive Platonist but insist on being called a scientific materialist is nothing but denial, and to make a habit of it is called a neurosis. In Part I [Wells (2011c)] the semantics context was made clear, and in the next paper in this series [Wells (2011d)] the metaphysical requirements implicated by this context were developed. The knowledge representation (somatic code) problem was shown to have the mathematical form of a topology problem, specifically, a topology generation problem. The task before us now is to further clarify and make distinct the particular form of this problem. It is, as Kant likely would have put it, to lay down the *Metaphysische Anfangsgründe* (metaphysical rudiments) of the solution *method*.

II. The Topology Problem in the Context of Organized Being

<u>A. State Space Models</u>. Solving the knowledge representation problem comes down to solving a topology generation problem. As mentioned earlier in this series, mathematicians approach this on a case-by-case basis and then generalize topology theory from what they learned by studying the special cases. Hocking and Young put it this way:

> Topology may be considered as an abstract study of the limit-point concept. . . In applying the unifying principle of abstraction, we study concrete examples and try to isolate the basic properties upon which the interesting phenomena depend. In the final analysis, of course, the determination of the "correct" properties to be abstracted is largely an experimental process. . . In many cases a "natural" topology exists, a topology agreeing with our intuition of what a limit point should be. . . In general . . . we require only a structure within a set which will define *limit point* in a simple manner and in such a way

[3] I am tempted to quip, "mere matters of semantics" but for the knowledge representation problem there is nothing "mere" about semantics.

> that certain basic relations concerning limits points are maintained. . .
>
> The study of topologized sets (or any other abstract system) involves two broad and interrelated questions. The first of these concerns the investigation and classification of the various concrete realizations, or models, we may encounter. This entails recognition of equivalent models, as is done for isomorphic groups or congruent geometric figures, for example. In turn, this equivalence of models is usually defined in terms of a one-to-one reversible transformation so chosen as to leave invariant the fundamental properties of the models. As examples, we have the rigid motions in geometry and the isomorphisms in group theory, etc. . . The second broad question in studying an abstract system such as our topologized sets involves considerations of transformations more general than the one-to-one equivalence transformation. The requirement that the transformation be one-to-one and reversible is dropped and we retain only the requirement that the basic structure is to be preserved. . . In topology, the corresponding transformations are those that preserve the limit points. [Hocking and Young (1961), pp. 1-3]

In Wells (2011d) all of the discussion was devoted to the *soma*-semantical topological space and no explicit mention was made about a second "equivalent model space." Where does that one come from? Finding it presents no difficulty when we remember that the applied metaphysic of the somatic code pertains to *psyche* [Wells (2009), chap. 1] and that its major acroam is the general transcendental Idea of Rational Psychology: absolute unity of the thinking subject. On the side of *soma* and the judicial-sensorimotor idea, the model space is a state space of somatic activity fields. All we have to do is remember the logical role of *psyche*, namely, to enforce thorough-going reciprocity between somatic representations and noetic representations. The second and equivalent model space is a state space of *noetic* representations. In the language of topology theory, the one-to-one reversible transformations Hocking and Young mention comprises what is called a homeomorphism between somatic state space X and noetic state space Y. The transformations are mapping functions – let's call them $f{:}X \rightarrow Y$ and $g{:}Y \rightarrow X$ – and these are *psyche* functions. Figure 1 illustrates this model-universe. Mapping function f, which takes somatic state space over into noetic state space, belongs to receptivity in *psyche* whereas mapping function g, which takes noetic state space over into somatic state space, belongs to motoregulatory expression in *psyche*.

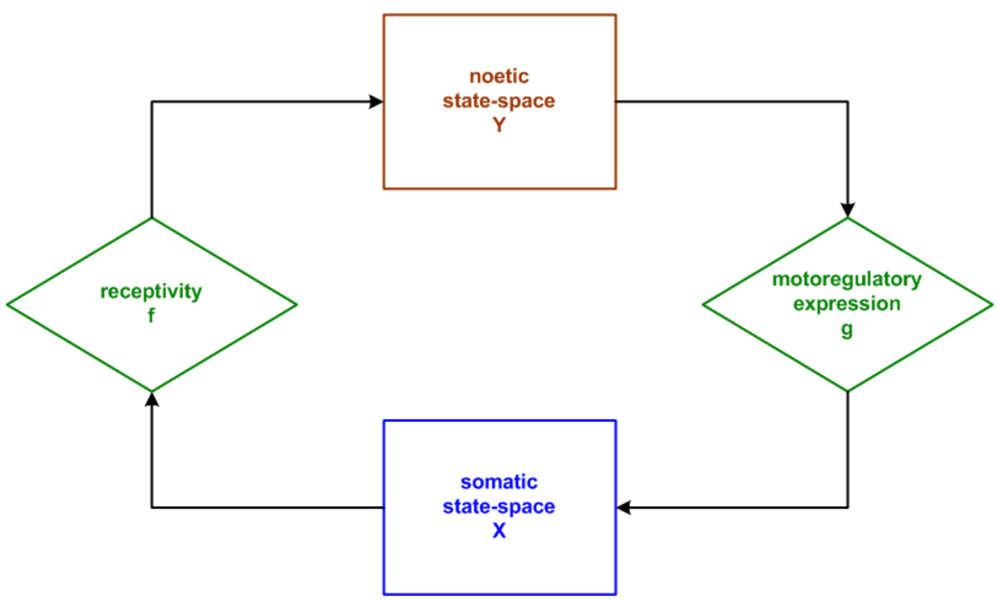

Figure 1: The homeomorphic two-model problem of topological organized being.

Now because the problem *solution* is of interest to a much broader community of scientists[4] than the relatively smaller community of system theorists, and because I wish to do what I can in this paper to help this broader community keep up qualitatively well and as quantitatively well as individuals' mathematics backgrounds permit, this term "state space" must be described[5]. On the side of *soma*, the representation universe is exhibited as a universe of somatic places that are characterized by quantitative measurements of somatic activities. This terminology was explained in the applied metaphysic [Wells (2011d)]. Each such place is called a ***state variable***. The aggregation of places and their activities in an activity field is called the somatic ***state***.

Here it is important to remember that somatic places have their boundaries set by measurement uncertainty so that a *physical* aggregation of biological cells in any particular place must be looked at as comprising just *one* set membership state variable. This is to say that the state variables of *soma* cannot be defined with objective validity by *arbitrarily* carving up somatic anatomy. Somatic state variables have to fall on the scientist's side of the horizon of possible experience [Wells (2011a)], and the measurement capabilities he has at his disposal *set* what can be identified as an objectively valid somatic place. This *epistemological* constraint on the *mathematics* is neither a new nor any longer a revolutionary thesis. Einstein said what amounts to the same thing in 1915:

> In classical mechanics, as well as in the special theory of relativity, the coordinates of space and time have a direct physical meaning. To say that a point-event has the X_1 coordinate x_1 means that the projection of the point-event on the axis of X_1 determined by rigid rods and in accordance with the rules of Euclidean geometry, is obtained by measuring off a given rod (the unit of length) x_1 times from the origin of the coordinates along the axis of X_1. To say that a point-event has the X_4 coordinate $x_4 = t$ means that a standard clock, made to measure time in a definite unit period, and which is stationary relatively to the system of coordinates and practically coincident in space with the point-event will have measured off $x_4 = t$ periods at the occurrence of the event. [Einstein (1915)]

The "rods and clocks" for somatic measurements are considerably more complicated, but this is not in the least relevant to the problem. In the facet A of experience, the physical capacities of observation and measurement *set rules for mathematics to obey*, and then physics is in its turn obliged to heed the constrained mathematical consequences in regard to *principal* quantities. Epistemology-constrained mathematics is nothing else than *Einstein's metaphysical axiom*.

The somatic state can now be represented by a ***state vector*** of finite dimension. The somatic ***state space*** is then nothing else than the vector space in which the state vector ranges. Here it is important to know that state definitions are not unique in the mathematical connotation of uniqueness [Nelson (2003)]. If one has two different-looking state spaces but all external observations of each are equivalent (in the context of set membership, i.e., they belong to the same solution set) then the two state spaces are *set membership equivalent* (indistinguishable). The Critical doctrine of method [Wells (2011a)] *mandates* set membership formulation of the mathematical system because this is epistemologically required for the possibility of principal quantities to be associated with physical appearances *with objective validity*.

Turning now to the noetic state space Y, everything that has just been said about the somatic state space applies equally to Y *except* that the state variables are no longer somatic places but, rather, noetic representations [Wells (2011c)]. This brings up a rather obvious question. Noetic representations, all of which belong to the *homo noumenon* aspect of being a human being, are *supersensible*. This means they cannot be measured by any physical measurement apparatus.

4 even though their interests likely do not extend so far as the mathematical gory details of system theory.

5 system theorists do not need the concepts explained; they already know them. Non-system-theorists need less than a full mathematical explanation, and that is what a "description" is.

How, then, are we to deal with *them* with objective validity? This is where epistemology-centered metaphysics is utterly crucial. *Noetic representations are supersensible but they are not isolated from experience*. The rigid and uncompromising first principle of transcendental analysis is that the only permissible constructs of *nous* are those *necessary for the possibility of experience*. Every aspect of the structure of *nous* (and of *psyche* as well) follows deductively beginning with the Critical major acroams of Kant's metaphysics through the minor Critical principles and, finally, through the *Metaphysische Anfangsgründe* provided by mental physics. The noetic state variables are objectively valid *because they are constructed to be objectively valid* and with their *transcendental* places strictly delimited by mental physics so that they stand *at* the horizon of possible experience as *noumena* and *never go beyond it*. If one cares to say that physical Nature is physics-determinable, one would equally well say mental Nature is mental-physics-determinable.

The non-uniqueness of state variable representation means that two system models with very different appearances can produce identical action responses, *y*, when stimulated by the same aliment input, *u*. Figure 2 illustrates an example of this. System 1 contains three integrators and its state variables have been defined as integrator outputs. System 2 contains only two integrators. It is obvious by inspection that these two models differ in appearance. The state of System 1 is the column vector $X = [x_1\ x_2\ x_3]^T$, and is described by a three dimensional state space. System 2, in contrast, has two state variables and is described by a two dimensional state space. However, direct analysis shows that both systems are described by the same differential equation. The two systems are therefore equivalent in terms of action responses to alimentary stimuli.

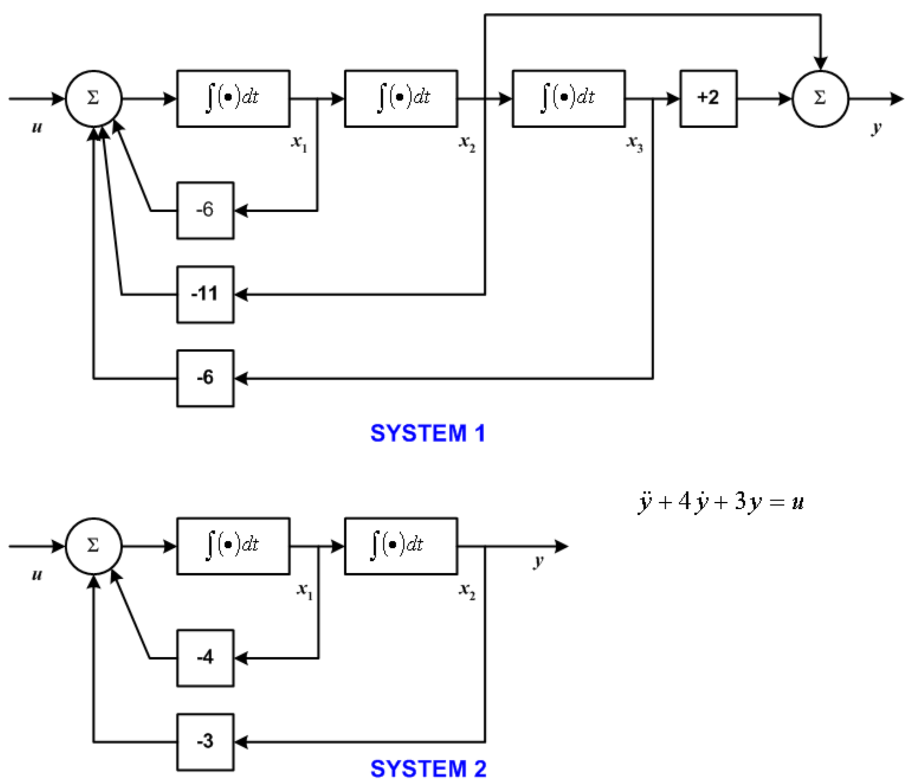

Figure 2: Two system models that are output-equivalent. *u* = alimental stimulus; *y* = system output action; integrator outputs x_j are used as state variables. A dot above a variable denotes time differentiation (Newton's notation). Although the model appearances are obviously different, both produce the same *y* action response when stimulated by the same *u* input and both have the same differential equation for *y*. The small square boxes denote scaling gain factors. The rectangular boxes are integrator functions. Circles with the notation Σ inside denote summations.

The state of System 1 of figure 2 is said to be "state unobservable" in the lexicon of system theorists. This means that no amount of observation of the system's input stimulus *u* and output action *y* can suffice for an observer to be able to determine the system's state *X*. If an observer were presented with systems 1 and 2 as two "black boxes" he would be unable to discern any difference between them. Mathematically, this means that the system state *X* is a *secondary quantity* in the mathematical model [Wells (2011a)]. It has no correspondent in the physical world (facet A) and no ontological significance whatsoever. In contrast, system action *y* is a principal quantity of the model and has ontological significance for facet A.

As another example, the wave functions used in quantum mechanics are state variables in a quantum mechanics model. They are mathematical secondary quantities and have *no* ontological import. Like all state variables, however, they do have *epistemological* import because the idea is used to *understand* physical appearances and behaviors of quantum mechanical systems. There is a long-standing debate in physics over what is called "entanglement" and it is a debate enlivened by experiments [Rohrlich (1983)]. Yet it is nothing but an argument concerning the ontological significance of state variables. But because the state variables *have no* ontological significance, the debate itself is empty – one of physics' modern day homologues to an old theological argument over how many angels can dance on the head of a pin. William James wrote,

> It is astonishing to see how many philosophical disputes collapse into insignificance the moment you subject them to this simple test of tracing a concrete consequence. There can *be* no difference anywhere that doesn't *make* a difference elsewhere – no difference in abstract truth that doesn't express itself in a difference in concrete fact and in conduct consequent upon that fact, imposed on somebody, somehow, somewhere, and somewhen. The whole function of philosophy ought to be to find out what definite difference it will make to you and me, at definite instants of our life, if this world-formula or that world-formula be the true one. [James (1907), pg. 25] □

<u>B. Continuity and Homeomorphism</u>. One pragmatic definition of topology theory is that topology theory is the study of continuity. What does "continuity" mean? Before getting into the mathematical definition, it is worthwhile to look at a pragmatic description of this idea. James wrote, "I can only define 'continuous' as that which is without breach, crack, or division" [James (1890), pg. 237]. James makes a mild (but not non-serious) ontological overstep in this statement. He would have better said "that which *I cannot perceive to have* a breach, crack, or division." It is quite obvious that the ability to perceive any "breach, crack or division" is limited by whatever empirical uncertainty characterizes the instrument used to make the observation, whether this instrument is a human being's capacities of external sense or is some measuring tool (scientific measuring instrument) that is used to provide his senses with a sensuous *dabile* ("givable"). We cannot perceive continuity; we can only perceive the appearance of a *lack* of the *property* of continuity. This means that continuity, like the wave functions of quantum mechanics or the state variables of a system model, belongs to the mathematical facet B of understanding and is a secondary quantity. Continuity is an object of mathematics (and so it is proper for hypothetical mathematics to study it) but it is not an ontological object of physical Nature.

We have here a very interesting case of epistemological vs. ontological significance. If a "breach or crack" can be perceived (and they can), then the concepts "breach" and "crack" are sensuous and belong to physical Nature with ontological significance of some kind. Yet their contrary, continuity, does not belong to sensuous Nature. Continuity has epistemological significance only; discontinuity has both epistemological and ontological significance. Is physical Nature continuous? That question is formally undecidable because continuity is a secondary mathematical quantity. What, then, does that mean for discontinuity? The answer is: discontinuity is a *noumenon* standing at the horizon of possible experience and its *practical* objectivity is that of a *condition* of appearance. This means, Critically, that discontinuity is not a substantive object but, rather, a concept of a relationship between appearances and perception. It *grounds* nothing *ontologically*. It *is*, however, a ground *for the orientation of human understanding*. One might call it a psychological stimulus because it leads, for example, to distinctions such as the me *vs*. not-me real division (a judgment with ontological significance). The perception of discontinuity and the Organized Being's reaction to this perception go to the governing acroam of reflective judgment (the principle of formal expedience in Nature) and the judgment stimulates evocation of one of the regulative principles of pure practical Reason, namely, the transcendental cosmological Idea: absolute completion in the series of conditions. What this means is that perception of discontinuity stimulates judgmentation *to understand the Nature of the discontinuity*. Simply put, it is a fundamental characteristic of human intellect that we understand Nature as *one* Nature-in-general and reality in terms of *one* substratum of Reality-in-general. If, as some physicists these days are inclined to do, one speculates about the possibility of "multiple" or "parallel" universes, all these "multiverses" are *understood* as *particulars embedded within some larger universal framework*. All multiverse theories end up positing *one* "hyperspace" to *hold* the universes.

All this is to say that the epistemological significance of discontinuity is *as stimulus* for reasoning about the ontology of Nature and the Objects in Nature. In other words, discontinuities *psychologically require* explanation. The mental physics of judgmentation in seeking for such an explanation is described in representational terms by four *negative* principles of judgmentation:

- in Quantity – *in mundo non datur saltus* ("a leap is not given in the sensible world");
- in Quality – *in mundo non datur hiatus* ("a gap is not given in the sensible world");
- in Relation – *in mundo non datur casus* ("chance is not given in the sensible world"); and
- in Modality – *in mundo non datur fatum* ("fate is not given in the sensible world").

"Not given" means that leap, gap, chance, and fate are not objects of sensuous experience but, rather, are only concepts employed in the judgment of appearances. For example, a young child does not think *anything* happens "by chance." He always pins some sort of causal or pseudo-causal explanation to everything he experiences [Piaget (1930)].

The mental physics of judgmentation pertaining to this is called *the synthesis in continuity* [Wells (2009), chap. 7 §3.2]. Among other things, this synthesis orients the process of perceiving and conceptualizing appearances. This is to say it contains an *objectivity function*, the function for understanding appearances *as* objects. This places "continuity" as an object in the mental logical division of organized being and thus makes "continuity" a *mathematical* object-of-reasoning.

Having established the *bona fides* for mathematics' sole custody of the idea of continuity, let us now look at what mathematics has to say about it. We begin with figure 1. Let us pretend we could look at this figure "frozen" at some particular moment in objective time, t_1. At any such time there is an epistemological relationship between the somatic state space and the noetic state space as a consequence of the principle of thorough-going reciprocity between *nous* and *soma*. The consequence is this: between the state of the somatic model and the state of the noetic model there is *necessarily* a homeomorphic mathematical relation. What this means is the following. Let X denote the somatic state space and let x_1 be the state of the somatic model at time t_1. Let Y denote the noetic state space and let y_1 be the state of the noetic model at time t_1. Let f be some transformation that maps state space X to state space Y (symbolically, f:X → Y). Let g be some transformation that maps state space Y to state space X (g:Y → X). Denote the mapping of state x_1 to Y as $f(x_1) \mapsto y_1$. Then $g(y_1) \mapsto x_1$. This is what is meant by homeomorphism between the two models. It means that at t_1 the states x_1 and y_1 represent *the same information*. Note that a trivial consequence of the homeomorphism is that $g[f(x_1)] \mapsto x_1$, $f[g(y_1)] \mapsto y_1$ for every x and y in the two state spaces. This is not a postulate. It is a *theorem* of mental physics.

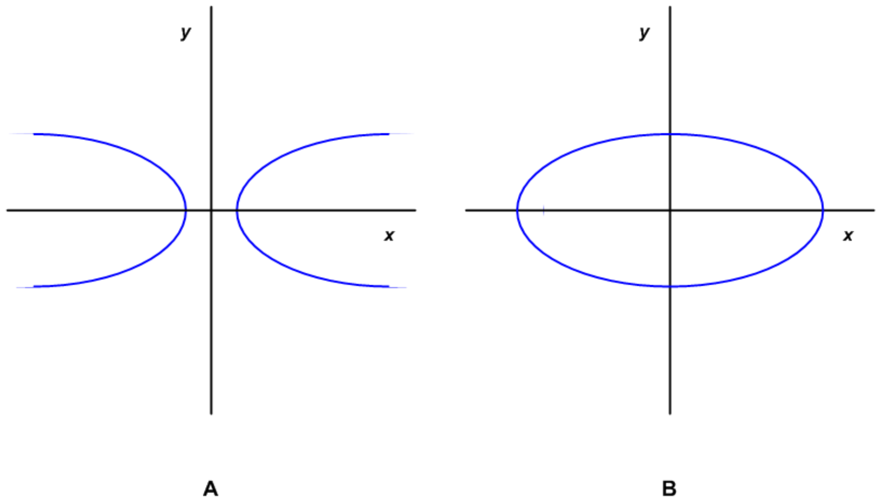

Figure 3: Cartoon-level illustration of trajectories in a two-dimensional state space with state $[x\ y]^T$. Each point on the blue lines denotes a particular state that lies on the indicated trajectory. 3A is a hyperbolic trajectory. 3B is an elliptic trajectory. Objective time is not illustrated in either figure.

Now, x denotes a somatic activity field at time t_1 [Wells (2011d)]. As for noetic state y, each state *variable* in y is some specific noetic *parástase* and the aggregation of all of these in y is the total state of representations in *nous*. Let us call this a ***parástase* field** by analogy to the model of the somatic activity field. Transformation f is a receptivity function of *psyche* and transformation g is a motoregulatory function of *psyche*. The principle of thorough-going *nous-soma* reciprocity then has for one of its consequences the mathematical requirement that the pair $[f, g]$ of mapping functions of *psyche* must form a homeomorphism at every instant of objective time t. From a well known theorem of topology, this means f is a continuous function in X and g is a continuous function in Y.

Because X is structured as a universe of topological neighborhoods in *soma* [Wells (2011d)], this theorem of mental physics tells us that Y is structured as a universe of topological neighborhoods in *nous*. However, a *universe* of topological neighborhoods is not yet a *system* of topological neighborhoods and, therefore, is not yet a topology. More is required for this. □

<u>C. Connectedness and Trajectories</u>. A somatic system of neighborhoods is a temporal sequence (in objective time) of activity fields that has been associated to form a somatic morpheme [Wells (2011d)]. Thus we must deal with the idea of a temporal sequence of activities. An organized sequence (one that represents a somatic morpheme) is called a *trajectory*. By the homeomorphic theorem above, the appearance of a somatic trajectory necessarily implies the (mathematical) existence of a noetic trajectory as its image in *nous*. Figure 3 illustrates this notion of trajectories. (Note that here x and y now represent state variables of somatic space X).

Now, there are three distinct *Existenz* cases of organized being with which the theory must deal. These are: (1) the *equilibrium* case; (2) the *non-equilibrium* case; and (3) the transition case between non-equilibrium and equilibrium, which I will call the *re-equilibration* case. As the equilibrium case is the simplest of these, I begin with it.

Figure 3B is the cartoon-level illustration of the equilibrium case. Critical analysis of the *Realerklärung* of "equilibrium" in organized being concludes that equilibrium means *a closed cycle of activity in which there are no innovations* [Wells (2009), chap. 4, §3.5)]. There are a number of names for this. Mathematicians often call such a cycle a *limit cycle*. Piaget calls it a *circular reaction* and identifies several psychological species of it [Piaget (1952)]. Mental physics calls it a *life cycle*.

Equilibrium imposes a mathematical requirement on trajectories in somatic space-time[6]. The requirement is called ***local path connectedness***. The mathematical explanation of this idea gets a bit detailed, e.g.: Hocking and Young (1961) pp. 14-17); Baum (1964) pp. 98-104; Wall (1972) pp. 41-47, but the idea itself is simple enough to grasp. Consider the ellipse of figure 3B. The usual topology of the real number plane $\Re^2$ [Baum (1964), pg. 22] defines the ε-neighborhood N_p of a coordinate point $p = (x, y)$ as the set of points $p' = (x', y')$ that satisfies the condition

$$N_p = \left\{ p' \,\middle|\, (x'-x)^2 + (y'-y)^2 < \varepsilon, \varepsilon > 0 \right\}.$$

Thus, each point p has for its neighborhood a "ball" of points within radius $\sqrt{\varepsilon}$ of p. Let S denote a specific shape (the hyperbola in figure 3A, the ellipse in figure 3B) defined by $S \subset \Re^2$. Call the shape H for the hyperbola and E for the ellipse and let $a, b \geq 1$ and $\varepsilon < a^2, b^2$. Then

$$\begin{aligned} H &= \left\{ p \,\middle|\, x^2 - y^2 = a \right\} \\ E &= \left\{ p \,\middle|\, (x/a)^2 + (y/b)^2 = 1 \right\} \end{aligned} .$$

The neighborhood U_p of a point $p \in S$ restricted to S is then $U_p = S \cap N_p$.

A space S is ***path connected*** if for any pair $p_1, p_n \in S$ there is a sequence of $p_j \in$ S such that

$$p_{j+1} \in U_{p_j} \;\; for\; j = 1, \cdots, n-1 . \qquad (1)$$

Similarly, a topological space S is ***locally path connected*** (l.p.c.) at $p \in S$ if its neighborhood U_p contains a neighborhood V such that any two points in V are connected by a path in U_p [Wall (1972), pg. 45]. Every path connected space is connected but not every connected space is path connected. If S is l.p.c. and connected then it is path connected.

Examining figure 3B, it is fairly obvious that E is path connected because it is l.p.c. at every p. In contrast, hyperbola H in figure 3A is ***disconnected***. Consider $p_1 = (a, 0)$ and $p_n = (-a, 0)$. There is no path in H connecting these two points because any route from p_1 to p_n must leave H. The hyperbola consists of two *disjoint* semi-hyperbolas defined by $x < 0$ and $x > 0$. If S can be partitioned into disjoint subsets, such as in this case, then S is defined to be a ***disconnected space***.

In the somatic state space X of figure 1, the neighborhoods of a somatic place p are those activity fields in which the somatic activity at p is determinably non-zero [Wells (2011d, e)]. In order to regard X as a vector space, we must allow its state vector X to contain all the somatic places in X. The somatic activity of p is determinable if it is measurable (that is, if the instrumentation for observing it can detect metabolic action). Let S denote a trajectory in X where a ***trajectory*** is defined to be a determinable actual succession of activity fields $N_1, N_2, \ldots$ with N_j a neighborhood of somatic place p_j and with p_1 regarded as the starting place of the trajectory. The Organized Being (OB) ***is in equilibrium*** if and only if the trajectory meets two conditions:

1. the trajectory is locally path connected; and
2. the trajectory eventually returns to N_1 after including at least one activity field $N \neq N_1$.

[6] For purposes of discussion, it is easier to explain this point by referring to somatic appearances. You should, however, bear in mind that for the somatic cycle being described there is a homeomorphic image of this cycle in noetic space-time. Topologists call the receptivity function f (and the motoregulatory function g) an *embedding*.

Such a trajectory is said to define a sensorimotor ***scheme of a circular reaction***. The ellipse in figure 3 is a cartoon-level illustration of this. Note that because X and noetic state space Y are related by a homeomorphism, if the trajectory in X is a circular reaction then the image trajectory in Y is also a circular reaction.

Next I consider the non-equilibrium case. In this case the initial trajectory is locally path connected but the trajectory fails to cyclically return to *any* N appearing in the initial sequence. The trajectory is said to ***rupture*** if in the sequence of somatic appearances an activity field N_{j+1} appears that is not locally path connected to the immediately antecedent activity field N_j. The rupture of the trajectory is called a sensorimotor ***cycle rupture***. The trajectory is said to ***terminate*** at N_n if after reaching N_n another equilibrium cycle is subsequently established.

There are some metaphysical conditions pertaining to the real possibility of the ruptured non-equilibrium case. However, it seems best to me to put off this discussion in favor of completing the mathematical exposition. Here I will just say that a ruptured trajectory is mathematically possible but its real *Existenz* is problematic outside of some special contexts such as, e.g., if the OB suffers a stroke. Figure 4 illustrates a non-equilibrium trajectory with cycle rupture.

The final case, the re-equilibration case, is the synthesis of the first two. It consists of two components, an initial non-equilibrium terminating trajectory and a subsequent equilibrium trajectory. Figure 5 is an illustration of this case. A non-equilibrium trajectory begins in neighborhood N_1 and travels by a locally-connected path to neighborhood N_{j+m}. At that point an equilibrium cycle (denoted by the ellipse) begins.

The re-equilibration trajectory is, again, mathematically possible. For it, too, there are some metaphysical considerations I will put off in order to maintain "local path connectedness" in the basic mathematics. Suffice it to say these contexts pertain to trajectories that appear "chaotic." □

D. Neighborhood and Neighborhood System Generation. Topological spaces are *a priori* in the context that they are mathematical objects, but no topological space is *necessary* a priori. What I mean by this is that every topological space is constructed by a human being and the way in which it is constructed depends on his personal *empirical* circumstances. The way in which he constructs it is *necessitated* to conform to either his *mental* or *biological* circumstances.

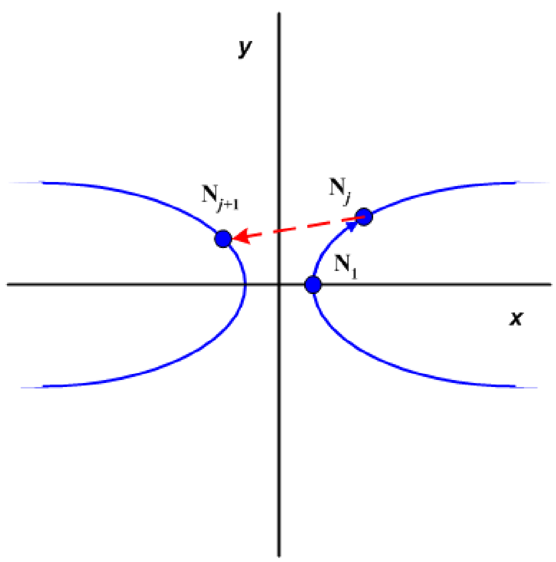

Figure 4: Cartoon level illustration of the concept of a non-equilibrium trajectory with cycle rupture. The initial trajectory travels unidirectionally from N_1 to N_j before jumping to N_{j+1}.

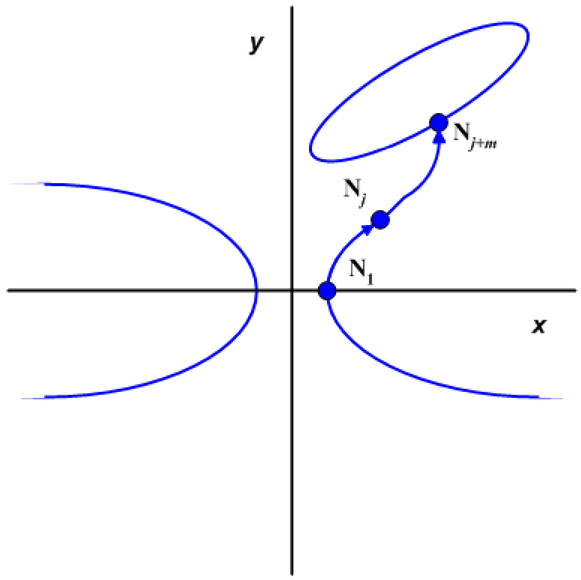

Figure 5: Cartoon-level illustration of the re-equilibration case. A non-equilibrium trajectory begins at N_1 and progresses to N_j. At N_j it encounters a disturbing innovation, departs from the hyperbolic path and travels l.p.c. along a different topological shape until reaching N_{j+m}, where a new equilibrium cycle begins.

When I say a particular construction is necessitated what I mean is that the construction is *made necessary* in order to satisfy some particular purpose the person doing it has in mind as the reason for engaging in this action. For example, if the purpose of the construction is to propose an hypothesis of the empirical Nature of an OB on the basis of observation and experiment, the topological model must be made in such a way that its principal quantities do not stand in contradiction with the scientific observations the hypothesis proposes to understand. There are a great many ways by which a topological neighborhood *could* be defined.

Research in developmental psychology has led to a model of human necessitation that is not-incongruent with mental physics and pertinent to the discussion in this paper. Piaget wrote,

> The principal results of the present research can be summarized in the following three points: (1) Necessity pertains to the composition carried out by the subject and is not an observable datum inherent in objects; (2) it is not an isolated and definitive state, but the result of a process (necessitation); and (3) it is directly related to the constituting of possibilities that generate differentiations, whereas necessity is related to integration – hence, the two formations [possibility and necessity] are in equilibrium. . .
>
> [Necessity] does not emanate from objective facts, which are by their nature merely real and of variable generality and therefore subject to necessary laws to a greater or lesser extent. They only become necessary when integrated within deductive models constructed by the subject. The necessity of *p* can thus not be characterized only as the impossibility of not-*p*, since new possibilities can always emerge, but must be described in Leibniz's manner as the contradiction of not-*p*, and this relative to a specific, limited model.
>
> What, then, is this endogenous origin of necessity? . . . Such a principle at the base of the necessitation process, and one having axiomatic validity, would be: "It is necessary that necessities exist," without specifying what they are. But why do there have to be necessities? It is because without them thinking would get lost in a Heraclitean flux, if it forgot or neglected them. And since thinking is always in development, it cannot do otherwise . . . than to integrate the past with the current state. Such integration, once complete, is the source of necessity. . .
>
> We thus define as necessary those processes the composition *C* of which cannot be

> negated without leading to a contradiction. It is obvious, and this confirms the role of assimilation, that only the subject's own actions (or operations) permit the verification of the contradictory nature of not-*C*. Reality can only indicate that not-*C*, in fact, [has not yet occurred[7]], which is insufficient to demonstrate its impossibility. . . In particular, the complete integration of the past within the current state, which is a condition for logico-mathematical necessity, can only be inferential in nature, as opposed to other subject activities such as the modification of habits . . .
>
> Being closely allied to integration, necessity thus consists in an auto-organization *causa sui*. It is not an observable datum in the real world. It is a product of systematic compositions that involves a dynamic of necessitating processes rather than being limited to states. This dynamic begins with the formation of concepts susceptible of and designed for mutual composition. It takes its departure from situations in which the organization of concepts is heterogeneous and includes only partial comparisons in terms of similarities and differences and where coordinations by reciprocal assimilations are not attained. . .
>
> What is to be learned from these situations is rather obvious: there exists no more an absolute beginning in the development of possibilities than one can determine an absolute end to necessity. Any necessity remains conditional and will need to be transcended. Thus, there do not exist any apodictic judgments that are intrinsically necessary. [Piaget (1983), pp. 135-143]

These specific findings are congruent with the theory of mental physics. There is no science without the scientist. Mental physics can delimit a scientist's theoretical *options* but it cannot, nor does it try to, do away with the role of empirical science, nor does it try to embrace a return to the failed metaphysics of rationalism. It does oppose embracing the failed metaphysics of pure empiricism and the pseudo-metaphysical prejudices of scientific materialism.

Now getting back to the topic at hand, the topologist's selection of criteria for defining a topological neighborhood generally follows a doctrine of method and quite often follows one equivalent to conforming with three "methodological axioms" known as Hausdorff's postulates for neighborhoods [Vaidyanathaswamy (1960), pg. 69]. These are:

1. Each element *p* of a set *S* (which is to be made into a topological space) has at least one neighborhood and is an element of every one of its neighborhoods;
2. If U_p and V_p are two neighborhoods of *p*, there exists a neighborhood of *p* which is contained in $U_p \cap V_p$;
3. To each neighborhood U_p of *p*, there exists a neighborhood V_p of *p* such that U_p contains a neighborhood of each point of V_p.

The first two Hausdorff postulates are concerned with neighborhoods of a single *p*, while the third links together the neighborhoods of different places. It is important to note that (3) does not say that all the members of V_p must be contained in the *same* subset neighborhood within U_p. It only says that at least one such a V_p must exist and that within U_p there must be subsets that taken *overall* constitute neighborhoods for each element $x \in V_p$. This postulate insures that V_p is locally path connected in U_p.

Sets are defined by associations of elements and associations are defined by properties. Sets, such as somatic activity fields, that constitute principal quantities are always *finite* sets[8] and the

[7] Piaget actually wrote "never occurs" here, but this was mere verbal carelessness. As the very old saying goes, "You can't prove a negative." Piaget knew that. "Never" is a *very* long time.
[8] The axiom of infinity in the Zermelo-Fraenkel-Skolem axiom system is not objectively valid [Wells (2006), chap. 23].

property sets that define the association must be systematically not-inconsistent. By this I mean the properties used to define a set cannot contain semantic antinomies (Russell paradoxes). This statement is an *axiom of subcontrarity* replacing the Zermelo-Fraenkel-Skolem (ZFS) axiom of substitution (also called the axiom of replacement)[9] in an axiom system of Critical mathematics[10].

Different property sets define different topologies. Because functions are often used to define properties, it is instructive to look at the practical definition of a mathematical function:

> [A function f is] a many-to-one correspondence [that associates] with each element r of [a set] R a definite element $s = f(r)$ of [a set] S. If every element of S occurs as a value of $f(r)$ as r varies in R, we say that f is a map of R **onto** S; in the contrary case we say it is a map of R **into** S. [Vaidyanathaswamy (1960), pg. 9]

A somatic activity field is an association of somatic places where somatic activity is measurable [Wells (2011d)]. The consequence of this definition is that the topological space of activity fields belongs to a mathematical classification called a Hausdorff[11] topological space [Baum (1964), pg. 40]. Hence, the use of Hausdorff's axioms of method is appropriate for this theory. For purely practical reasons, the formal (hypothetical) somatic state space X is defined as the universe of somatic places *regardless* of whatever activity might be found in each specific place. Therefore a topological space is a subspace of X. This is analogous to defining the universe set of a topological space to be the set of rational numbers while defining its state space to be the set of real numbers. This permits activity fields to be constituted as what topologists call "open sets" and to be regarded as topological neighborhoods.

At the date of this writing, it seems very likely that future *empirical* work will lead to the setting of additional properties for defining activity field neighborhoods and subsets. Qualitative analysis of brain scan images is an example of empirical research that in a very practical context can be regarded as an investigation of neighborhood properties exhibited in brain function. For now, though, what is discussed in this paper is all that we are justified in asserting *a priori* in regard to somatic neighborhoods and neighborhood systems on the basis of the metaphysical analysis carried out to date.

One last item needs to be discussed in regard to the formal structuring of the topological problem. To this point, the topological ideas being used have been illustrated by means of two-dimensional cartoon representations. It is quite obvious that the somatic state space is of a dimension much higher than two, and so the appropriateness of the cartoon examples is called into question. To respond to this question, it is to be noted that in the development of most structures in mathematics, functional notation in terms of a single variable is used in defining multi-variable concepts and constructs. Mathematicians do this by means of a rather neat trick,

---

[9] for definition of the ZFS axiom system, see Bernays (1968), pp. 3-26.

[10] The axiom of substitution is not an axiom but, rather, an axiom schema for an indefinite number of axioms. Its formal statement using the Russell-Whitehead formal language system of mathematical notation is so complex that it is even difficult to precisely render it in English. ZFS, like other usual axiom systems that appear in mathematics, formulates all its axioms as positive assertions so that they basically could be constituted as machine-readable instructions. (This is because of Hilbert's failed program of formalism). If we are willing to admit that computers and Turing machines are not mathematicians and mathematicians are people, then an axiom stated using subcontrarity, "*is* not-inconsistent," will do because an OB can judge *in*consistency from finite examples. The axiom of substitution attempts to make a *positive* definition of "consistency," which is an altogether more difficult task but one that *aims* at *preventing* inconsistency. Is it not simpler just to say "if I find an inconsistency in my axiom system, I must change my axiom system"? Persons who embrace the metaphysics of rationalism cannot make this admission, but the rest of us can.

[11] Named after Felix Hausdorff, the German mathematician who pioneered topology theory. Hausdorff and his wife committed suicide in 1942 in order to escape being sent to a Nazi extermination camp.

namely, to regard this single variable as being the representation of a set of variables. In this way, they can define a function of one variable and then use this as a basis for expanding the definition to cover functions of two, three, or however many variables one might require. Topologists, for example, use this trick to develop the notions of connectedness and path-connectedness in terms of mathematical "points" (so-called "zero-dimensional properties") and then extend these concepts to concepts of "n-path-connectedness" by replacing points with polyhedra [Wall (1972), pg. 47]. The analogous operation here is to regard figures 3-5 as planar projections of some higher-dimensional state space. This is done, for example, by regarding the axis variables *x* and *y* in the figures as being representations of *vector partitions* of the state vector, i.e. $S = [\mathbf{x} \mid \mathbf{y}]^T$ rather than just $S = [x\ y]^T$. This trick can be applied to produce state vectors of however high a dimension as one might require. Such are the practical tricks-of-the-trade employed by mathematicians in working their scholarly craftsmanship.

III. Graph Theory and Embedding Field Theory

It is a well known empirical fact that the phenomenon of *soma* exhibits physiological changes in appearance over time (biological maturation, growth and development, re-organization of brain structures, etc.). It is also a well known fact that human behaviors exhibit a pattern of psychological development that objectively grounds positing the *Dasein* of *kinesis* for the phenomenon of *nous* ("mental development"). Furthermore, the principles of mental physics present the objective actuality of mental development as a theorem of *a priori* necessity for the possibility of experience as human beings come to know the phenomenon of experience. When the phenomenon of being human is modeled mathematically in objective time, these facts lead to an apodictic consequence, namely, that the functions *f* and *g* depicted in figure 1 vary in objective time. This theoretically necessitated variation, however, is tightly constrained by the *a priori* condition that the pair [*f*, *g*] must always comprise a homeomorphism.

On the other hand, mental physics does not and cannot specify specific formulas of the system [*f*, *g*] because such formulas present *knowledge of experience* – and are for that reason contingent concepts. Understanding the [*f*, *g*] system in its specifics is one of the tasks within a special sub-discipline of a broader natural science called Critical anthropology[12]. I call the special science taking this task under its topic ***Critical psychophysics***. Critical psychophysics is the natural science of *nous-soma* reciprocity. Mental physics provides its *Metaphysische Anfangsgründe* (metaphysical rudiments) and the sensorimotor idea is its applied metaphysic.

Psyche is the faculty for enforcing thorough-going *nous-soma* reciprocity in organized being and so Critical psychophysics is a science of *psyche*. As regards the [*f*, *g*] system, though, its task is one of mathematical psychophysics and it must, therefore, be concerned with objectively valid mathematical modeling of both the noetic and somatic systems. (Mathematical neuroscience is a *framework* science for it insofar as its theoretical constructs deal *jointly* with those objects of experience we classify as belonging to physical body and those we classify as mental). Its doctrine of method, therefore, requires a mathematical schema suitable to both *nous* and *soma* for understanding the temporal dynamics of each in objective time. This series of papers cannot be brought to a conclusion without discussing what is needed for and pertinent to this. It is to fulfill this need that another topic within mathematics is brought into the overall context of the knowledge representation problem, namely, the topic known as ***graph theory***.

As of the date of this writing, there has not yet been enough time for graph-theoretic treatment of *nous* to yield up enough experience with sufficiently significant pertinence to be employed in

[12] Critical anthropology is "a doctrine of the range of knowledge of the human being possible through observation" [Kant (1800), 7: 119].

this paper for understanding the dynamical schema issue. What I mean by this is that this aspect of Critical mathematical-neuroscience is still in its primarily qualitative phase of development, and even those ventures into the topic that have been produced by psychology, e.g. Piaget *et al*. (1968) and Piaget (1975), are too qualitative to provide very much assistance. However, the same cannot be said of ***embedding field theory*** [Grossberg (1968, 1969a, 1971)]. For this reason, and because embedding field theory has not become widely known within the community of neural network theorists, it is pertinent and important to briefly discuss it here.

The use of graph theoretic constructs called mathematical neural network models has been in practice for half a century. For the first several years (and, to a great extent, to the present day), the earliest essays in this new technical art were primarily forays of engineering said to be inspired by biology (biological neural science in particular) and did give some passing notice to psychology. Yet it would be both fair and accurate to say that these linkages were more romantic than scientific. The first serious efforts to scientifically link graph-theoretic neural network theory, neural science and cognitive psychology were made by Grossberg in the latter half of the 1960s. It was Grossberg who named this new doctrine embedding field theory, and the name is particularly well-suited to the problem of knowledge representation.

Furthermore, neural network constructs are particularly good implements for set membership mathematics, satisfying a key requirement of the Critical doctrine of method [Wells (2011a)]. For example, a neural network classifier can be regarded as a function representing a set membership solution set. An ART network[13], e.g. ART 2 [Carpenter and Grossberg (1987)], can be regarded as a function for dynamically updating set membership solution sets. The viability of SMT (set membership theory) as a method for implementing adaptive systems has long been demonstrated (e.g. McCarthy and Wells (1997) and the citations therein), and there is today no doubt that embedding field theory and ART provide a powerful structure of applied mathematics for SMT.

Although it has long been known that topology theory finds useful applications in graph theory, e.g. Wall (1972) pp. 90-91, the synthesis of topology theory and graph theory historically has been an underdeveloped (or, at least, under-emphasized and under-taught) topic in applied mathematics. The fundamental significance of topology theory for the problem of knowledge representation has, I hope, been amply demonstrated in this series of papers. The fundamental significance of applied graph theory for neural networks – and, therefore, for the dynamical problem of knowledge representation – has had a long history of awareness. However, it is probably more accurate than not to say that the link between knowledge representation and neural network graphics has not been sufficiently brought out. To the best of my knowledge, the most clear linkage between them is provided in embedding field theory, to which we now turn.

Viewed in terms of knowledge representation in general, the general reduction problem is one of going from a semantic representation to noetic and somatic representations of knowledge. By the term *semantic representation* I mean any concrete instantiation of an example schematized to represent an object appearance for which an Organized Being has formed meaning implications. For example, the written letter sequence "ABC . . ." has as one of its meaning implications "the alphabet" for an OB who is a literate English-speaking person. In grammar theory, representation using sentence structure pattern diagrams, e.g. figure 6, can be regarded as a semantic representation to someone who has been trained in how to properly construct and interpret them. What all such representations have in common is presentation in terms of a spatial structure representation and an ordering structure representation. Metaphysically, what all such representations have in common is that within the structure of the Organized Being they are "dual coded" in a noetic and a somatic representation.

[13] The acronym ART stands for "adaptive resonance theory." ART was discovered by Grossberg in 1976 [Grossberg (1976)] and was an important outcome of embedding field theory [Grossberg (1978)].

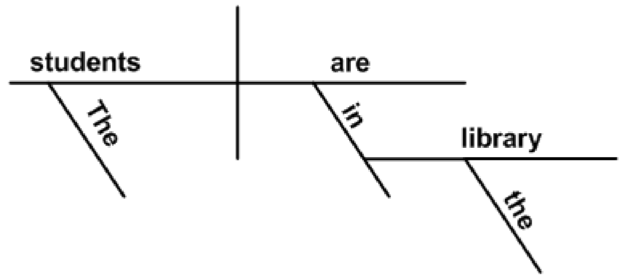

Figure 6: Example of a sentence structure pattern diagram for "The students are in the library."

Furthermore, the mathematical representation of a semantic *parástase* in *nous* is spatially and temporally *discrete*. This is illustrated in numerous places throughout Wells (2009) by the various diagrams of noetic representations. On the side of *soma*, observations of biophysical phenomena recorded by measuring instruments are usually presented mathematically as spatially and temporally continuous. It cannot be over-stressed, however, that such representations are not yet representations of signals (not somatic representations) because the raw observational *data* by themselves carry no *inherent* meaning implications. Meanings are noetic objects, not physical objects. The passage from raw observational data to the somatic *parástase* of a *signal* likewise involves a "chunking" of the *data* into a phoronomic form that is spatially-discrete even if its dynamic mathematical representation remains temporally continuous *in toto*. However, even such a dynamic temporal continuity of representation becomes mechanically[14] discrete, i.e. takes on a re-presentation in terms of objective time *intervals*[15] as soon as one begins talking about signal*s* (in the plural) rather than the general signaling complex.

This semantic property of interaction between spatio-temporally continuous and spatio-temporally discrete depictions was noted long ago and linked to the phenomenon of learning by Grossberg. He wrote,

> [Although] each sensory modality seems to provide us with essentially different varieties of experience, the very same language tools are adequate for describing at least the rudiments of all of these various modalities. Thus, the discrete representation of continuous processes must be a *universal representation* of some kind. For this reason, we expect conclusions about the dynamics of language behavior to generalize to many other psychological phenomena.
>
> The centrality of the connection between relatively discrete and continuous phenomena in behavior is better understood by considering several simple examples. Consider the phenomenon of walking for specificity. When a child begins to learn how to walk, he must concentrate much effort on the endeavor, and must attend continually to his efforts. An observer is struck by the many motions of the child that are inessential to the walking process, and by the total absorption of the child in the process. In an adult, walking takes on a different appearance. A first step is automatically followed by a second, the second by a third, etc. Once the decision to walk is made, the walk essentially takes itself, and one can pay attention to other matters so long as a minimal amount of obstacle avoidance is accomplished. After walking to one's destination, one "decides" to stop walking and the walking comes to an end. . . The very process of learning how to walk involves a passage from a relatively continuous representation of voluntary efforts at walking to a relatively discrete representation of these efforts.

[14] In using the terms phoronomic, dynamic and mechanic, I refer to Kant's system of phoronomy, dynamics, mechanics and phenomenology in representational Quantity, Quality, Relation and Modality. This was explained in Wells (2011d).

[15] A time interval serves as a discrete *quantum* in set membership theory. For a mathematical example of the use of intervals as "numbers" in set membership theory, see Wells (1996).

> The intuitive significance of such a passage is easy to see. Once the saying of a verbal unit seems to the performer to be a simple act rather than a tremendously complicated juxtaposition of delicately poised muscular motions, he can proceed to integrate several of these units into more complicated composite units constructed from sequences of seemingly simple acts. After these composite units also seem to be simple, the composite units themselves can be organized into still more complicated composites, and so on. Without the reduction of continuous (and complicated) acts to discrete (and simple) acts, the integration of more complicated behavior based on these acts would seem hopelessly complicated. . . The passage from initially continuous representations of behavioral controls to asymptotically discrete representations is thus no casual event. It makes possible the emergence of new organized behavior patterns and is a prerequisite for effective learning. . .
>
> Properties of discreteness and continuity coexist at every stage of learning. The continuous background is never wholly eliminated. We must study how certain processes superimposed on this background become increasingly discrete relative to an initially prescribed standard of continuity, and will have at our disposal at least two different levels of dynamical graining such that the degree of continuity of one level takes on a meaning only relative to the degree of continuity of the other. [Grossberg (1969a)]

Grossberg demonstrated how these dynamical "grains" can be represented by a graph structure. Although earlier mathematical neural network theorists had used graphical forms as representations of their networks, the significant new idea Grossberg introduced was a general concept of what each vertex ("point") in a graph *could* represent depending on the *level* of representation L. This is a *schematizing* idea that was absent in the work of earlier neural network theorists. He referred to the collection of such "points" (vertices) in the structure as the "field of points" and noted that the actions of different field "points" were affected by the actions at other field points. The representation of these interactions is presented by functional arcs in the graph.

Figure 7 illustrates this concept. For the sake of clarity only one field interaction is depicted and it is to be understood that a field interaction is presumed to exist between every pair of vertices in the graph. Vertices p_j denote spatially discrete places in the field space. A depiction of

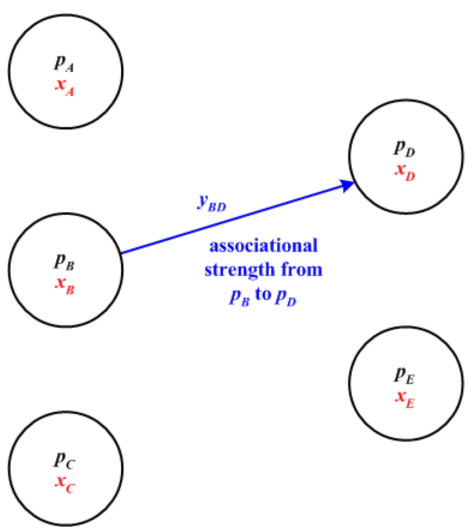

Figure 7: Graphical representation of a field at some level of description L. Vertices p_j represent discrete spatial places where local actions $x_j(t)$ occur in objective time t. The numerical value of x_j represents the degree of action occurring at p_j at time t. $x_j = 0$ denotes "no action." Action at a point p_j affects the actions occurring at all other points. This is a field interaction and is represented by a measure of the amount of the effect produced, called the associational strength y_{jk} from p_j to place p_k.

of the field space in terms of discrete places ("field points") *is a requirement of Critical mathematics*. This is because these points do not denote an idealized geometric point-in-space but, rather, a *solution set* of spatial *intervals* defining a volume in the space within which further fine discriminations of the local character of the space *are not made*. This is what is meant by the term *level of description*, L. Naturally, making a graphical depiction at level L does not preclude the possibility of making a lower-level depiction L-1 provided that L-1 does not call for more empirical certainty than can be provided by the measuring instruments used to associate physical phenomena with mathematical principal quantities. If L-1 is not precluded by this empirical consideration, then each vertex in figure 7 is replaced by a subgraph of the same schematic form as figure 7 illustrates. The practice of doing this is called ***scientific reduction*** (SR). Contrariwise, the theorist might choose to re-present the structure in less specific detail, as when he wishes to embed a part of his overall system into another graph, again schematized as in figure 7. In this case, all the places illustrated in figure 7 might be combined into one integrated place p at a level of description L+1. This p is then used as a specific vertex in a level L+1 graph. The practice of doing this is called ***model order reduction*** (MOR).

At each place p_j it is posited that "something is happening" in objective time t. This something is called the ***action*** x_j at p_j. It is a mathematical quantity represented by a numerical value that is in some way a measure of "how much is going on." This measure is an ***intensive magnitude***, which means that it is a unity in which the idea of a multiplicity of values can be represented only by an approximation to negation ("no action occurring"). In Kant's terminology, x_j is said to "fill the space" at p_j [Kant (1786), 4: 496]. The numerical value assigned to x_j is called the ***degree of matter*** at p_j and is a *parástase* of dynamic Quality at p_j. It is again important to note that if this degree of matter is to be associated with any sensible physical appearance then x_j must be depicted as a set membership solution set, i.e., as an interval and not a "point" solution such as the transcendental number π[16].

The action at each place p_j is posited to interact with (have an effect on) the actions at every other place p_k. This is the field effect and is represented by a function y_{jk} called the ***associational strength*** of the action effect at p_k by action x_j. This y_{jk} is a mechanical idea of Relation, and it is the idea by which we regard x_j as a ***moving power*** of matter [Kant (1786), 4: 497]. By convention, y_{jk} has a minimum degree of 0 and a maximum degree of 1.

Next the phenomenon of learning has to be taken into account. Let us call the graphical model of the system M.[17] Let us designate an aliment of the system (e.g., an "input") at place p_j by the symbol r_j and call a temporal sequence of aliments a *list*. We assume each r_j is drawn from a set of possible aliments, R, with $r_j = 0$ denoting the sensible absence of an aliment at p_j. We may further model the aliment effect as a vector $\mathrm{r} = [r_1\ r_2\ .\ .\ .\ r_n]^T$ if there are n vertices in the graph. We now come to the essential core idea of Grossberg's embedding field theory. *At the semantic level representing phonemes, each vertex in the graph represents a specific phoneme*. We will call a graph at this level of representation a ***semantic phoneme graph***. In Grossberg's words,

> Consider a machine M before it has learned anything. Suppose that M is capable of learning any list chosen from [a sequence] $r_1\ r_2\ .\ .\ .\ r_n$ in which no symbol r_i occurs more than once. Suppose also that M is unbiased for specificity. . . Since M begins in a state of maximal ignorance . . . x_1 grows momentarily and large signals are transmitted to all the

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[16] In classical electromagnetic field theory and the semi-classical "picture" of an electron, failure to treat the space-place p_j as a set membership interval causes a famous antinomy. If the electron is regarded as an ideal Euclidean "point" in space, then a straightforward calculation shows that its electric field produces an *infinite* amount of electron mass. Because we know that the mass of an electron is really very, very tiny, this is a rather odious paradox. Quantum electrodynamics solves this problem by a method that, in practical effect, treats the principal space quantity as an SMT interval. The process is called renormalization.
[17] Grossberg called this graph a "machine."

> other points p_j, $j \neq 1$. If r_2 then occurred, p_2 sends large signals to all the other points p_j, $j \neq 2$. And so on. Before learning occurs, therefore, the entire "field" of points is influenced by an event at a single point, i.e., a kind of "Gestalt" effect "in space" occurs . . .
>
> Now let us consider M after it has learned the list $r_1\, r_2 \ldots r_n$. Then, by definition,
>
> $$y_{12}(t) \cong y_{23}(t) \cong \cdots\ y_{n-1,n}(t) \cong 1$$
>
> for all times t during which M knows the list, and all other $y_{ij}(t)$ are approximately zero. Thus, a chain of associational strengths leads from p_1 to p_2, from p_2 to p_3, and so on until p_{n-1} and p_n are reached. This chain has been embedded into the field of M's alternatives – hence the name "embedding fields" for our theory. [Grossberg (1969a)]

Expressed in graph-theoretic terms, the outcome of adaptation dynamics appears within the topology of a graph as structures that have semantic interpretations. If the vertices of the graph represent semantic phonemes, the arc structures y_{ij} that adaptation dynamics develop constitute the formation of higher level semantic structures. Higher level structures become ***embedded*** in the graphical field. This is the significance of the earlier statement we saw where Grossberg said the OB can "integrate several of these units into more complicated composite units constructed from sequences of seemingly simple acts." Herein lies the ingenious insight of embedding field theory.

IV. Topological Embedding Field Theory and Mind-Body Science

A depiction (*parástase*) at the level of a semantic phoneme graph can be re-presented as a higher level graph in which the vertices represent phoneme *structures* rather than individual phonemes. Likewise, a semantic phoneme graph can be synthesized from a lower level graph in which the vertices depict semantic phones. (Phones are combined to form phonemes). These are both examples of MOR synthesis. In the language of Critical Logic, such a synthesis is called a prosyllogism [Wells (2011b)]. Contrariwise, the procedure can be reversed. One can descend from a semantic phoneme graph to a semantic phone graph or from a phoneme structure graph to a phoneme graph by scientific reduction. This kind of synthesis is called an episyllogism.

Embedding field theory provides not merely a model but a *framework science* of modeling schemata applicable to moving up and down a ***ladder*** of MOR/SR levels. The crucial constraint, and principal thing to bear in mind, is the general *semantic context* of this paradigm. It is this context that fuses specific mathematical procedures to the topological theory in Wells (2011d) and its further discussion in this paper. However, for all this to be useful in understanding mind-body phenomena, one must have a way to *contextualize* a semantic graph representation in terms of somatic and noetic graph representations.

The practical utility of science for society is achieved because knowledge can be organized at different levels of abstraction. Bridges are made of atoms but a civil engineer does not (and does not need to) think about atoms when he designs a bridge. The MOR/SR process in science can be regarded as forming a ladder in which the rungs represent knowledge at different levels of abstraction. However, rungs in a ladder are held together by *railings*. This is a point taken too much for granted except by general system theorists. A typical scientist, when he has to move between levels of abstraction, often uses the term "modeling" to describe a MOR ascent of the ladder and the term "hypothesis" when he makes a descent by SR. A model (MOR outcome) is often looked upon with a trace of disdain as being somehow "less basic" as a description of Nature. In science education it tends to be treated as craftsmanship and students are usually expected to pick up on it – by osmosis, perhaps? – as they learn their special discipline. But this is a mere ontological prejudice. Hypothesis-making, on the other hand, tends to be venerated as if it were a mystic capacity scientists possess and non-scientists do not. The result is that both skills are treated as arts rather than as disciplined parts of a canon of scientific practice.

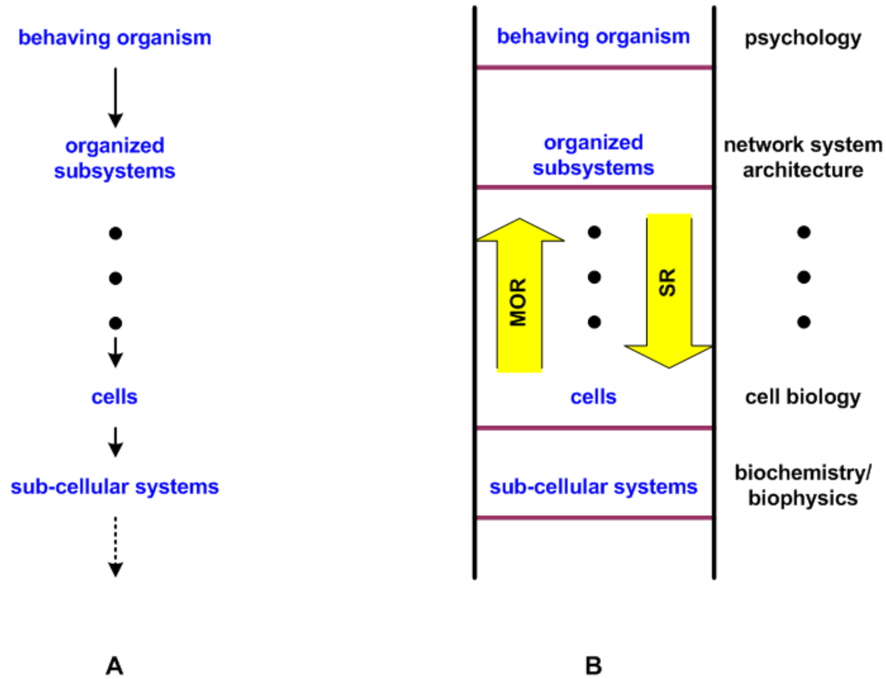

Figure 8: Typical depiction of levels of organization (A) and its re-depiction in ladder form (B).

The ladder issue is appreciated in theoretical neuroscience but not as clearly appreciated as it needs to be. Figure 8A illustrates a typical depiction of what computational neuroscience textbooks usually call "levels of organization" in brain theory. The arrows are meant to denote the process of scientific reduction but in effect all they say is "and then we go to here." There is no explicit indication given in 8A that says "but make sure every pair of neighboring levels is a self-consistent whole." There is, in other words, no overt indication that the ladder of knowledge has rails. Figure 8B is a better mathematical representation of what 8A is supposed to be saying. The rungs of the ladder are typically viewed as being "owned" by one or a few of the special sciences or sub-disciplines within one of the special sciences. However, if these sciences (or their sub-disciplines) become "silos of knowledge" and do not effectively communicate with the others, what we have is not a unified scientific system but, rather, a disintegrating *potpourri* of knowledge fragments. The rails of the ladder are concerned with the structural integrity of the overall canon of science in general and its *disciplined* construction can be called *general system theory*. Note that the ladder depicts MOR and SR as *co-equals* and jointly essential to the unity of neuroscience overall. This section of the paper is about the rails of the ladder.

By one widely-accepted estimate, there are around ten thousand distinct classes of neurons, around 100 billion neuron cells, 100 trillion synapses, and somewhere in the range of 500 billion to 1 trillion glial cells in the anatomical brain. This staggering complexity is so vast that its true scope is as practically unimaginable as the notion of the distance from the earth to the Andromeda galaxy. One consequence of scientific materialism is that "brain science" begins by making a specious real division between the phenomenon of mind and the phenomenon of body and then discards the former with a romantic hope that mental phenomena will somehow re-emerge as a property of this unimaginably complex dead-matter system of cells. Even psychology tends to fall victim to this ontology-centered prejudice. Reber tells us,

> **mind** This term, and what it connotes, is the battered offspring of the union of philosophy and psychology. At some deep level we dearly love and cherish it and see behind its surface great potential but, because of our own inadequacies, we continually

> abuse it, harshly and abruptly pummeling it for imagined excesses, and occasionally even lock it away in some dark closet where we cannot hear its insistent whines.
>
> The history of the use of this term reveals two conflicting hypotheses: the tendency to treat mind as a metaphysical explanatory entity separate and apart from mechanistic systems, and the tendency to view it as a convenient biological metaphor representing the manifestation of the still-not-understood neurophysiological processes of the brain. [Reber (2001)]

Where this state of affairs leaves biological neural science is this: the bio-mass of the brain carries in it no nameplates telling us "the function of this part of the anatomy is x." Only at the periphery (sensory cells and motor neurons) is the practical function of the neuro-glial anatomy discernible. Within this outer boundary, all guesses as to what *function* the cells are implementing (and, therefore, what the anatomical structure and physiological processes *mean*) is speculation and nothing more. The practice of science under the prevailing paradigm of pseudo-metaphysical materialism is what Kant labeled "natural science improperly so-called." The knowledge that neural science needs to succeed in understanding the central nervous system is the knowledge it threw away at the very first step when scientists make the fictitious real division between the phenomenon of mind and the phenomenon of body. The *real object* of the science is the entire human being – the Organized Being. No more, and no less.

Biological objects regarded merely in and of themselves *have* no meaning in the context of knowledge. It is, though, intuitively clear that these objects have something to do with knowledge and so the key question is "what?" This is where a methodology of cross-linking semantic ladders provides a route to answering the key question. Figure 9 illustrates this idea. This series of papers has revealed that semantics can be mathematically expressed as a synthesis of topology and embedding field theory (center ladder in the figure). Let us explore the implications of this.

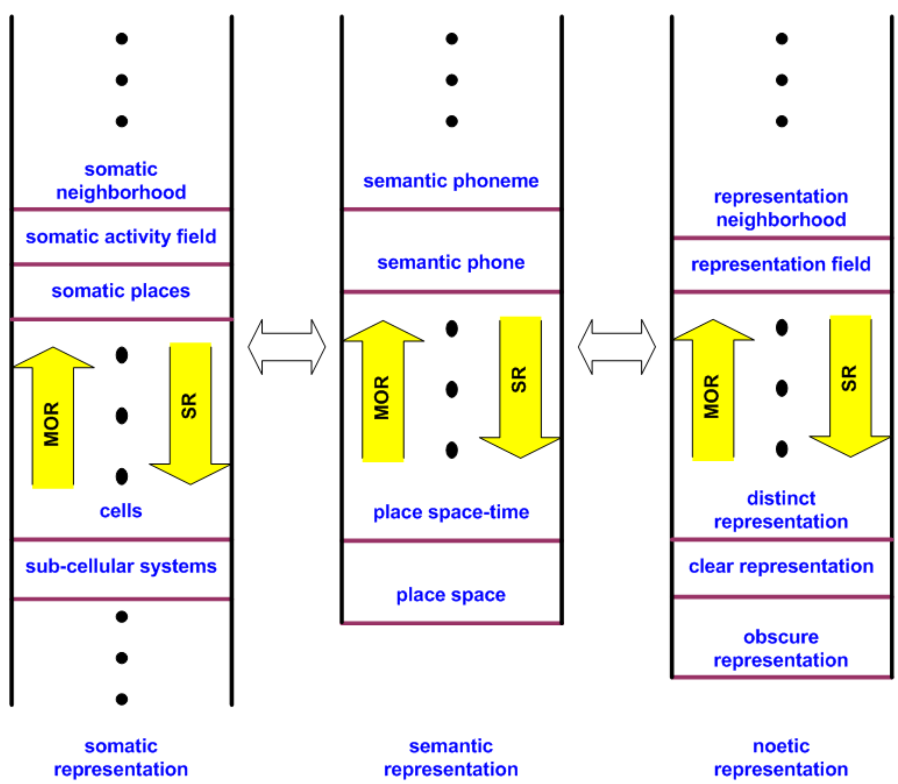

Figure 9: Somatic and noetic ladder organizations of knowledge linked through topological semantics.

Meaning for biological organization is achieved by synthesizing a horizontal coordination of ladders of the form semantic organization ⇔ somatic knowledge representation ⇔ systematic neuroscience (figure 8B). The intermediary structure is somatic knowledge representation by means of topological embedding field theory. The intermediary representation is used to identify state-space forms of organization (structures embedded in a bio-somatic field) and to define what constitutes signals and signaling complexes in terms of somatic state-space (figure 1). Structures so defined *mathematically*, with identified principal quantities, provide the clues for what capacities must be presented in biological appearances. Anatomical and physiological research then looks for homeomorphic relationships of this kind. Biologists try to do precisely this already, but the system discipline required to provide adequate tools for the search has not been there to support the effort. The practical consequence is that merely having very smart people working on it isn't enough; a genius must appear on the scene from time to time or else some fortunate experimental accident must occur that jars the working paradigms and premises.

If, on the other hand, the science works from practical (meaningful) ideas of mathematical organizations of biological function, then, hypothetically at least, one can descend from a somatic activity graph to an embedding field graph in which the vertices depict some level of biological anatomical structures, descend from these to successively lower-level anatomical structures, and so on until one runs out of instrumentation capability to provide for objectively valid association of principal quantities in the graph with sensible physical phenomena. Here I say "hypothetically" because even with adequate tools the mind-body system is still enormously complex and the contingency of empirical knowledge provides plenty of opportunities for very smart people to make mistakes. No one should underestimate the challenges of science.

This is the biological *logical* division of neuroscience. We must also account for the mental logical division. Here the same methodology applies except that the horizontal structure goes semantic organization ⇔ noetic knowledge representation ⇔ noetic state-space embedding field topology (right-hand side of figure 9). With semantic structure re-expressed as noetic embedding field topological structure, one proceeds methodically in the same way as would be done in the biological division *except* that in this case we are dealing entirely within the framework of *nous*. An embedding field graph depicting noetic representation structures is deduced at some level. From there, one can either continue the SR process into progressively lower levels of noetic representations or undertake MOR into higher levels of noetic representations until reaching the ***behavioral level*** studied by psychology.

V. Critical Psychophysics

The point of conjunction between biology and psychology is semantic topological embedding field theory and its coordinated connections to the ladders on either side of it in figure 9. This arena of research is what is properly called Critical psychophysics. The psychophysicist need not be a master of all science; the empirical details of corporal *Existenz* belong to biology and the empirical details of mental *Existenz* belong to Critical psychology. The psychophysicist must, however, take *psyche* as his logical object of research and know enough about biology, psychology and mathematics to be a synthesizer. In this, the focal point is the sensorimotor idea (the applied metaphysic of *psyche*), which grounds the types of functions that must be employed for objective validity in semantic constructs. On the side connecting to biology is the judicial-sensorimotor idea (J-SMI) [Wells (2011d)]. On the side connecting to psychology is the theoretical-sensorimotor idea (T-SMI) [Wells (2009, 2011f)]. The J-SMI and T-SMI combine to form the applied metaphysic of *psyche*. The combined structure is a 3LAR providing 6,561 general synthetic functionals for the Critical expression of *nous-soma* reciprocity. Figure 10 illustrates the 3LAR structure of the sensorimotor idea.

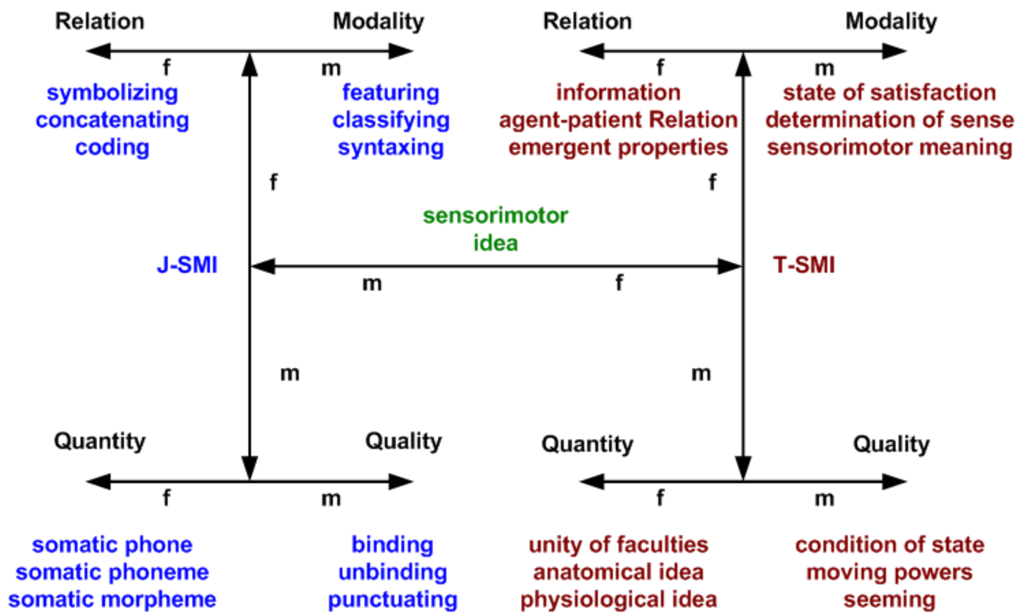

Figure 10: General 3LAR structure of the sensorimotor idea of *psyche*.

The twenty four *momenta* in figure 10 have been explained elsewhere [Wells (2011d), (2011f) and (2009), chap. 4 §3]. In this paper I speak to the homeomorphic functions *f* and *g* of figure 1. The first (and *practically* most fundamental[18]) task is to represent the semantic Nature of signals and signaling on the side of *soma* in a somatic state space model *along with* a homeomorphic image of this, the semantic Nature of noetic representation on the side of *nous* in a noetic state space model. In both cases, topological embedding field theory is the mathematical doctrine required. The empirical Object of psychophysics is the adaptive *psyche* [Wells (2009), chap. 4].

Representation theory for *nous* has been and is being developed. It is an integral part of mental physics. Still left to be filled is a doctrine for knowledge representation in *soma*, and this brings us back around to the subject where Part I of this series left off: signals and signaling.

In Critical metaphysics, root explanations for objects of Critical ontology always reduce to explanations of abilities, processes and functions because it is the manifestations of these in the *homo phaenomenon* aspect of organized being that are the aliments for the possibility of real experience as human beings come to know experience. As was deduced in Part I, understanding of signals and signaling reduces to understanding the Nature of *signaling ability*. We have now come to the point where the Critical answer to the question, "What is the Nature of signaling ability?" can be obtained.

Part I reached the stage where it could be concluded that signaling is the *nexus* of signals. It also noted the defining characteristic of the logical essence of what it means for some appearance to *be* a signal. For an object of appearance to be a signal it must be regarded as an outcome or consequence of some action. We can now specify what kind of action this must be. It must be *a change in accidents of appearance understood as a physical phenomenon in which temporal variations are reciprocally connected in understanding with concepts of semantic objects*. This is Critical ***signaling action***. To make this idea distinct and usable in practice, we must present it in

[18] By "practically fundamental" I do not mean "almost fundamental." When I use the term "practically" it is as an adjective that refers to *reduction to practice*. A science that cannot be reduced to practice has no uses and is, therefore, a *useless* science. A theory that cannot be reduced to practice has no uses and is a *useless* theory. Platonic theories are always useless unless one wishes to practice fantasizing.

terms of a 2LAR of an ability, namely, signaling ability.

Composition of an ability in general is called a power (*Kraft*); therefore the matter of signaling ability is appropriately called *signaling power*. *Nexus* of an ability is the form of that ability regarded as an idea of a *potential* power (*Vermögen*) of organization, i.e. the potential power *to organize*, and this idea in Critical terminology is called a *faculty*. Faculty depicts *how* an ability is exhibited in experience; *Kraft* depicts *what* matter-of-ability is exhibited. A ***signal*** is *an object of appearance understood as the outcome of the action of realizing (making actual) a moving power that stands combined with a signaling faculty*. To understand an object *as* a signal is to understand a signaling power (*Kraft* of a signaling ability); to understand an object *as* an act-of-signaling is to understand the signaling faculty of a signaling ability.

Now, there are two contexts for this, one of which is superfluous to the aim of this paper and the other of which is the constituted aim of the series. The superfluous context is the context of an observer observing some phenomenon and naming it a signal. Here the connection between the phenomenal object and its meaning *as* a signal is made in the judgment of the observer in context with whatever his purpose for observing might be. *This is always an inference of analogy with the object of the second context*. If the second object is understood (with objective validity) then the object of the analogy is understood *in the context of the observer's purposes*. For example, I understand a frog to-be-an-animal *because* I understand myself to-be-an-animal. The frog is: (1) presented (to me) as an appearance of something that is biologically organized; my corporal self is presented as biologically organized appearances of me; (2) the phenomenon of the frog appears to exhibit spontaneity of locomotion; I exhibit spontaneity of locomotion; (3) the frog eats things; I eat things (property of taking nourishment); (4) frogs biologically reproduce; I can biologically reproduce; & etc. An ***inference of analogy*** is *an inference of judgment by which marks of one object concept are made part of the representation of the concept of another object* [Wells (2011e)]. Analogy proceeds under the rule of the principle of specification: things of one genus that agree in many marks agree in all marks as they are known in one object but not in the other. I put the frog in the same genus as myself ("animal") because the frog's appearances sufficiently resemble my own in *specific* contexts.

Thus, to properly judge by analogy that some physical phenomenon is a "signal" or is "signaling" we must first understand these ideas in the context of the Organized Being. This is the second context mentioned above, in which anything to be regarded as a signal or as signaling *must* be subsumed in judgment. This second context of signaling ability provides the *empirical basis* for signaling ability in contexts other than that of an Organized Being[19].

As one might possibly expect from the fact that the applied metaphysic of *psyche* (the sensori-motor idea) is understood as a 3LAR structure, understanding of signaling ability likewise calls for a 3LAR structure. Figure 11 illustrates the skeleton of this 3LAR. The remaining task at hand for this paper is, in a manner of speaking, to put some flesh on the bones.

Signaling faculty is the more straightforward part of the idea. *Nexus* of signaling ability is the representation of manifold of heterogeneous things that combine together "because of the nature of Nature." In our specific context, this is the organization of *objects said to generate signals.*

[19] This is one of the consequences of Kant's Critical philosophy that ontology-centered thinkers have the most difficulty grasping or accepting. "It is too subjective," they might protest. "Real things cannot depend on subjectivity like this." But ask them what they mean by "real thing." An ontology-centered person can not *ground* an answer to this on the human side of the horizon of possible experience. Critical epistemology defines *what it means* "to be real." This definition is practical, not ontological. Again, as Protagoras wrote long ago, "Man is the measure of all things, of things that are that which they are, and things that are not that which they are not" [Diogenes Laertius (*Lives of Eminent Philosophers*), vol. II, IX. 53]. If you *can't* say what you mean then you don't *mean* what you say and you can't communicate anything *meaningful*.

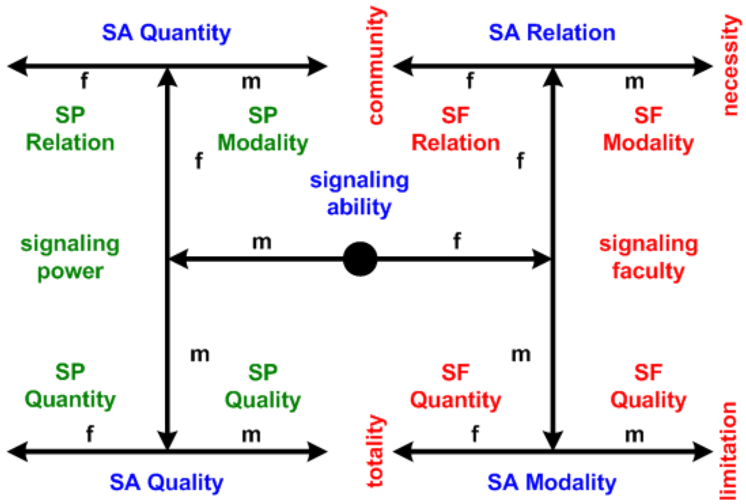

Figure 11: Skeletal outline of the 3LAR structure of signaling ability. SA denotes signaling ability; SP denotes signaling power; SF denotes signaling faculty; m denotes matter; f denotes form.

Now, such an object requires a complete context in terms of object appearances and in terms of objective semantic implications. The former pertains to *composition* of signaling faculty, which is Quantity and Quality in this 2LAR substructure of signaling ability. The latter pertains to the *nexus* of signaling faculty (Relation and Modality in the SF sub-2LAR). As the idea of the organization of signaling ability, SF Quantity is understood by the primitive category of totality, i.e., as the *entirety* of a signaling complex of objects regarded as generating a signal. The composition in Quality is judged by the category of limitation because we are considering the organizational faculty that is regarded as generating a *specific* signal. In terms of physical *nexus* (SF Relation), the idea of signaling faculty is understood by the category of community because saying a physical phenomenon stands in a semantic relationship is equivalent to saying it is homeomorphic to some noetic representation. The SF Relation is an idea of the form of *nous-soma* reciprocity. Finally, because we wish to understand somatic signals as somatic events that *actually depict* something, the idea of SF Modality is judged by the category of necessity since signaling theory is the theory of a *system* of signaling, thus unity of its metaphysical manifold is a unity *made necessary* by the purpose of the theory. This is the Critical Logic of signaling faculty.

The *synthesis* of every objective idea of a signaling faculty is governed by the *momenta* of the theoretical-sensorimotor idea = {physiological idea, seeming, emergent properties, sensorimotor meaning} (refer to figure 10)[20]. The first three *momenta* are functions of the theoretical data of the senses, the last a function of the transcendental sensorimotor idea. The objective functions (Quantity, Quality and Relation) pertain to "spanning the gap" between empirical science and rational science (because that is the role of the T-DOS *momenta* in the applied metaphysic of *psyche*). The Modality function falls under the transcendental sensorimotor idea in the theoretical Standpoint as the subjective function pertaining to *how* the object concept is held-to-be-true in meaning implications (rational *grounding* of *empirical* understanding). In terms of mathematical object representation, these *momenta* correspond to representation of {integration, subcontrarity, transitive Relation, determining factor} [Wells (2011c)].

This is the metaphysical context for a *faculty* of signaling. Now we must turn to the more subtle context of signaling power, which must explain the context in which a phenomenal object can be regarded to be the manifestation of a signal. We are inquiring into "what is signal-matter?"

[20] For the technical explanation of these *momenta* refer either to Wells (2011f) or Wells (2009), chap. 4.

This inquiry, though, is inquiry into epistemologically correct *context*. The *essential* context of signaling power is *semantic* context, i.e., context deduced from the judicial sensorimotor idea. To *realize* (make actual) a semantic content requires construction of structures of self-organizing transformations that *in combination* constitute the performances of:

- **in Quantity** – acts of aggregation assimilating somatic activities into the form of a specific topology $\mathfrak{I}$ in some subspace X of the universe X of somatic places;
- **in Quality** – acts accreting or dissipating somatic activity fields such that locally path connected trajectories are formed in some topological space $(X, \mathfrak{I})$;
- **in Relation –** acts of constitutive somatic place *coordinators* that effect a union of topological structuring and order structuring through coordinations among activity fields in such a way that homeomorphism between somatic state space and noetic state space is possible;
- **in Modality** – acts constituting systematic global coordination of somatic spatio-temporal activities *that effects a somatic marking of empirical apperception*.

That these four 2LAR headings of synthetic combination derive from the J-SMI sub-2LAR in figure 10 is not immediately obvious but it is inherent in the applied metaphysic of the J-SMI [Wells (2011d)]. That these acts are the somatic substratum (or, better put, *co*-substratum) of noetic context is likewise not immediately obvious although, again, this is inherent in the Critical idea of context when this idea is subsumed under the Critical principle of thorough-going *nous-soma* reciprocity[21]. Perhaps the best way to explain this *Realerklärung* of signaling power is to present and discuss *in concreto* some of its specific manifestations that have emerged from researches in mathematical neuroscience, empirical psychology and neurobiology.

A. Manifestations of the Quantity Transformations. Functions of Quantity always pertain to aggregations and, indeed, Critical Quantity grounds the mathematical notion of sets. In the context of general physics (that is, "physics" in the ancient Greek connotation of *φψσικησ*), the functions of Quantity pertain to Kantian phoronomy [Kant (1786), 4: 480]. As mathematic, the functions pertain to the constituting of topological spaces. Empirical psychology, too, has been able to unearth a somewhat unexpected linkage to this otherwise quite abstract context. Biologists tend to remain unaware of the Quantity function, presumably because historically the study of biology has employed little or no mathematics beyond elementary algebra and the bare rudiments of calculus and statistics. I find no concretely useful illustrations of concepts of Quantity in the principal biological literature[22] and only abstract mathematics making hypothetical reference to biological structures in the literature dedicated to mathematical biology. Biologists' *intuitions* of Quantity, though, are *manifested* by those biological concepts called *anatomy* and *organelle*.

Mathematically, the self-organizing transformations of Quantity are those which construct topological spaces. I think the first thing that must be emphasized here is that these acts of self-construction by the OB are not acts initially leading to the construction of just *one* topological space but, rather, of a *multiplicity* of topological spaces. It falls as a task to the functions of the T-SMI and the signaling faculty to eventually construct the coordination of these divers spaces. The *empirical* evidence of topological construction capacity by the OB was unearthed in studies of

[21] Context is the sphere of concepts, combined by judgment with the concept said to have the context, which delimits the applicable scope involving that concept in Reality [Wells (2011e)].

[22] In my own opinion, biological science severely handicaps the discipline by the widespread aversion almost every biologist I know displays towards mathematics. However, I also am of the opinion that this is not the fault of the biologists. It is the fault of the dreadfully bad pedagogy used to *teach* mathematics. If mathematics education *taught* mathematics productively, I have no doubt biologists would *use* it more.

developmental psychology. Piaget and Inhelder reported,

> [Abstract] geometrical analysis tends to show that fundamental spatial concepts are not Euclidean at all, but 'topological'. That is to say, [geometrical concepts are] based entirely on qualitative or 'bi-continuous' correspondences involving concepts like proximity and separation, order and enclosure. And, indeed, we shall find [in this book] that the child's space, which is essentially of an active and operational character, invariably begins with this simple topological type of relationship long before it becomes projective or Euclidean. [Piaget and Inhelder (1948), pg. vii]
>
> It is also quite obvious that the perception of space involves a gradual construction and certainly does not exist ready-made at the outset of mental development. . . The first two stages of development are marked by an absence of co-ordination between the various sensory spaces, and in particular by lack of co-ordination between vision and grasping . . . The most elementary spatial relationship which can be grasped by perception would seem to be that of 'proximity', corresponding to the simplest type of perceptual structurization, namely, the 'nearby-ness' of elements belonging to the same perceptual field. . . A second elementary spatial relationship is that of *separation*. Two neighboring elements may be partly blended and confused. To introduce between them the relationship of separation has the effect of dissociating, or at least providing the means of dissociating them. But once again, such a spatial relation corresponds to a very primitive function; one involved in the separation of units, or in a general way, the analysis of elements making up a global or syncretic whole. . . A third essential relationship is established when two neighboring though separate elements are ranged one before another. This is the relation of *order* (or spatial succession). It undoubtedly appears very early on in the child's life . . . A fourth spatial relationship present in elementary perception is that of *enclosure* (or *surrounding*). . . Lastly, it is obvious that in the case of lines and surfaces there is right from the start a relationship of continuity. [*ibid*., pp. 6-8]

The research of Piaget and Inhelder established as conclusively as empirical research ever can that: (1) the infant's earliest perceptual capacities are topological in nature; (2) that in the infant there is initially not just one construction of topological space manifested but, on the contrary, more than one; and (3) that these divers initial topological constructions gradually become more and more coordinated and unified with the growth of experience during mental development. Thus we have the necessary evidence in real experience for concluding with objective validity the *Dasein* of the topological construction capacity of the OB.

There is an additional bit of interesting evidence for concluding that the innate capacities of an OB include innate capacities for constructing mathematical structures. Note with care that the capacity is for *constructing* them, and I am *not* saying the structures themselves are innate (they are not). This bit of evidence came out of an accidental meeting between Piaget and the Bourbaki mathematician J. A. Dieudonné. Piaget described the discovery:

> As you know, the aim of the Bourbaki was to find structures that were isomorphic among all the various branches of mathematics. . . This search led to three independent structures that are not reducible to one another. By making differentiations within each one of these structures or by combining two or more structures, all the [other mathematical structures] can be generated. For this reason the structures were called the mother structures. . . The first is what the Bourbaki call the algebraic structure. The prototype of this structure is the mathematical group. . . The second type of structure is the order structure. This structure applies to relationships, whereas the algebraic structure applies essentially to classes and numbers. . . The third type of structure is the topological structure based on notions such as neighborhood, borders, and approaching limits. . .
>
> A number of years ago I attended a conference outside Paris entitled "Mental Structures and Mathematical Structures." This conference brought together psychologists and

> mathematicians . . . Dieudonné gave a talk in which he described the three mother structures. Then I gave a talk in which I described the structures that I had found in children's thinking, and to the great astonishment of us both we saw that there was a very direct relationship between these three mathematical structures and the three structures of children's operational thinking. We were, of course, impressed with each other, and Dieudonné went so far as to say to me: "This is the first time that I have taken psychology seriously. It may also be the last, but at any rate it's the first." [Piaget (1970), pp. 24-26]

Now let us look at a precise mathematical definition of a topological space [Baum (1964), pp. 20-21]. The basis of the set membership equivalent of a mathematical "point" in the somatic universe X of "points" x is the somatic place. Any *topological* space is a subspace in terms of a subset $X \subset$ X combined with a specific topology $\mathfrak{I}$. This is formally symbolized by writing it as a pair $(X, \mathfrak{I})$ for each topological space construction. Let $\mathcal{U}_x$ denote a system of neighborhoods of x. Let U_x and V_x denote different specific neighborhoods of x (recalling that a somatic neighborhood of a somatic place is a somatic activity field; for an illustration see figure 12). A system of neighborhoods $\mathcal{U}_x$ is defined by three conditions:

1. if any V_x is such that $V_x \supseteq U_x$ for some $U_x \in \mathcal{U}_x$ then $V_x \in \mathcal{U}_x$;
2. if U_x and $V_x \in \mathcal{U}_x$ then $U_x \cap V_x \in \mathcal{U}_x$;
3. if $U_x \in \mathcal{U}_x$ and there is some $V_x \in \mathcal{U}_x$ containing a somatic place $y \in V_x$, then $U_x \in \mathcal{U}_y$.

The topology $\mathfrak{I} = \{ \mathcal{U}_x$ such that $x \in X\}$ is an *assignment* of neighborhood systems. $(X, \mathfrak{I})$ is the topological space. Note that a topology contains a multiplicity of neighborhood systems. The acts of Quantity *constitute* the neighborhood systems and *assign* them to specific topologies. □

B. Manifestations of the Quality Transformations. In order to more clearly explain the concept of somatic neighborhoods, figure 12 presents a cartoon illustration of a neighborhood sequence in

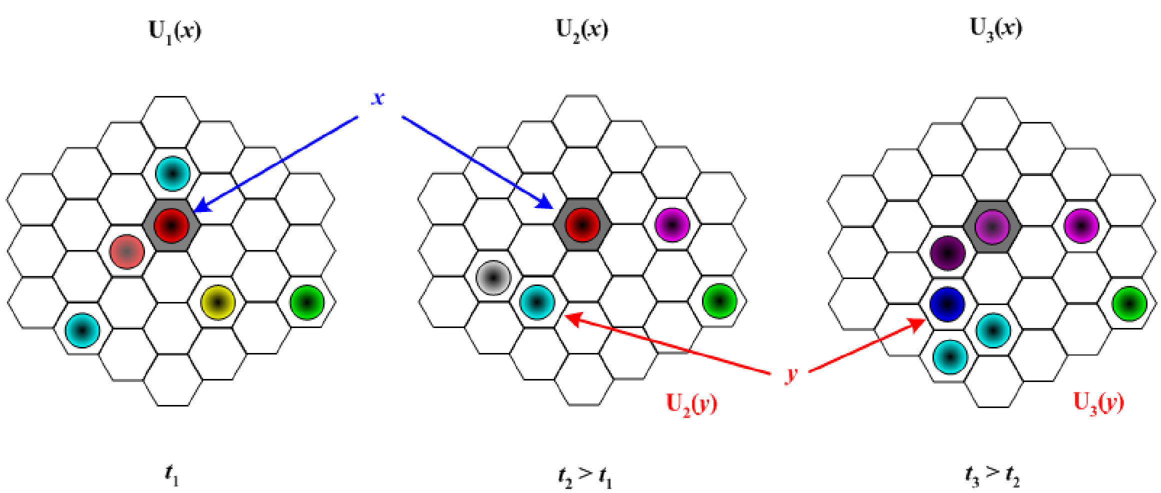

Figure 12: Illustration of a trajectory of locally path connected neighborhoods $U_t(x)$ in objective time t. Hexagons denote somatic places in universe X. The darkened hexagons denote somatic place x. Colored circles denote measurable activity in the somatic place containing them. Hexagons without colored circles denote places with zero-measures of activity. The illustrations are meant to be analogous to, e.g., brain images obtained by, e.g., PET scan or fMRI scan. The center and rightmost figures also illustrate a second somatic place, y, and these two neighborhoods are also neighborhoods of y, denoted as $U_2(y)$ and $U_3(y)$. Since both neighborhoods contain x as an active somatic place, $U_2(y)$ and $U_3(y)$ both also belong to the neighborhood system $\mathcal{U}_x$ of place x. The *amount* of activity at an active place is topologically irrelevant.

objective time *t*. The temporally-ordered neighborhoods $U_t(x)$ are locally path connected because active somatic place *x* is contained in all three and all three belong to a neighborhood system of *x*, $\mathcal{U}_x$. Refer to the figure caption for explanation of the figure symbols.

In the metaphysics of Kantian dynamics [Kant (1786), 4: 496-500], non-zero measurable activity at a place is regarded as a moving power. This means it is regarded as having a capacity for diminishing activity at other somatic places. In graph theoretic terms, this is what is represented by an arc y_{xv} (refer to figure 7) from place *x* to another place *v* where the activity at *x* has a dissipating effect. The moving power at *x* is also regarded as having a capacity to accrete other activities (i.e., cause activity at other places), and this, too, would be represented by an arc in a graphical depiction. Quality transformations of signaling power are transformations that coalesce formation of a locally path connected trajectory through a topological space ordered in a succession in objective time. Possible trajectories that might be stimulated due to activity at *x* can be either assisted or opposed by accretion or dissipation actions of other active somatic places. A graph theoretic representation of this is often called a *competitive network* in the lexicon of neural network theory and frequently takes the form of what are called *on-center off-surround* network structures [Grossberg (1978), §14-15].

Biological manifestations of neuronal organizations exhibiting competitive network action and on-center off-surround structures are legionary. Figure 13 provides a schematic illustration of competition among neurological networks in the thalamic-neocortical interface in the brain. Similarly, competitive neurological networks are found to exist in abundance in the ventral horn anatomy of the spinal cord. There competition serves to coordinate flexor and extensor muscle activation and relaxation during skeletal muscle movements.

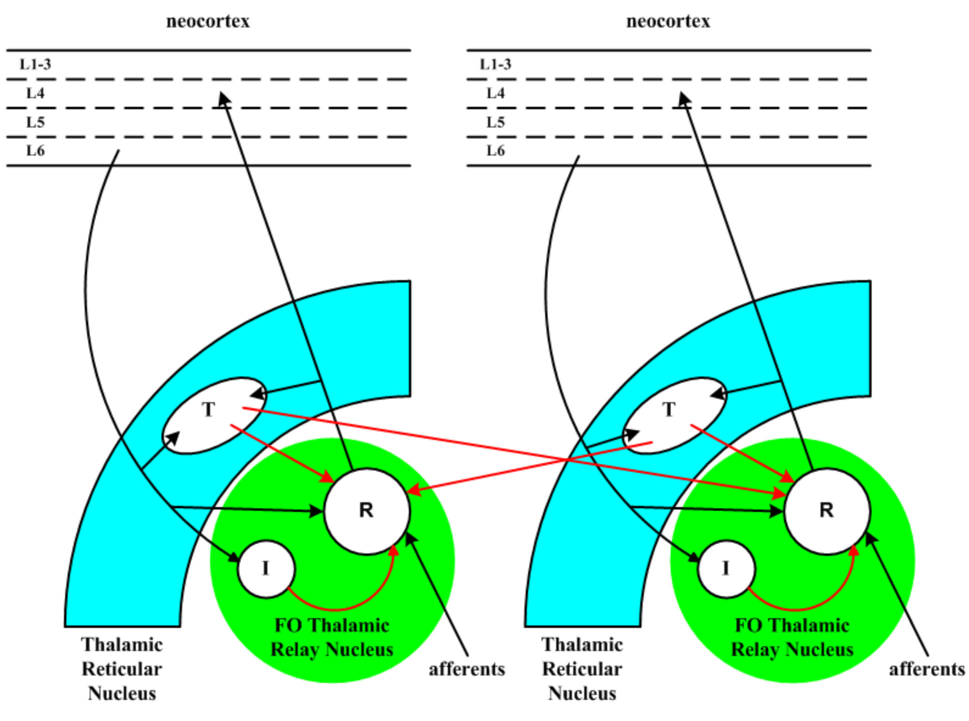

Figure 13: Schematic illustration of cross-coupled first order thalamic relay nuclei showing thalamical-neocortical connections in the brain. The figure was developed from neurological material presented in Sherman and Guillery (2006), chapters 3 and 6. Red lines denote inhibitory connections (dissipation) while black lines denote excitatory (accreting) connections. The two thalamic relay nuclei compete with each other via projections from the thalamic reticular nucleus in establishing neuronal action in the system.

The formation of accretion and dissipation arcs in a graph theoretic representation of the topological space performs an association function (which is a type of coalescing action; Quality pertains to coalescence of intensive magnitudes). Empirical psychology recognizes association as an important psychological phenomenon, and one that has proved challenging for theoretical psychology to tightly grasp. Reber provides us with the following psychology definition:

> **association 1.** Most generally, any learned, functional, connection between two (or more) *elements*. Identifying precisely what these elements are (e.g. ideas, acts, images, stimuli and responses, memories) and specifying the mechanisms underlying their connection is a theoretical exercise that has occupied many a philosopher and psychologist for many a year. [Reber (2001)]

It reflects the importance of the idea that Reber's Dictionary devotes three and one-half pages to technical terms used in psychology pertaining to the phenomenon of mental association. The topological semantics methodology I am discussing in this paper provides an organized schematic approach to psychological association as well as to neurological association. □

C. Manifestations of the Relation Transformations. The performances that transformations of Relation manifest are acts constituting the somatic basis of the psychological phenomenon of ***context dependent choice*** in the execution of sequences of sensorimotor actions [Grossberg (1978), §1.B-F]. Suppose a network graph contains two embedded paths that branch from a common vertex as illustrated in figure 14. Further suppose different sensorimotor actions result from propagating activity down the different paths and that these different paths denote different somatic manifestations of meaning implications. The possibility of *soma*-syntactical generator functions then requires the graph to contain other vertices (p_2 and p_3 in figure 14) that perform the task of selecting which path is activated by common vertex p_1. In figure 14 the trajectory paths are denoted by blue colored associational strength arcs while the *soma*-syntactical generator transformations are denoted by green arcs, here called ***non-specific arousals***. This network schematic was used in the first neural nets *demonstrating* the capacity of spatio-temporal learning

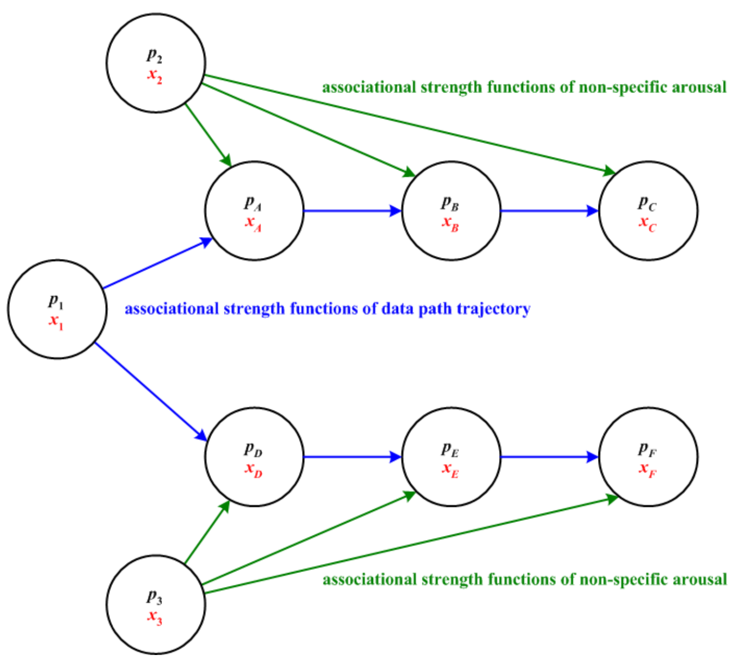

Figure 14: Embedding fields manifesting context-dependent choices of trajectories. The capacity for context-dependent choice functions is due to the Relation transformations called non-specific arousals.

action in 1969 [Grossberg (1969a,b, 1970, 1978)]. We may call networks of the type shown in figure 14 *avalanche networks*. A computer engineer would call the two different types of arcs illustrated in figure 14 by the names *data flow path* and *control signal*.

The non-specific arousal schematism illustrated in figure 14 has known neurological counterparts. One example is provided by the reticular formation [Felten and Józefowicz (2003), pp. 170-172]. Metabotropic processes in sub-cellular biology similarly stand as neurological manifestations of this "control function" schematism [Levitan and Kaczmarek (2002), chap. 12]. One simple yet particularly vivid example of metabotropic Relation transformations performing what Malsburg calls "dynamic links" is provided by the stomatogastric ganglion that controls muscles in the stomach [*ibid*., chap. 19, pp. 512-519].

Empirical psychology likewise has its analogous correspondents to Relation transformations. It seems likely that Piaget would call these "pre-operations" or, perhaps, "proto-operations" because the *a priori* psychological structures that reflect Relation transformations do not innately provide for all the behavioral capabilities that define full "operations" in the Piagetian context [Piaget (1953), pp. 8-22]. Piaget *et al*. discovered four basic and apparently innate types of compensation behaviors, which they called *constitutive* coordinator functions because these functions "constitute" (build) more complex learned behavior functions. Mathematically, these constitutive coordinators require the Relation transformation for the possibility of their sensorimotor expression. The four Piagetian functions are called the association coordinator, the repetition coordinator, the permutator coordinator and the identifier coordinator [Piaget *et al*. (1968), pp. 172-173]. Somatic transformations of Relation are the co-substrata necessary for the possibility of homeomorphism functions *f* and *g* in figure 1. □

D. Manifestations of the Modality Transformations. Modality transformations pertain to the constituting of somatic signaling ability that corresponds to (is the reciprocal phenomenon of) *empirical apperception*. Empirical consciousness is, functionally, "the representation that another representation is in me and is to be attended to" [Kant (1783), 29: 878]. The *soma*-phonological coordinator function is constituted by Modality transformations.

Empirical consciousness and its descriptive adjective ("conscious") are ideas where empirical psychology labors with great difficulty and a pronounced lack of community agreement. Reber provides the following usages of these terms in psychology:

> **conscious 1**. adj. In its most general sense the term is used to characterize the mental state of an individual who is capable of (a) having sensations and perceptions, (b) reacting to stimuli, (c) having feelings and emotions, (d) having thoughts, ideas, plans and images, and (e) being aware of (a) to (d). [Reber (2001)]
>
> **consciousness 1**. Generally, a state of awareness: a state of being conscious (1). This is the most general usage of the term and is that intended in a phrase such as "he lost consciousness." [*ibid*.]

Most (likely, all) psychologists agree with these connotations of the terms. Beyond that the science shatters into a rainbow of *ad hoc* nominal definitions by fiat and clashing mini-theories of consciousness, almost all of which are ontology-centered speculations with no *real* grounding. Critical epistemology distinguishes between empirical consciousness and apperception, which we are discussing here, and the faculty of *pure* consciousness, which is the organized capacity of *nous* [Wells (2009), chap. 4].

The neurology of the somatic co-substratum of empirical consciousness is far from understood but there is essentially no doubt of its *Dasein*. The many speculative disputes about the subject center upon the Nature of its *Existenz*, i.e., its appearances in *soma*. One illustration of reasonable and evidence-based speculation regarding the neurology of consciousness has been provided by Damasio. He conjectures that brain structures including the reticular formation, the intralaminar nuclei of the thalamus, the cingulate cortex and the superior colliculi are foundationally involved in the phenomenon of empirical consciousness [Damasio (1999), pp. 234-276].

Piaget also found it necessary to posit consciousness-producing transformations to ground his empirical conclusions concerning what he called interaction structures [Piaget (1975), pp. 42-77], although it is not-unfair to say his description of these structures was far too vague to do more than supply a rather equivocal idea of equilibration processes. Piaget posited a more primitive (Type I) and a less primitive (Type II) schematic of interaction structures. These are illustrated in figure 15. In these schematics, the functions *a* and *b* denote consciousness functions pertaining to, respectively, awareness of resistance to the subject's actions and awareness of success/failure in comparisons of outcome appearances of the action in relationship to the subject's anticipations of these outcomes. Process *OS* denotes an awareness function of the insufficiencies or adequacies of the subject's actions. The point I wish to emphasize here is that the *Dasein* of something like these awareness transformations were found to be necessary for explaining Piaget's empirical observations of the actual behaviors of young children.

Mathematical neural network theory also finds it necessary to posit constructs pertaining to awareness transformations within, e.g., ART networks [Carpenter and Grossberg (1987)]. In ART terminology, these are referred to as attentional and orienting subsystems. Grossberg has provided an excellent thematic overview of the issues of consciousness, learning and attention and their role in mathematical modeling in Grossberg (1999).

These sorts of considerations carry implications for somatic activity fields. The most specific implication of import for this paper is this: within any activity field we must find some active somatic place where activity is indicative of *soma*-phonological Modality transformations at work in the overall functioning of the somatic system. For example, the reticular formation is deeply implicated in OB actions that are characterized in terms of consciousness and so we must expect reticular formation activities to be found within any activity field that can be regarded as semantically relevant. A greatly simplified illustration of this is provided in figure 12 by the somatic place denoted by the green activity indicator. *Two* or more active somatic places are necessary in every activity field association regardable as manifesting signaling power. □

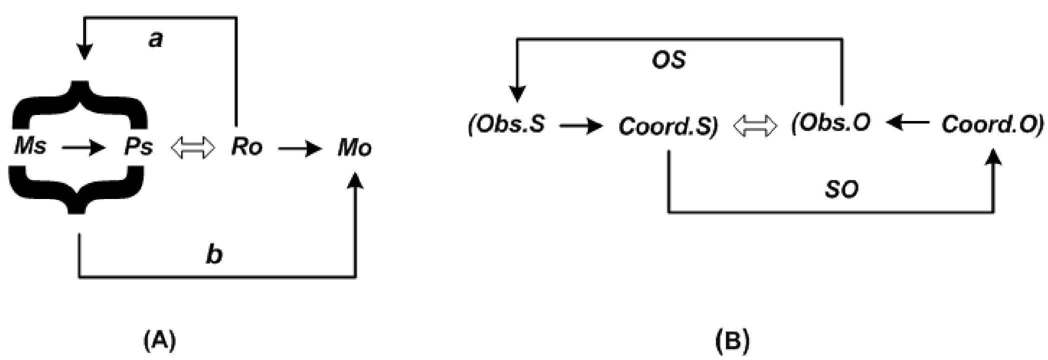

Figure 15: Schematic illustrations of Piaget's interaction structures. (A): Type I interaction. (B): Type II interaction. Detailed explanation of these structures is superfluous to the purposes of this paper (the interested reader can consult the Piaget (1975) citation). The pertinent idea contained in these schematics is the presence of *necessary* transformations pertaining to consciousness functions as integral parts of these interactions. In the specific, functions *a* and *b* in (A) denote functions of the subject's awareness of resistance to its actions (*a*) and awareness of success/failure comparisons of the actual outcome of the action vs. the subject's anticipation of the outcome (*b*). Process *OS* in (B) denotes a transformation function of the subject's awareness of the insufficiencies or adequacies of his actions.

Signaling ability represented in somatic topological space is necessary for regarding that topological space in the context of Critical semantics and somatic coding. This puts us in a position to now present the *Realerklärung* of what a somatic signal is regarded from the judicial Standpoint of Critical metaphysics: *A* ***biological signal*** *is a spatio-temporal biological event insofar as, and only insofar as, that event is understood as a delimited spatio-temporal trajectory in a system of activity field neighborhoods*. With this explanation, one of the fundamental questions raised in Part I [Wells (2011c)] has now been addressed with objective validity. Although the patient reader might well wonder if all the development that has transpired between the posing of the question in Part I and its answer now was really necessary, the need for this lengthy development is easily grasped when one remembers that the idea of a "signal" is treated as a mathematical primitive in theories of brain-object function and in theories of knowledge representation. If the primitive being employed is to have *real objective validity* and is to be suitable to stand as a principal quantity of Critical mathematics (the linkage point between mathematical theory and actual experience), then the metaphysical rudiments of the idea of "signal" *must* be rigorously established and linked without gap or *saltus* to the Critical foundations of science. Only then can the real organic unity of the *homo phaenomenal* and *homo noumenal* aspects of an Organized Being be understood as *natural* science.

The general problem of knowledge representation has also now been supplied with a doctrine of an objectively valid schema for knowledge representation in natural science. This is not to say that all knowledge representation problems are solved *in the specific* by this series of papers. What is presented here is the *objectively valid* schematization *method* in general. To apply it to any specific Object in Nature, one must incorporate the special properties and appearances that distinguish that Object from other Objects in Nature. Because these are precisely the factors that are not *a priori*, the specific task rightly belongs to empirical-rational natural science and cannot belong to metaphysics. The metaphysical rudiments presented in this series pertain to the epistemological pre-conditions for objective truth in natural science properly so-called.

VI. A Few Closing Metaphysical Observations and Remarks

In bringing this series of papers to a close, a few final remarks and comments are in order with regard to a few of some more detailed specifics of the implications in this theory that are easy to overlook. These implications become *very* visible during the *practice* of this methodology. I wish to deal here with the more predictably troublesome of these.

First, let us settle a potential question glossed over earlier. The activity of a somatic place was defined in terms of measurement capability. Now, only a moment's reflection should suffice to understand that what is measurable by scientific instrumentation and what is perceivable by an Organized Being are not equivalent ideas, nor is there any objectively valid ground to expect the lowest degree of measurable activity to correspond to *any* sort perceptual thresholds in the empirical apperception of an Organized Being. The practice of science employs instruments to *extend the horizon of possible experience*. Before the invention of the microscope bacteria were not objects of possible experience. Afterwards they were and the *Dasein* of bacteria and other microorganisms is no longer a matter of mere speculation. The same is true of the use of scientific instruments to measure somatic activity. What is the implication of this relatively obvious fact?

I deem it likely that a biologist reading this series greeted the definition of somatic activity with a feeling of ridicule. "Doesn't this guy know that of course there will always be metabolic activity in a cell?" he might have thought, "Otherwise the cell would be dead." There is one very well known criterion for distinguishing between "living" and "no-longer-living" cells, namely the presence or absence of cellular respiration. Viewed in this context, any "living" cell and any association of "living cells" represented by a somatic place will by definition have some non-zero metabolic rate. The issue isn't that. The experimental issue lies in the determinability of intensive magnitudes of metabolic rates in an aggregation, a factor that is always limited by the measurement range of the instrumentation involved. Furthermore, there is no *absolute gauge* by which metabolic rate, or any other empirical appearance, can be absolutely determined[23]. In science this is known as the *gauge* issue. It is a matter of scientific *practice*, not of ontology.

More pertinent is the issue of measurement uncertainty vs. perceptual threshold. Viewed from the *homo phaenomenon* aspect of organized being, the Organized Being's biophysical sense apparatus can be viewed with objective validity as merely another kind of recording instrument for physical appearances. From the *homo noumenon* side of *nous*, noetic representations fall generally into two *logical* divisions, namely obscure representations (non-perceptions) and perceptions. "Perceptual threshold" is an object of *nous*, not *soma*. Probably no one did more than Freud to establish the theoretical fecundity of dividing "the unconscious" (obscure representation) and "the conscious" (perception = representation with consciousness). Piaget wrote,

> In general, when a psychologist speaks of a subject being conscious of a situation, he means the subject is fully aware of it. The fact that he has become aware of it neither modifies nor adds anything to the situation . . . Freud even compares consciousness to an "organ of the internal senses," it being understood that for him a sensation can only receive and not transform an external matter. However, no one has contributed more than Freud to make us consider the "unconscious" a continually active dynamic system. The findings of this book lead us to claim analogous powers for consciousness itself. In fact, and precisely insofar as it is desired to mark and conserve the differences between the unconscious and the conscious, the passage from one to the other must require reconstructions and cannot be reduced simply to a process of illumination. [Piaget (1974), pg. 332]

Hence we conclude that somatic matters must be measured with measurement instruments (with the attendant gauge considerations) and not by noetic acts, and so this first issue is in fact merely a minor confusion over transcendental place. Somatic activity must be assayed by physical instrumentation and here noetic objects such as "perception thresholds" have no transcendentally-valid part to play in it.

A second issue, this one of a purely mathematical character, is raised by the distinction in figure 1 between somatic state space and noetic state space. That state variable representations are not unique and that two different homeomorphic systems are functionally equivalent even though in appearance they have markedly different depictions is, I trust, already clear enough. There is, nonetheless, an issue of function vs. mechanism that sooner or later tends to intrude upon the reflections of a researcher. Grossberg stated the issue very clearly many years ago:

> The distinction between a network's emergent functional properties and its simple mechanistic laws also clarifies why the controversy surrounding the relationship of an intelligent system's abstract properties to its mechanistic instantiation has been so enduring. Without a linkage to mechanism, a network's functional properties cannot be formally or physically explained. On the other hand, how do we decide which mechanisms are crucial for generating desirable functional properties and which mechanisms are adventitious? Two seemingly different models can be equivalent from a functional viewpoint if they both generate similar sets of emergent properties. . .
>
> In summary, the relationship between the emergent functional properties that govern behavioral success and the mechanisms that generate these properties is far from obvious. A single network module may generate qualitatively different functional properties when its parameters are changed. Conversely, two mechanisms which are mechanistically

[23] not even the "absolute zero" of the Kelvin temperature scale. Absolute cessation of atomic motion is not physically observable. Therefore the zero-point of the Kelvin scale is a speculative and *mathematical* destination (a secondary quantity) and is not a possible principal quantity of Critical mathematics.

> different may generate formally homologous properties. The intellectual difficulties caused by these possibilities are only compounded by the fact that we are designed by evolution to be serenely ignorant of our own mechanistic substrates. The very cognitive and learning mechanisms which enable us to group, or chunk, ever more complex information into phenomenally simple unitized representations act to hide from us the myriad interactions that subserve these representations during every moment of experience. Thus we cannot turn to our daily intuitions or to our lay language for secure guidance in discovering or analyzing network models. The simple lesson that the whole is greater than the sum of its parts forces us to use an abstract mathematical language that is capable of analyzing interactive emergence and functional equivalence. [Grossberg (1987)]

The mechanism vs. function issue Grossberg described belongs to a common issue class that is raised in scientific reduction as well as in model order reduction. For example, in seeking for knowledge "closer to mechanism" from a higher level functional starting point, SR proceeds from a modeling level where activity is concretely represented by real numbers (such as the cases graphically depicted in this paper) to a level where activity is represented by pulse-mode waveforms (e.g. "pulse-coded neural networks"). To date there hasn't been very much SR work of this sort reported in the literature, but it is an obvious route in service of the "ladder rail making" I discussed earlier. What can happen during this sort of SR work – and I am inclined to think it is likely the rule rather than the exception – is that *functional* interpretations, so seemingly obvious at the higher level as to be mistaken for *mechanistic* properties, are demonstrated at the lower level as being of *only* functional and *not* mechanistic character. That this can be so was recently demonstrated by Sharma (2011). As it so happens, this demonstration came quite unexpectedly and in manners not especially pleasing to Sharma (or, for that matter, to me) except in *retrospect* after the issue was resolved. It demonstrated that there is a great deal of context and content work that has to go into "building the rails" of the science ladder, i.e., *a doctrine of general systems*.

But there is an even more potentially perplexing consequence of the non-uniqueness of state variable models that the semantics properties of knowledge representation are *sure* to raise in Critical psychophysics. This one *promises* to catch the Critically unprepared researcher entirely with his ontologically-centered mainsheets unbelayed and with the force of a gale. It is an issue that goes straight to the heart of Grossberg's last sentence above. The issue is *objective time*.

Almost everyone, including physicists who ought to know better if they had actually studied *Einstein's* relativity papers, *reifies* objective time. It is treated as if it were a real-thing-in-nature rather than the *purely* mathematical object that it is. It is true that physicists, since the discovery of quantum electrodynamics theory, now treat objective time as an object that can just as well "run backwards" from "future" to "past" as in the other direction. But instead of taking this as a clue pointing squarely at the *epistemological* Nature of objective time, they instead engage in speculative fantasies regarding this "nature of time *per se*." As Einstein pointed out in 1905, objective time is a matter of measurement convention and not an object of possible experience. The *Existenz* of objective time is determined by clocks and practical measurement procedures.

Objective time has *no* appearances other than the mathematical ones that spring from these conventions. The very act of making a *parástase* of an *object* named "time" brings this object entirely under the laws that govern objective validity in Critical epistemology. Suppose you were to construct a somatic state space model X as per figure 1. You would very likely incorporate "time" into that model in its usual "one-dimensional" form and, again most likely, make it an "arrow" pointing "into the future" from "right now." And there is nothing wrong with this unless you should happen to be working with what system theorists call the "backwards in time" problem. Here's what will happen when you then turn to modeling noetic state space Y in figure 1: You will be immediately confronted with the fact that, in regard to *nous*, it is *objectively necessary* for objective time to be represented as a time*scape*; noetic objective time *cannot be* one-dimensional because if it is so depicted then it is impossible for *H. sapiens* to represent objects of appearance in the way that we *do* come to know experience [Wells (2006), chap. 21]. If you liked Einstein's relativity of objective time, you should love mental physics' relativity of objective time. If not . . . well, there's always laboratory work to be done in biology.

We are thus confronted straightaway with the fact that mathematical *parameterization* of order relationships will not be the same for the noetic state space model as for the somatic state space model. And this isn't even just a simple matter of "time scale." The t_{nous} and t_{soma} *secondary quantities* are unlikely to have the same number of mathematical dimensions in their respective depictions. *Principal* quantities of objective time must, of course, be homeomorphic, but these quantities will, as Einstein said they must, come down to the processes that *determine* them.

Fortunately, the possibility of concordance between the principal quantity (or quantities) of mathematical somatic objective time and noetic objective time is guaranteed *a priori* for any objectively valid model of somatic time. This is because all objective ideas of mathematical time owe their transcendental point of origin to the same source, namely the synthesis of the pure intuition of ***subjective*** time in the synthesis of sensibility of *nous*. This synthesis is the synthesis of mathematical *ordering structures* [Wells (2009), chap. 3 §3] and mathematical representation of noetic *objective* time is merely a *parástase* (depiction) of this process.

Figure 16 illustrates a representative timescape structure making an objective depiction of order structuring in *nous* synthesized in the pure intuition of subjective time. Moments in time are specific markings of depictions of intuitions[24]. Multiple timelines in a timescape arise from the presentation in sensibility of a *manifold* of intuitions. Unity of this manifold is effected by the *affective* perceptions that span the multiplicity of possible timelines in the timescape. A classic one-dimensional "time line" is, mathematically, a set membership solution set comprised of timescape points spanned by their common affectivity. This guarantees the possibility of a homeomorphism between principal quantities of noetic and somatic objective time. The depiction

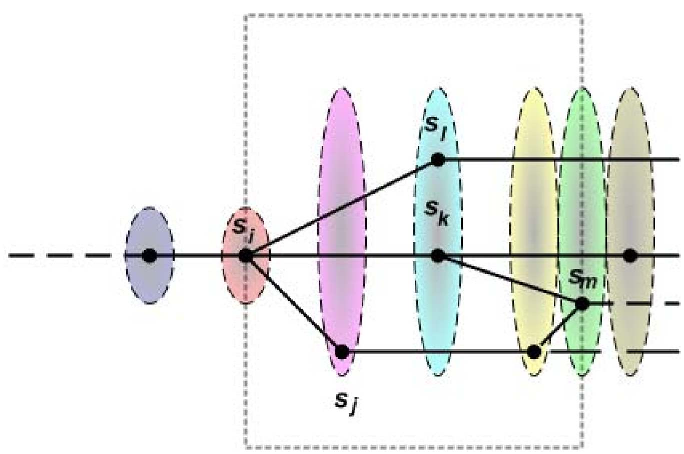

Figure 16: Illustration of a timescape in the synthesis of the intuition of time in *nous*. The depiction is necessarily an objective depiction and, therefore, an illustration of *objective* noetic time. A *moment in time* is the marking of the perception of an intuition by the process of reflective judgment. Black dots denote intuitions, colored ovals denote affective perceptions spanning the multiple timelines contained in a timescape. The dotted box denotes a submanifold in time bounded by intuitions s_i and s_m. A moment in subjective time "grows out of" its direct cover (predecessor) moment in time in the synthesis of subjective time. Affective perceptions have the mathematical effect of defining solution sets that stand as principal quantities for a classic one-dimensional *objective* timeline. The *computer* analogy here is *clock distribution*.

[24] An intuition is an objective perception.

of noetic objective time (which is a spatio-temporal depiction because it is the depiction of a *mental* object) has the logical *momenta* structure {universal, infinite, categorical, necessary}. It is interesting to compare this with the logical *momenta* of a mathematical depiction of the structure of *subjective* space, which is {universal, infinite, disjunctive, necessary}. The pure intuition of subjective space is a synthesis of topological structuring [*ibid*., chap. 3 §2].

Another issue I want to touch on here is the issue of progressive coordination of different topological spaces to eventually put together a unitized topological somatic space-time system. As noted earlier, the infant's earliest mental constructions of space-time-object representations are initially uncoordinated, i.e., sensorimotor schemes for the different sensory modalities in external sense are initially developed in their own distinct semantic space-times. Thus, e.g., vision and prehension are initially uncoordinated by common schemes. Empirical evidence gathered from studies of early development of sensorimotor intelligence in children supports this *theorem* of mental physics [Piaget (1954)].

The infant gradually comes to be able to coordinate his divers sensorimotor schemes because of two practical factors. The first is that different sensorimotor schemes initially involve a syncretic object representation, denoted *Obs.OS*, in which the infant's actions (*S*) and the object upon which he acts *O* (both as viewed by the psychologist-observer) are fused in single clear but indistinct objective perceptions. However, different schemes, S_1 and S_2, acting upon the same external object, *O*, contain *materia* of depictions in mental representation capable of serving as a minimal *practical* point of intersection. Mathematically, this is to say $Obs.S_1O \cap Obs.S_2O = Obs.S_mO$ where S_m denotes whatever intuitive *parástase* of objective perceptions is common to both S_1 and S_2. This makes two things possible: (1) the eventual distinction of the phenomenal object, *Obs.O* in Piaget-like Type II interaction structures; and (2) the development of what Piaget called ***mobile schemes***, $Obs.S_m$, as distinctive sensorimotor schemes that can subsequently be more *globally* applied as part of the child's practical assimilation of new objects.

Mathematically and semantically, this process is one of coming to coordinate and integrate the initially diverse topological space-times of knowledge representation. The Organized Being does not lose any of its previous capacities for assimilation during this unitizing synthesis. The process of accommodation is *conservative* in this regard, a point Grossberg stressed using different words in Grossberg (1976) and which is a fundamental mathematical principle of ART research.

Like every semantic synthesis, this synthesis process is essentially practical, i.e., is based on action schemes and interactions among action schemes. Within *nous* this corresponds to the process of judgmentation [Wells (2009), chap. 9] and the synthesis of the motivational dynamic [*ibid*., chap. 10]. Its appearance on the side of *soma* is exhibited by schemes of behavior indicative of the coordination of interacting Piagetian schemes [Piaget (1975)]. Figure 17 provides a schematic illustration of how the Organized Being is able to carry out this practical synthesis. The OB becomes capable of coordinating two diverse semantic space-time structures only when each has developed to the point where interactions of the sort Piaget termed "Type II" are possible for it. *Practical* sensorimotor interactions between the semantic space-time structures is then and only then possible for the OB to carry out. *The very same central process of practical equilibration that synthesizes higher and better levels of equilibrium within an individual space-time structure is also the central process that makes this coordination and unification possible*.

This is a basic metaphysical rudiment for Critical psychophysics. Knowledge development is an essentially practical process that Piaget described as "proceeding from the periphery to the center," i.e., *from* pursuit of a practical goal *to* conceptualization and understanding (cognizance) [Piaget (1974), pp. 332-337]. Elsewhere Piaget described in Kant-like language the effect of this process as bringing on "a kind of Copernican revolution" in the child's development of intelligence [Piaget (1975), pp. 87-88].

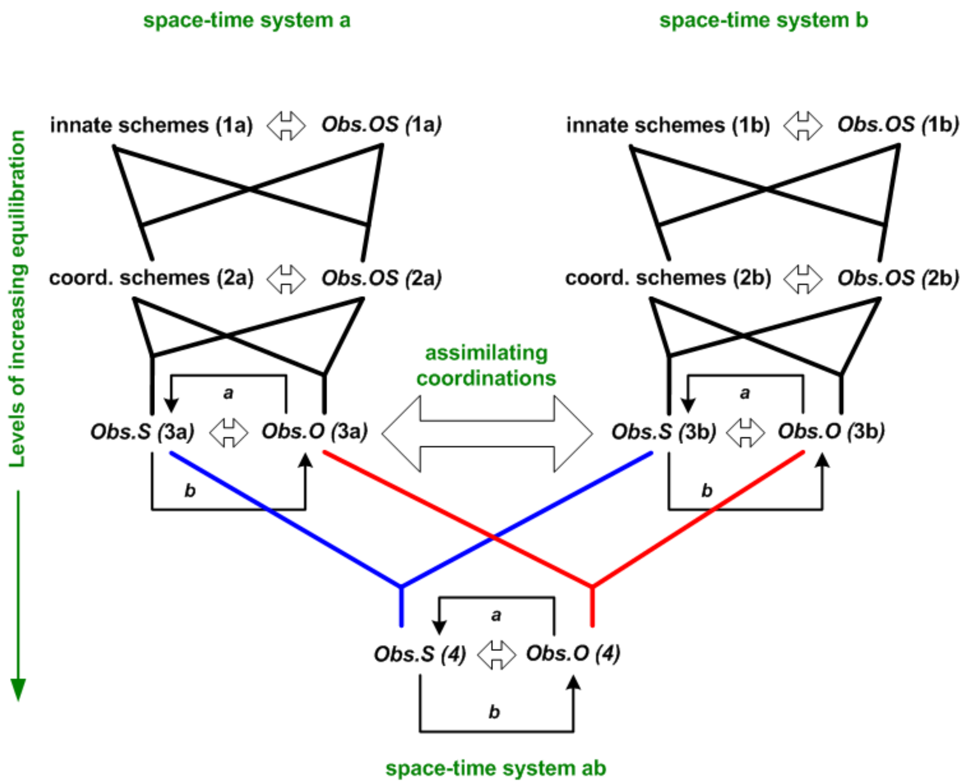

Figure 17: Schematic illustration of the synthesis of unified semantic space-time structures through the practical exercise of sensorimotor scheme interactions. Isolated semantic space-times become capable of being synthesized into a unified topological space-time ***system*** once the more primitive space-time systems are developed to the point where Piaget-like Type II interaction structures have been formed in the OB's practical manifold of rules and supported by concepts in its manifold of concepts.

All of this, then, is the objective for theory and model development in Critical psychophysics. The theory is (or, more accurately, will be) mathematical and grounded in the topology of Critical semantics discussed in this series of papers. There remains uncovered here only better clarification of methodological details of how one can carry out this *empirically*-based scientific work. That topic is beyond the scope of the present series of papers, but stating clearly a connection pathway to it is not. The foundation is embedding field theory supported by mathematical topology theory and graph theory. Practical methods employed in systematic neural network theory present what I see as the best presently known starting point for theory and model development *provided* that methodology is brought under the discipline of the set membership paradigm (a constraint necessary for producing mathematical principal quantities). There are numerous technical tools system theory has developed over the years, including but not limited to the subdisciplines of system identification theory, estimation theory and set membership based optimization theory, and implemented using, e.g., Hamilton-Jacobi-Bellman optimization and the techniques of dynamic programming [Werbos (1997)].

As a final recommendation for pragmatic tactics, I cannot improve upon a sage method stated long ago by Grossberg, which he named ***the method of minimal anatomies***:

> The theory introduces a particular method to approach the several levels of description that are relevant to understanding behavior. This is the method of *minimal anatomies*. At any given time, we will be confronted by particular laws for individual neural components, which have been derived from psychological postulates. The neural units will be interconnected in specific anatomies. They will be subjected to inputs that have a psychological

> interpretation which create outputs that also have a psychological interpretation. At no given time could we hope that all of the more than 10^{12} nerves in a human brain would be described in this way. Even if a precise knowledge of the laws for each nerve were known, the task of writing down all the interactions and analyzing them would be bewilderingly complex and time consuming. Instead, a suitable method of successive approximations is needed. Given specific psychological postulates, we derive the *minimal* network of embedding field type that realizes these postulates. Then we analyze the psychological and neural capabilities of this network. An important part of the analysis is to understand what the network cannot do. This knowledge often suggests what new psychological postulate is needed to derive the next more complex network. In this way, a hierarchy of networks is derived, corresponding to ever more sophisticated postulates. [Grossberg (1972)]

To this practical prescription I have not one more word to add except: it must all take its basis for objective validity from mental physics.

VII. References

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