Chapter 6

The Logical Functions of Determining Judgment

§ 1. The Doctrine of Logic

It is a lasting tribute to Kant's shortcomings as a writer that many professional logicians and other philosophers think Kant's formal transcendental Logic is nothing other than the classical logic of his day with some shadings and refinements added. There is little doubt that at least part of the reason for this was Kant's practice of retaining the traditional terminology of the old logic even as he redefined the meaning of these terms to make them epistemology-centered. There is also little room to doubt the rest of the reason lies in Kant's notoriously opaque style of writing. But however well-based the reasons are for this common misinterpretation, the fact remains that it is a misinterpretation. In this chapter we discuss Kant's formal logic of structuring in determining judgment.

Before doing so, however, there are some important remarks that need to be made concerning the topic of logic as a discipline in general. Almost everyone who is acquainted with logic in one or more of its various forms tends to regard it as a finished or essentially finished doctrine. There was some wrangling over this in the 1960s and 1970s when "fuzzy logic" was introduced by Lofti Zadeh, but since that time fuzzy logic has gradually been assimilated into the mainstream of at least engineering and applied mathematics. There is today some tinkering about with "modal logics" by some mathematicians and computer scientists, but again this is by and large seen by most people familiar with the topic as refinement, an evolution rather than a revolution. Those whose professional endeavors require them to be on rather intimate terms with the discipline of formal logic can recite a list of particular logics; this list will include propositional logic, predicate logic, equational logic, predicate calculus, and so on. Most modern logicians tend to regard "Kant's logic" as a short-sighted anachronism that time and the development of mathematical logic have rendered irrelevant and quaint.

In this intellectual environment, the statement that logic as a doctrine and a discipline is far from being either adequate or even fundamentally complete is likely to be regarded as being at the least surprising and probably controversial. It likely will be seen as being the sort of thing only someone who is either ignorant of the grandeur of mathematical logic or else is a crackpot would say. One need only look at the heated opposition Zadeh's idea aroused to feel comfortable in predicting the reception this statement is likely to receive. Nonetheless, it is necessary to make precisely this statement. And if the label of crackpot is to be applied, then we must apply it to some very famous men whose contributions to science in the twentieth century no one denies.

In the introduction to his *Mathematical Logic* W.V. Quine wrote,

> Mathematical logic differs from the traditional logic so markedly in method, and so far surpasses it in its power and subtlety, as to be generally and not unjustifiably regarded as a new science. Its crude beginnings are placed with George Boole . . . but it was from Boole onward through Peirce, Schröder, Frege, Peano, Whitehead, Russell, and their successors that mathematical logic underwent continuous development and reached the estate of a reputable department of knowledge.
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> The traditional formal logic, dating in its essentials from Aristotle, is nevertheless the direct progenitor of mathematical logic. The striking differences between the two must not be allowed to obscure the fact that they are both "logic" in the strictest sense of the word. They both have, vaguely speaking, the same subject matter. Just what that subject matter is, is not easy to say; the usual characterizations of logic as "the science of necessary inference", "the science of forms", etc. are scarcely informative enough to be taken as answers. . .
>
> Mathematical logic has been applied, but the most important applications are surely still to come. The usefulness of a theory is not to be measured solely in terms of the application of prefabricated techniques to preformulated problems; we must allow the applicational needs themselves, rather, to play their part in motivating further elaborations of the theory. The history of mathematics has consisted to an important degree in such give and take between theory and applications. Much of the promise of mathematical logic for science lies in its potentialities as a basis from which to construct subsidiary techniques of unforeseen kinds in response to special needs. [QUIN]

An empiricist, Quine was probably the first major logician and philosopher – certainly the first major American philosopher – to break with the philosophy of language descended from logical positivism. He also held that the application of logic and mathematics to science does, in important ways, drive the on-going development of both, although he seems to have held, as evidenced by the quote above, that this development evolves from an existing basis in mathematical logic rather than as a radical reformation of that existing basis. In any case, it is evident that he did not regard logic as already existing in a finished form.

Another major twentieth century figure was John von Neumann, whose mathematical works made major contributions to quantum theory, the theory of operators, ergodic theory, logic, axiomatics, and proof theory. Von Neumann also founded mathematical game theory and pioneered the development of the theory of automata. He is widely regarded as the father of the digital computer, and his insights on the relationship between automata theory and the brain have in many ways proven remarkably prescient. Von Neumann saw "complexity" as an issue of very fundamental importance in science – so much so that in his view it called for a major reformation of logic at its roots. He referred to this vision of a new logic as "probabilistic logic." In a series of lectures delivered at the University of Illinois in 1949, later published as Part I of *Theory of Self-Reproducing Automata*, von Neumann said,

> [In] no practical way can we imagine an automaton which is really reliable. If you axiomatize an automaton by telling exactly what it will do in every completely defined situation you are missing an important part of the problem. The axiomatization of

> automata for the completely defined situation is a very nice exercise for one who faces the problem for the first time, but everybody who has had experience with it knows that it's only a very preliminary stage of the problem.
>
> The second reason for the importance of statistical considerations in the theory of automata is this. If you look at automata which have been built by men or which exist in nature you will very frequently notice that their structure is controlled only partly by rigorous requirements and is controlled to a much larger extent by the manner in which they might fail and by the (more or less effective) precautionary measures which have been taken against their failure. And to say that they are precautions against failure is to overstate the case, to use an optimistic terminology which is completely alien to the subject. Rather than precautions against failure, they are arrangements by which it is attempted to achieve a state where at least a majority of all failures will not be lethal. There can be no question of eliminating failures or of completely paralyzing the effects of failures. All we can try to do is arrange an automaton so that in the vast majority of failures it can continue to operate. These arrangements give palliatives to failures, not cures. Most of the arrangements of artificial and natural automata and the principles involved are of this sort.
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> To permit failure as an independent logical entity means that one does not state the axioms in a rigorous manner. The axioms are not of the form: If *A* and *B* happen, *C* will follow. The axioms are always of this variety: if *A* and *B* happen, *C* will follow with a certain specified probability, *D* will follow with another specified probability, and so on. In other words, in every situation several alternatives are permitted with varying probabilities. Mathematically it is simplest to say that anything can follow upon anything in accordance with a probability matrix. You may put your question in this manner: If *A* and *B* have happened, what is the probability *C* will follow? This probability pattern gives you a probabilistic system of logics. Both artificial and natural automata should be discussed in this system as soon as there is any degree of involvement. I will come later to the question as to why it is just complexity which pushes one into this kind of axiomatization instead of a strict one.
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> Now this inclines one to view probability as a branch of logics, or rather, to view logics affected with probability as an extension of ordinary rigorous logics. The view that probability is an extension of logics is not trivial, is not generally accepted, and is not the major interpretation of probability. It is, however, the classical interpretation. The competing interpretation is the frequency interpretation, the attitude that logic is completely rigorous, and with respect to phenomena about which you are not completely informed, you can only make statements of frequencies.
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> This distinction was, I think, quite clear to Laplace, who pointed out that there are two possible attitudes toward probability: the frequency and the logical. . . There's a great deal in other modern theories, for instance, in quantum mechanics, which inclines one very strongly to take this philosophical position, although the last word about this subject has certainly not been said and is not going to be said for a long time. Anyway, one is also tempted in the case of quantum mechanics to modify one's outlook on logics and to view probability as intrinsically tied to logics. [NEUM1]

When von Neumann talked about "failures" he was primarily talking about physical failures such as a vacuum tube burning out or a neuron dying. Since then computer science has extended the scope of the concept of "failure" to include software failures ("program bugs"). We can take von Neumann's concept even farther, however, and regard errors in human judgmentation and deduction as being yet another species of failure.

From the viewpoint of Critical epistemology, man as phenomenal object in Nature is an automaton. As *noumenal* object in the intelligible world he cannot be so regarded, but neuroscience, quite properly, studies man as phenomenal object and so here von Neumann's remarks about "natural automata" are wholly relevant.

In Chapter 5 (§4.3) I remarked that, "At the moment of its making every judgment of cognition has complete coherence with all other concepts in the manifold of concepts" *so far as the Organized Being knows*. It is the contingency of empirical knowledge that opens the door to the possibility of error, and it is this possibility of error that introduces uncertainty in knowledge. The mathematical treatment of the consequences of this uncertainty is where the role for a new probabilistic logic lies. It seems likely such a logic, when we finally have it, will be closely related to, or perhaps even an extension of, set membership theory. I say this because formal logic, like mathematics, belongs to facet B of human knowledge and our ability to apply mathematics (and formal logic) to the phenomenal world of facet A *with objective validity* is restricted by Critical epistemology as we discussed earlier in this book. It was this restriction under the fundamental requirement for objective validity in our ideas and concepts that brought the set membership paradigm into play.

So it is that I said earlier that logic as a doctrine and a discipline is neither adequate nor fundamentally complete. Von Neumann's early death aborted the program to develop probabilistic logics and, other than for some nascent contributions by fuzzy logic, this work remains untouched to this day. It cannot be over-emphasized that any objectively valid system for such a logic must itself begin with axioms that are themselves objectively valid at the point where principal quantities of facet B come into play. The foundations for such objective validity can be drawn from nowhere else than the Critical acroams.

Kant's set of logical functions of judgment, which we are about to treat, is not the starting point for such a reformation of mathematical logic at its roots, much less this logic itself. These twelve logical *momenta* of judgments are not even fundamental in and of themselves. Rather, they take their basis from the categories of understanding and the regulative principles of the transcendental Ideas. Neither do they contain any reference to probability (which is a *noumenal* object; a statistic is a measurement and so belongs to the phenomenal world; probability is merely the object of the idea of a *noumenon* whose *Dasein* is implicated by the phenomenon of statistical regularity). The role and significance they have for this problem is that of an outcome. In Critical epistemology a **function** is the unity of the act of ordering different representations under a common one. The logical functions of judgment define the structural unity produced in the manifold of concepts by acts of the process of determining judgment. Therefore they do not tell us the theory for describing the details of how this structure is brought about but merely what it looks like when it is brought about by thinking and judgmentation. They speak to the *Existenz* of what is produced rather than to the doctrine of genesis for this *Existenz*. They are, in this context, the objects of a new and yet-to-be developed mathematical doctrine of Critical Logic.

§ 2. Determining Judgment

The manifold of concepts can be called the Organized Being's structure of understanding. As a system it is an open system and this structure is constructed through acts of determining judgment. The process of determining judgment is represented in 2LAR form in Figure 6.2.1 below. The exposition of the manifold of concepts properly begins with the exposition of what the process of determining judgment does and the knowledge it works with and produces.

Deduction of the 2LAR of Figure 6.2.1 is provided in Chapter 7 of *CPPM*. The making of determinant judgments is an activity. The objectivity of determining judgment is vested in what it does (make determinant judgments) and the manner in which it does it (infer according to criteria of truth). The former is the matter of determining judgment, the latter is its form. From the logical reflective perspective and viewed as a function,

> A judgment is the representation of the unity of consciousness of diverse representations or the representation of their relationship insofar as they constitute a concept. [KANT (**9**: 101)]

The determinant judgment is the matter of the process of determining judgment. We make a further analytic division of the idea of a determinant judgment according to its matter and form:

> Matter and form belong to every judgment as essential stockparts of it. The *matter* of the judgment subsists in the given cognitions that enter into the unity of consciousness in judgment, the *form* in the determination of the way and manner that the various representations belong, as such, to one state of consciousness. [KANT (**9**: 101)]

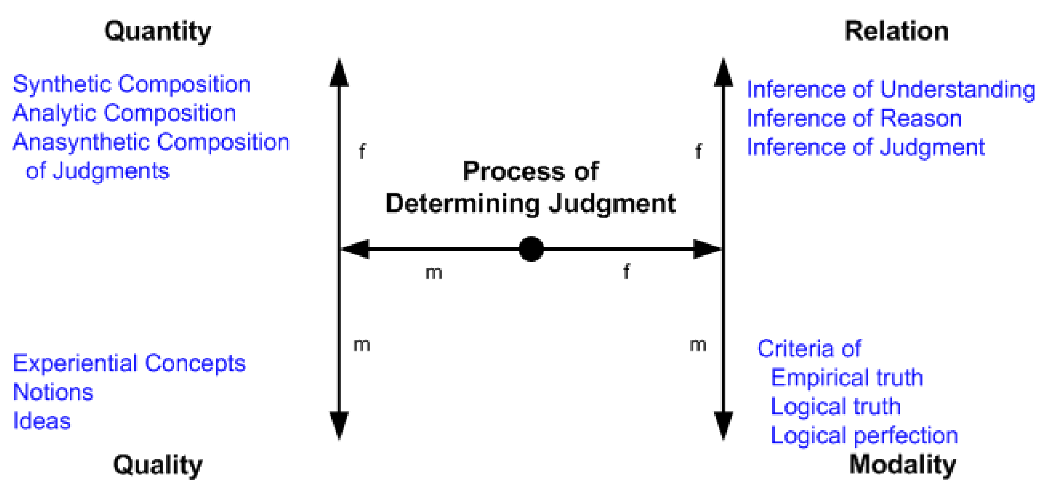

Figure 6.2.1: The process of determining judgment.

§ 2.1 Quantity in Determining Judgment

For Quantity in the process of determining judgment, we combine the idea of a judgment with the general ideas of Quantity in representation in general. Identification denotes a unit of representation. A judgment has precisely this character of identification when concepts combined in the manifold of concepts are *employed* as a unit. The form of composition in this case is a single representation and so the act of determining judgment in this case is called a **synthetic composition** of a judgment. The process of determining judgment begins with a given general concept (e.g. concept 1 of Figure 5.2.1) and produces a new concept by combining it with other particular concepts already present in the manifold to make a new representation of a distinct identified Object [KANT (**9**: 64)]. It takes in the parts and makes a new rule of unity.

The general idea of differentiation is the idea of the multiplicity of detail in a representation. The making of a representation of the multiplicity contained in a representation is called **analytic representation**. The act of composing such a representation is called an **analytic composition** of judgment. This act employs the category of plurality in the making of the composition. While a synthetic composition makes the concept of a distinct Object (category of unity), the analytic composition *makes a concept distinct*.

> When I make a concept distinct . . . my cognition grows not at all as to its content through this mere analysis. The content remains the same, only the form is altered, in that I learn to distinguish better, or to know with clearer consciousness, what lay in the given concept already. As nothing is added to a map through the mere illumination of it, so a given concept is not in the least increased through its mere clarification by means of the analysis of its marks. [KANT (**9**: 64)]

Figure 6.2.2 illustrates the synthesis of an analytic composition. In this synthesis, concepts (2)

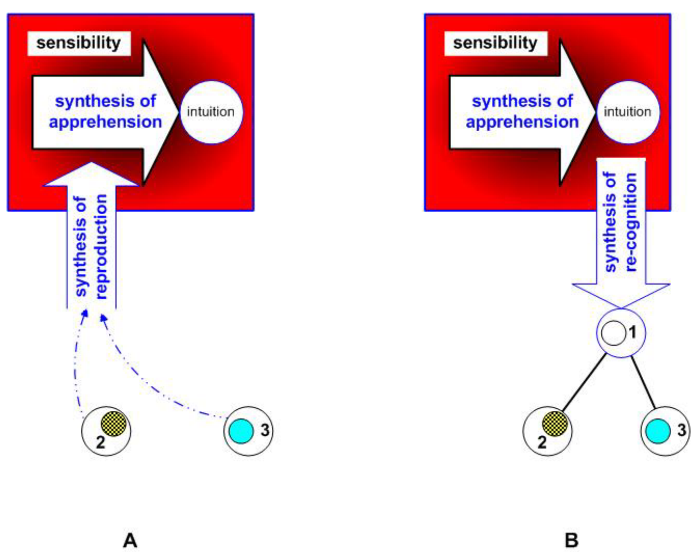

Figure 6.2.2: Making an analytic composition. (A) Summoning of comparates. (B) Combination under a distinct concept.

and (3), which had previously been placed in the manifold of concepts, are drawn into the synthesis of apprehension as comparates during the synthesis of an intuition. Neither concept, reproduced in sensibility, survives the synthesis of the *Verstandes-Actus* in total, but some common part of their sensible makeup does survive the abstraction to be contained in the intuition that results. This intuition, re-cognized as a concept (1), is contained in both (2) and (3). Determining judgment combines these three concepts according to the rule of the category of plurality. Concept (1) is contained in concepts (2) and (3); they are contained under concept (1). Nothing "new" is contained in the manifold of concepts by this action except the clear distinction of concept (1) as a mark of the other two concepts, each of which previously contained what is represented in concept (1) as an obscure representation of what was obscurely in the manifold of their intuitions originally. The synthesis of apprehension in this case can be called a synthesis of comprehension. The synthesis is a prosyllogism (synthesis *a parte priori*).

The general idea of integration in the context of making determinant judgments is called the **anasynthetic composition**. The name is chosen to signify that we view this act of composition as a synthesis of the previous two ideas of Quantity in determining judgment. We will use Figure 6.2.3 to illustrate what is meant by the idea of anasynthetic composition. During the construction of the manifold of concepts, any particular concept (1) will come to have representation in terms of a multiplicity of coordinate marks, represented by concepts 2 – 5 in Figure 6.2.3(A). In this multiplicity, some of these coordinate concepts may be relatively homonymous, i.e. the relationships of the coordinate marks to concept (1) may be objectively different from each other.

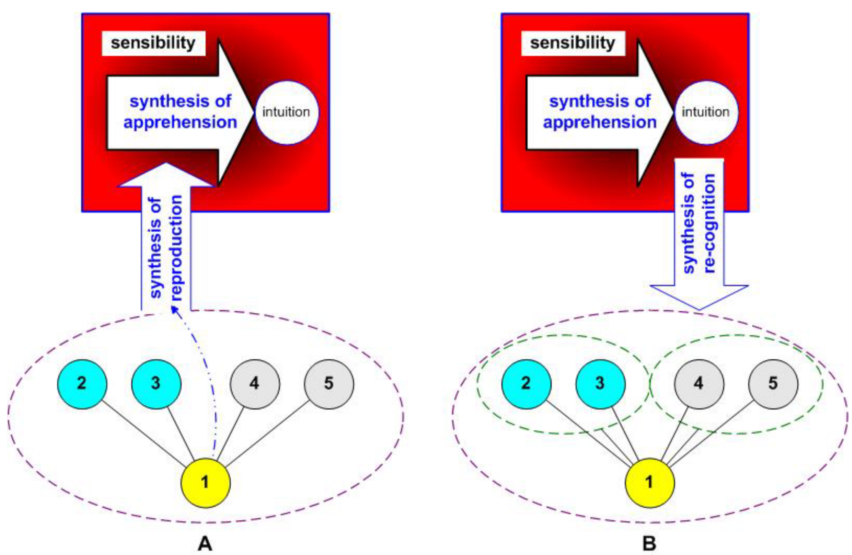

Figure 6.2.3: Making an anasynthetic judgment. (A) Concept (1) and all that is contained in it (the coordinate concepts 2 – 5) are taken into the synthesis of apprehension. (B) The composition of judgment produces a partitioned representation in which concept ($2\wedge3$) and concept ($4\wedge5$) become distinct coordinates, relative to each other, of concept (1).

Now let the totality of concept (1) and the coordinate concepts (2 – 5) contained in it undergo the synthesis of reproduction. Let us further suppose concepts (2) and (3) are homonymous with respect to concepts (4) and (5) but not with each other. Let the same be true for concepts (4) and (5). Because of this relative homonymity, concepts (2) and (3) must produce a different intuition from that produced by concepts (4) and (5), and so the synthesis of apprehension leads to a synthesis of re-cognition in which concept ($2 \wedge 3$) is made distinct from concept ($4 \wedge 5$). Here we will recall that a combination of concepts from the manifold of concepts is also a concept. Nonetheless, these two new concepts both remain coordinate concepts of concept (1). This is what Figure 6.2.3(B) depicts.

To make this illustration a bit less abstract, let us use a more concrete example. Suppose concept (1) is the concept "United States of America." One set of possible coordinate concepts is the set of individual states making up the United States. A second set of possible coordinate concepts is the set of individual citizens of the United States. These two sets are homonymous in the sense that both sets are grounds for thinking "United States" but they are not compatible marks. In other words, they are coordinate marks *of distinct contexts*. **Anasynthetic composition *makes the contexts distinct*** without adding anything more to the manifold of concepts other than a new form of composition of Quantity in the form of the manifold. We may regard the judgment as the judgment of *how to integrate* the parts of concept (1) *in* concept (1). This judgment of Quantity is made under the rule of the category of totality.

§ 2.2 Quality in Determining Judgment

The matter of every determinant judgment is a concept. Now, a concept has only two sources of possible origin; it can be givable through experience or it can be made as an act of spontaneity. In either case, a concept remains nothing else than a rule for the reproduction, or the original creative production, of an intuition. Quality in the process of determining judgment pertains to what can be experienced through concepts.

The general idea of agreement in representation in the context of determining judgment is the idea of agreement of the matter of determining judgment with the conditions of sensation. It was said earlier in this book that a concept must in some manner contain sufficient information to permit the synthesis of reproductive imagination to provide to sensibility the matter of intuition. This matter is nothing other than sensation. If the information for doing so was originally given by means of receptivity in the synthesis of apprehension, then the concept matter agrees with the matter of an actual sensuous intuition and we call such a concept an **experiential concept** because it arises immediately during the synthesis of real experience.

Contrary to this idea of agreement is the general idea of opposition within the context of determining judgment. Opposition (*Widerstreit*) in this context is that in determining judgment which can never be part of any real experience. Now, empirical concepts are constructed representations and stand as *empirical* rules for the reproduction of intuitions. However, because concepts are constructed, there must also be within determining judgment rules governing this construction and these rules cannot themselves ever become part of any actual experience. This is because such rules are pure, *a priori*, and their only ground of objective validity is that they are *necessary for the possibility* of experience itself. Kant called such a rule a **notion** [KANT1: B377]. The categories of understanding are, of course, the pure *a priori* notions of determining judgment. Notions correspond to the general idea of opposition because a notion contains nothing corresponding to sensation and its relationship to intuition is strictly a relationship of form. It pertains not to the appearance in time but, rather, to the synthesis of the pure intuition of time itself.

Finally we come to the general idea of subcontrarity in the context of determining judgment. The experiential concept is always a concept that represents experience. The notion is a pure form of representation that modern philosophers might prefer to call a "meta-concept." A notion is transcendental because it can never be the concept of an object of actual experience but, rather, is a rule necessary for the possibility of experience. Now, in our general 2LAR of representation, the third idea in each of the four titles can always be viewed as the synthesis of the other two under its title. The synthesis of notion with experiential concept is that in determining judgment which, on the one hand, pertains to possible experience yet, on the other, can never itself be part of actual experience. As a concept – that is, as a rule for the production of an intuition – the *cognition* to which it corresponds is nothing else than the cognition of a supersensible object. We call the concept of a supersensible object by the name **idea**.

An idea is a made, not a given or givable, concept. All objects of mathematics are objects of ideas because these objects can never be part of any actual experience. Examples include Euclid's geometrical point, the triangle, the transcendental number π, mathematical infinity, and all secondary quantities in Slepian's facet B. Numerous other examples of ideas abound: the "rational man" of economic theory; Toynbee's "societies"; the Greek god Apollo; the "manifest destiny" of nineteenth century Western political ideology; Descartes' *res cogitans*; etc. The object of an idea can never itself be presented in any intuition. Rather, intuitions arising from ideas can never be anything other than *examples*, *similes*, *metaphors*, or *exhibitions* of what the idea represents. One can travel to anywhere on earth and yet never anywhere encounter Great Britain, but one can encounter a great many things that are "British" (including a small island near Ireland).

Merely because the object of an idea is supersensible and can never be met with in any actual, sensuous experience, this does not mean that ideas and their objects are unimportant. Science, for example, could not be science without the ideas that unify divers phenomena under scientific laws. Science itself, for that matter, is a supersensible object and is exhibited by scientists in the work they do. One can never experience a sensuous encounter with a right triangle *per se*, but very little physics and very little engineering would be possible without the triangle. Without the capacity of determining judgment to produce ideas, mathematics itself – that is, the things mathematicians do – would not be possible. The tremendous human capacity to create ideas sets humankind far apart from all other forms of life on earth and is our most distinctive trait.

§ 2.3 Relation in Determining Judgment

Quantity and Quality in determining judgment pertain to composition of the manifold of concepts. We now turn to the connecting (*nexus*) of this manifold and we begin with the form of this *nexus*, Relation in determining judgment. The six ideas of *nexus* are ideas of the manner in which determining judgment constructs the manifold of concepts. Those of Relation pertain to the acts of judgment. The name we give to these acts generally is inference (*Schluß*). An **inference** is the derivation of one judgment from another [KANT (**9**: 114)]. Our three ideas of inference are ideas of the internal, external, and transitive representation in the context of determining judgment.

Each of the ideas of Relation addresses a particular "how" dealing with the *Existenz* of the manifold of concepts. Kant named the first of these the **inference of understanding**:

> If the inferred judgment already lies in the first one, so that it can be derived from it without the mediation of a third representation, then this is called "immediate inference" (*consequentia immediata*). I would rather call it an inference of understanding. [KANT1: B360]

It is clear from this description why such an inference is entitled to be called the internal idea of Relation in the process of determining judgment. The new judgment is drawn immediately from "within" the proposition (connected concept) that serves as its ground. We have seen an example previously, depicted in Figure 6.2.3, that can serve equally well as an example of the inference of understanding since the process of determining judgment did not have to involve any matters (concepts) not already present within the manifold of concepts in order to derive its connection of judgment. Kant tells us,

> The fundamental character of all immediate inferences and the principle of their possibility subsists simply in a change of the *mere form* of judgments, while the *matter* of the judgments, the subject and predicate, remains *unaltered, the same.* [KANT (**9**: 115)]

Why, though, should we call this internal Relation an inference "of understanding" rather than merely an inference "of determining judgment"? The answer to this question goes back to the meaning of "understanding" in Critical epistemology. In the first edition of *Critique of Pure Reason* Kant tells us,

> We have above explained understanding in various ways: through a spontaneity of cognition (in contrast to the receptivity of sensibility); through a capacity to think; or likewise a power of comprehension, or also of judgments – which explanations, if one looks at them properly, amount to the same thing. Now we can characterize it as the *faculty of rules*. This distinguishing mark is more fruitful, and comes closer to its essence. Sensibility gives us forms (of intuitions), but understanding gives rules. It is always busy peering through appearances with the aim of finding some sort of rule in them. Rules, so far as they [interpret *Existenz* as necessary][1] (and thus necessarily pertain to the knowledge of objects) are called laws. Although we learn many laws from experience, these are only particular determinations of yet higher laws, the highest of which (under which all others stand) come *a priori* from understanding itself and are not borrowed from experience, but rather must provide appearances with their lawfulness and by that very means make experience possible. Understanding is thus not merely a capacity to make rules through the comparison of appearances: it is itself the legislature for nature, i.e. without understanding there would not be given any nature at all. [KANT (**4**: A126)]

Kant scholars and readers who are familiar with *Critique of Pure Reason* will have, I am confident, already taken note of the non-appearance of any box labeled "understanding" in my diagram of the logical organization of *nous* (Figure 1.5.1). This is because "understanding" is not a process (much less a "faculty" as "faculty psychology" uses that term), but rather an outcome of acts of *nous*. The phrase "faculty of rules" does not mean "rule factory"; it means, quite literally, the power of rules *to accomplish something*. This something is the transformation of information (that which is persistent in representations of both *soma* and *nous* and stands as their substance) into *knowledge*. Kant distinguished seven grades of knowledge according to the scope of its objective content [KANT (**9**: 64-65)]:

1. to represent;
2. to perceive (= to represent with consciousness);
3. to be aware (= to represent in conscious comparison with other things);
4. to recognize (= to be aware with consciousness, i.e. to conceptualize awareness);
5. to understand (= to know by means of concepts through thinking);
6. to recognize by means of reason (= to understand by means of Ideas);
7. to comprehend (= to recognize through reason *a priori* to a degree sufficient for one's aim).

These levels describe Quality of logical perfection attained by the Organized Being's capacity for cognition. The transformation from mere information of appearance to knowledge of phenomena occurs at the fifth level of Kant's hierarchy. Note that these grades are expressed as *verbs*.

The idea of the external Relation in the process of determining judgment is the **inference of reason**. As external Relation, an inference of reason requires two distinct judgments for the inference to take place. Kant explained this in the following way:

[1] Inserted in Kant's personal copy of the first edition, [KANT (**23**: 46)].

> In every inference of reason I first think a rule (the *major*) through understanding. Second, I subsume a cognition under the condition of the rule (the *minor*) by means of the power of judgment. Finally, I determine my cognition through the predicate of the rule (*conclusio*[2]), therefore *a priori* through reason. Thus the relationships between a cognition and its condition, which the major premise represents as the rule, makes up the various kinds of inferences of reason. They are therefore threefold – as all judgments in general – so far as they are distinguished in the way they set forth the relationships of cognition in understanding, namely: *categorical* or *hypothetical* or *disjunctive* inferences of reason. [KANT1: B360-361]

The logician's first impression at this point is quite likely going to be to see in this quote the classical syllogism. However, this would be a misinterpretation of Kant's meaning. Kant's word for "syllogism" was *Syllogismus*, he rarely used it, and he did not use it here. We must distinguish between "a line of argumentation" (the logician's context) and the actual *making* of the judgment as an inference. The process of determining judgment does not "argue"; it makes judgments. It is worth our while to be mindful that every normally healthy human infant can make determinant judgments, infantile though the concepts involved are, but no human infant can form a syllogism or mount a logical argument. We must stick to our context and resist the temptation to make any specious overgeneralization.

Why is the external Relation here called an inference of reason? To understand this we must look at the relationship between the Organized Being's power of pure Reason and its understanding. Kant tells us,

> First, the inference of reason does not operate on intuitions, to bring them under rules (as does understanding with its categories), but rather on concepts and judgments. If, therefore, pure reason also works on objects, it yet has no immediate reference to them and their intuition, but only to understanding and its judgments, which apply *directly* to the senses and their intuition in order to determine their object . . .
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> Second, reason in its logical use seeks the universal condition of its judgment (of consequents), and the inference of reason is nothing but a judgment mediated by the subsumption of its condition under a universal rule (the major premise). Now since this rule is once again exposed to this same attempt of reason, and the condition of the condition thereby has to be sought (by means of a prosyllogism) as far as we may, we see very well that the proper fundamental principle of reason in general (in its logical use) is: to find the unconditioned for conditioned cognitions of understanding, with which its unity will be perfected.
>
> But this logical outcome cannot become a principle of pure reason unless we assume: when the conditioned is given, then so also is given the whole series of conditions subordinated one to the other, which is itself unconditioned. [KANT1: B363-364]

Determining judgment does not determine its own employment. It is rather at the beck and call of the process of pure speculative Reason by means of orienting regulations imposed on it through ratio-expression (Figure 6.2.4). The inference of reason is the Relation of determining judgment to Reason. Determining judgment does all the work and operates only with its manifold

[2] "the act of finishing or completing"

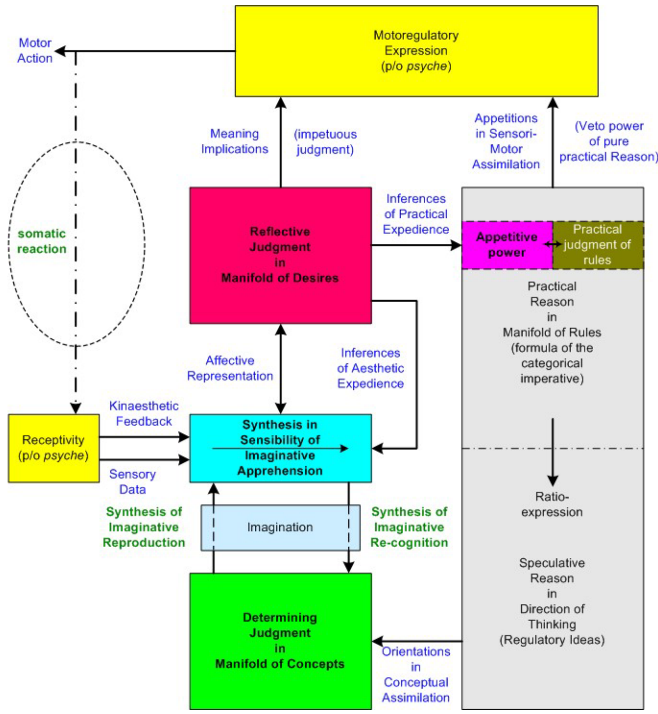

Figure 6.2.4: The logical organization of *nous*.

of concepts, but *which part* of this manifold it operates on is directed by the orientations it receives under the regulation of pure Reason. This regulation cares nothing for objects or even for understanding, but aims only at perfecting the form of the manifold of concepts according to a criterion of perfection contained in the process of pure Reason itself. Reason's orientations *bias the selection* of which parts of the manifold of concepts are to participate in the making of the determinant judgment. Thus it is that the inference of reason operates "on concepts and judgments." In this way, Reason brings the free play of understanding and imagination under the master regulation of the outer loop of judgmentation that passes through pure practical Reason to produce acts of ratio-expression. The regulative principle of judgmentation is, of course, a transcendental Idea of pure Reason.

The process of pure Reason is the master regulator of *nous*. As we may note in Figure 6.2.4, the direction of orientation flows from speculative Reason to determining judgment and flows in only one direction. This is why the inference of reason is external Relation in the 2LAR of determining judgment. The situation is otherwise when it comes to the relationship between the process of determining judgment and that of reflective judgment. Here, as we may note from the figure, determining and reflective judgment affect each other through the intermediary of the synthesis of apprehension. *Materia ex qua* enters into sensibility from determining judgment and this *materia* affects reflective judgment; but reflective judgment in its turn marks intuitions that enter into determining judgment through the synthesis of re-cognition and thus reflective judgment affects determining judgment. This is a transitive Relation and the corresponding inference in this case is called the **inference of judgment**. Kant tells us,

> Inferences of the power of judgment are certain modes of inference to come from particular concepts to general ones. They are not functions of the determining, then, but rather of the reflecting power of judgment; hence they also do not determine the Object, but only the mode of reflexion on it, in order to gain cognizance of it. [KANT (**9**: 132)]

Kant distinguishes between reflective judgment (*reflectirenden Urtheilskraft*, reflecting power of judgment) and the mode of reflexion (*die Art der Reflexion*) in his theory. We first saw this distinction when we introduced the *Verstandes-Actus* in the synthesis of apprehension. In the earlier quote from *Critique of Pure Reason* Kant said that reasoning seeks for higher conditions for every concept contained in the manifold of concepts. However, pure Reason cares nothing about objects or their representations, and so while it is within the power of Reason to regulate for logical perfecting of the manifold of concepts, it does not fall to Reason to judge the condition of this manifold. Such a judgment cannot be made objectively because the power to make a judgment of this sort must necessarily presume the *Dasein* of objective "innate ideas" and the Organized Being possesses no such knowledge *a priori*. This leaves only a *subjective* source for the origin of this kind of judgment, and that power lies with reflective judgment.

The *a priori* principle of all acts of reflective judgment is the principle of formal expedience. Reflective judgment itself cannot and does not judge objectively, but through its power to mark or not mark a representation of sensibility as an intuition, it provides *materia* to determining judgment and by this means brings objective thinking under the influence of the principle of formal expedience. That is what the inference of judgment does. *It is also through this Relation in determining judgment that the possibility for objective errors in thinking is first introduced as a consequence of the subjective nature of acts of reflective judgment*.

The modes of reflexion to which Kant refers are three-fold: (1) inferences of ideation; (2) inferences of induction; and (3) inferences of analogy. The **inference of ideation** requires no contribution to the *materia ex qua* of sensibility by determining judgment. It is the synthesis of the original intuition of an appearance and provides to determining judgment the first conceptual representation of the *Dasein* of an object. The **inference of induction** is a mode of reflexion by which marks that many object concepts have in common are generalized and united under the representation of a genus. As Kant put it,

> Induction thus concludes from particular to general . . . according to the principle of *generalization*: *What belongs to many things of a kind belongs to the remaining ones, too.* [KANT (**9**: 133)]

The **inference of analogy** is a mode of reflexion by which marks of one object concept are made marks of another object as well. While induction expands the empirically given from particular to general with respect to *many objects*, analogy expands from the given *properties* of a thing to divers properties *of the very same thing* [KANT (**9**: 133)]. As Kant put it,

> *Analogy* concludes from *particular* to *total* similarity of two things according to the principle of *specification*: Things of one kind which we know to agree in much also agree in the remainder as we know it in some of this kind but do not perceive it in others. [KANT (**9**: 133)]

Mathematicians tend to regard induction as reliable and firmly objective, whereas everyone past the age of childhood who cultivates the habit of critical thinking tends to regard analogy as much less trustworthy. However, the deep trust accorded to induction, especially by mathematicians, is based on a sandy foundation because, like analogy, induction arises from acts of reflective judgment and these acts, while they enjoy a subjectively sufficient condition for the making of the judgment, *do not possess an objectively sufficient ground of determination*. Like analogy, induction is only subjectively valid and not objectively certain. Proofs by induction – in particular, those extending to the mathematically infinite – do not possess objective validity and so, while suitable for speculative mathematics, cannot be used by Critical mathematics. Their products can never be anything other than secondary quantities of facet B.

Finite induction, and finite analogy as well, that produces judgments of possible objects of experience, which can therefore be *tested* in possible experience, can produce principal quantities but the objective validity of such a speculation does not carry real necessity *a priori* in its application to objects but merely logical necessitation in understanding Nature (one's world model). And this logical necessitation serves only as a means for judging the congruence of the manifold of concepts with objective experience. Every science must admit to dubitability and risk refutation of its theories in the arena of actual experience. A theory for which this cannot be done is not a *scientific* theory at all.

§ 2.4 Modality in Determining Judgment

The manifold of concepts takes the form of its connection from inferences. We must now ask: What is the matter of this *nexus*? The acroamatic principle of the process of determining judgment is the **principle of conformity to law:** All objects of Nature conform necessarily to the *a priori* laws which are the conditions of the possibility of experience. What is *in an inference* that entitles it to be called, as we do, a *local law of Nature*? Here one is tempted to say the answer to this question is "truth." But this answer raises at once an issue of profound and fundamental urgency. Truth is the congruence of the cognition with its object. But, as Kant was quick to point out,

> Now I can compare the Object[3] with my cognition *by re-cognizing it*. My cognition shall thus endorse itself, which is not by a long way sufficient enough for truth. For since the Object is outside me and the cognition is in me, all I can ever pass judgment on is whether my cognition of the Object agrees with my cognition of the Object. The ancients called such a circle in explanation a *diallelon*. And actually the logicians were always reproached for this mistake by the skeptics. . .
>
> The question here is, namely, whether and in how far there is a criterion of truth that is sure, general, and useful in application. For that is what the question, *What is truth?* means. [KANT (**9**: 50)]

Note carefully Kant has not said it is truth that is to be "sure, general, and useful in application." He said this of the *criteria* by which we determine whether or not a judgment is *held-to-be-true*.[4] Every inference contains a factor Kant called the **argument** (*Schlußfolge*). The argument is nothing else than *a criterion of truth*. Modality in a determinant judgment is always a judgment *of the judgment*, and Modality for the process of determining judgment is nothing less than what determines the criterion of truth for the arguments of inferences.

The general ideas of Modality are the determinable, the determination, and the determining factor. The idea of the determinable in the context of the process of determining judgment we call the **criteria of empirical truth**. In all representations it is the matter of representation that represents what is determinable. For determining judgment, this matter is the concept (the matter of composition) and of these we have the experiential concept, the notion, and the idea. Only the notion is pure and *a priori*, a part of the system of rules for the construction of concepts. The notions of understanding are primitives and of them it is without meaning to inquire if they are true because the representation of truth, even by criteria, is itself an idea and ideas are constructed from notions. If the primitive notions of understanding are not held to be true *de facto* then the very idea of truth is incapable of any explanation of its meaning whatsoever. The situation is quite different for experiential concepts and for ideas. These are not primitives.

Now, the basic explanation of truth is congruence of the *cognition* with the object. Criteria of empirical truth must be criteria by which we judge truth in regard to the object because to say we seek a criterion to judge empirical truth in regard to the cognition is to say we seek to see if the cognition is congruent with itself – the *diallelon* of which Kant spoke. However, as Kant showed in his *Logik*, the idea of a *universal material* criterion of truth is self-contradictory. To be universal it would have to apply to all objects, which means it would have to make abstraction of

[3] Kant is speaking here of comparing the transcendental object (matter) with its representation (form).
[4] Hegel championed the transcendent illusion of an Absolute Truth. The Object of this Platonic idea was the product of induction *ad infinitum* that could terminate only in god, the Rescuer of false theories. The determinant judgments of an Organized Being are *held to be true* or *held to be false* at the moment of their making and we cannot say of them they are absolutely true or absolutely false for in this the idea of **the absolute** (that which is valid in every respect and without restriction) lies far beyond the horizon of any possible experience.

all material differences among objects and, at the same time, deal with just this difference. Thus there can be no universal material criterion of truth.

However, a material criterion of truth is not the same thing as a criterion for empirical truth. In Critical epistemology an object is **empirical** when the representation of the object is so constructed that its concept is signified as thinglike and its marks are characterized by thinghood. **Thinglike** is that which is signified in the structure of the manifold of concepts by the category of unity. **Thinghood** is that character of the structure within the manifold of concepts meeting the conditions that this manifold is structured under the categories of both reality and unity. The criteria for empirical truth are therefore threefold:

1. It is possible to make an intuition of the object such that this intuition is reproduced from a thinglike manifold of concepts. Such an intuition is possible when the free play of imagination and understanding can be brought into harmony, a state that is adjudicated by the process of reflective judgment;
2. The experience represented in this intuition must be real, i.e. meet the condition of thinghood; and
3. There must be a conceptual ground for thinking the *Dasein* of the object. In general this means the concept of the object must spring from a prosyllogism where the object concept stands as cause of another concept, i.e. that it is connected to that concept through the Relation of the category of causality & dependency.

These criteria dictate specific conditions the manifold of concepts must meet in order to establish the empirical congruence of the cognition with the object. However, it is important to note that the first criterion brings the process of reflective judgment into the picture and all acts of reflective judgment are subjective and non-objective. In a manner of speaking, the inference is held to be empirically true "because it *feels* right." Therefore *objective certainty never is attained for the empirical truth of any inference*. Truth and certainty are not the same thing. The empirically true is also and always the contingently true. The *degree* of empirical holding-to-be-true can be very great, as we will later discuss, but it can never be absolute. Logicians have long known this and that is why empirical truth is not dealt with in formal logic. The criteria of empirical truth are metaphysical (supra-logical) rather than logical.

The situation is otherwise when we turn to the idea of the determination because here we are dealing with the congruence of a cognition with the whole structure of the manifold of concepts. For this reason we call the idea of the determination in the context of determining judgment the **criteria of logical truth**. For this it is possible to state *formal* universal rules. There are again three principles and logicians will recognize their names. They are:

1. the principle of contradiction;
2. the principle of sufficient reason; and

3. the principle of the excluded middle.

Logicians will recognize these names, but it is important for them and for us to bear in mind that these principles apply under the terms of Critical epistemology. Present day logic is not founded upon epistemology-centered metaphysics and so one should not assume one already knows the meaning of these principles merely because the same names have been in use since the days of the European Scholastics in the Middle Ages.

The first criterion concerns logical possibility. Cognition cannot contradict itself. This is a negative criterion for judging logical truth. Non-contradiction as a criterion for the holding-to-be-true of the judgment is consequently problematical in character and falls under the rule of the category of possibility & impossibility. It is the *sine qua non* of truth and it must hold not only for the immediate inference being made but also across the entirety of all inferences of judgment concerning the same object at the same moment in time. However, this aspect takes us into the second principle so we will come back to it in a little while.

The **principle of contradiction** is: To no (logical) subject whatever does there belong a predicate that contradicts the subject itself. Kant called this "the principle of negative truth" [KANT (**1**: 389)]. This is a fairly trivial principle when we are dealing with inferences of simple logical predications of the form S-P but is less so when the inference concerns remote marks of the subject concept. If a remote mark contradicts a less remote mark (one that has fewer intermediary concepts between itself and the subject concept in the manifold of concepts), then that remote mark contradicts the subject concept as well and cannot be connected to the subject concept as an affirmation. Here it is important to bear in mind that contradictories and contraries are not the same thing and the principle applies to contradictory predicates rather than merely contrary ones. The criterion marks *contradictories* as inexpedient in thinking.

There is an affirmative pole in this principle that Kant often called the "principle of identity." It merely states, in effect, that if the predicate does not contradict the subject then the predication might be true. In this case the inference is expedient but affirmed only as merely possible. In contrast, the negative principle (contradiction) is definitive and assertoric. Kant often lumped these two forms together and called them, jointly, "the principle of contradiction and identity."

The **principle of sufficient reason** likewise has two poles and, likewise, one of them is the "weak" pole. This principle is one for the logical grounding of inferences. The strong pole is: From the truth of the consequence we may embrace the truth of the cognition as ground, but only negatively; if one false consequence flows from a cognition then the cognition itself is false. For if the ground were true, so must the consequence also be true because the consequence is determined by the ground [KANT (**9**: 52)]. The weak pole of this principle merely states that if all the consequences of a cognition are true then the cognition is true too. This is a weak form of the principle because it is not possible to know *all* of the consequences apodictically, and therefore the weak pole only admits as expedient a hypothetical inference, i.e. is merely the basis for making hypotheses. A hypothesis is a *guess* based on facts and sometimes judgment just guesses.

The **principle of the excluded middle** states: From the negation of one contradictory opposite to the affirmation of the other, and from the positing of one to the negating of the other, the consequence is valid [KANT (**9**: 130)]. The pairs of inferences referred to in this principle are contradictory predications posited of the subject; that the inference is a cognition is to be understood. Kant's statement of this principle is the same as that used by classical logicians and the inherent reference it makes to cognition and determinant judgments is easy to miss if one forgets that our context in all of this is the process of determining judgment. It is also important to keep in mind that the principle deals with contradictory, not merely contrary, inferences. Modern day fuzzy logicians, as well as mathematicians from Brouwer's "constructivist" school of mathematics, often assail the principle of the excluded middle. These objections are misplaced. In the case of the fuzzy logicians, their examples involve contraries rather than contradictories and one can only assume the objectors are calling something else the "law of the excluded middle." In the case of the constructivists, their objection is usually based on being unable to find a finite construction for a mathematical theorem. Again, however, this does not involve contradictories and so is not a situation having the proper context for the principle. At best they prove lack of objective validity for a theorem rather than *determine* its truth or falsity.

We may note that the principles for the determinable and the determination both permit the making of inferences where "truth" can be merely problematical (possible) or hypothetical (asserted without a sufficient proof of necessity). After centuries of practice under the misguidance of non-Critical pseudo-metaphysics, both cases are likely to seem quite uncomfortable to logicians and mathematicians alike. Poetically speaking, determining judgment is not afraid of errors. When one considers how complex is the task of understanding Nature, this character of determining judgment seems likely to have a high survival value for the Organized Being within the context of the hypothesis of natural selection in evolution. This is not to say the Organized Being "likes making mistakes"; the discovery of error is dissatisfying so far as the process of *reflective* judgment is concerned because "errors feel wrong" (are inexpedient). But this character of the process of determining judgment is one that introduces a real need for the development of something along the lines of what von Neumann spoke of in terms of "probabilistic logic" – whatever the character of the developed form of this new logic eventually turns out to be. Our challenge is, as von Neumann correctly foresaw, developing the mathematical and logical *tools* to better understand and deal with complexity.

This brings us at last to the idea of the determining factor in the process of determining judgment. In a manner of speaking, this is the idea of the "Why?" contained in the manifold of concepts. But what is this "why?" idea? The answer here involves an old idea from logic that has dropped out of sight in our modern era: the idea of logical perfection. Here the term "perfection" means "making perfect" and the idea is *practical* (as we might have anticipated from Reason's regulation of the process of determining judgment). The determining factor is a "because" factor we name *the criteria for judging logical perfection*.

Perfection in general is divided into three types: logical, aesthetical, and practical. Aligned with each of these are the three processes of judgment: determining for logical perfection, reflective for aesthetical perfection, and practical for practical perfection. With regard to cognition,

> Logical perfection goes to understanding and is knowledge of objects by way of them. The aesthetical [perfection] goes to feeling and to the state of our Subject, namely: how we come to be affected by the Object . . . Practical perfection goes to our appetites, through which activity comes to be brought about.
>
> The perfection of a cognition rests on four principal points:
>
> 1. For the Quantity of a cognition, as it is a universal. A cognition which serves as a rule must be more perfect than one that holds only in particular cases.
> 2. . . . Quality, distinctness of the cognition. Contains the *quo modo?*[5] Logical perfection as to Quality is *distinctness*, the aesthetical is *liveliness*.
> 3. . . . Relation, truth of the cognition. Truth is the Relation of the cognition to the Object . . . Logical perfection according to Relation is *objective truth*. The aesthetical is *subjective truth*.
> 4. . . . Modality, so far as it is a certain and necessary cognition. Logical perfection according to Modality is the *necessity of cognitions according to understanding.* The aesthetical is *empirical necessity*. [KANT (**24**: 809-810)]

Perfection can be called the aim of judgmentation. Each of the three processes of judgment has its particular brand of perfection that serves as an ideal for judging. For this to be objectively valid for the theory of *nous*, we cannot look at perfection as some innate idea residing within judgment but, rather, as a kind of set-point analogous to the reference signal employed in a feedback control system. This requires that the state of perfection be measurable in some way and that the outcome of such a measurement is capable of being evaluated:

> The logical perfection of cognition rests on its congruence with the Object, hence on *universally valid* laws, and thus likewise suits itself to be judged according to norms *a priori*. [KANT (**9**: 36)]

These norms can be called ideals for judgment in the sense that the greater the measure of

[5] "in what way?"

something is, the closer that something approaches its ideal. The measures of logical perfection are: (1) the measure of how universal a judgment is; (2) the measure of the distinctness of a cognition; (3) the measure of truth in the judgment; and (4) the degree of holding-to-be-true the judgment commands. These measures go to the magnitude of the sphere of a concept, the depth of understanding contained in the concept, the fecundity of the scope of the concept, and the degree of certainty in the concept. The **criterion of logical perfection** is then *to maximize the measures of perfection* in the structure of the manifold of concepts. We will discuss this in Chapter 12.

§ 3. The Logical Functions of Understanding in Judgments

The technical term "function" means the unity of the act of ordering different representations under a common one [KANT1: B93]. The act undertaken in the process of determining judgment is the making of a determinant judgment and so, viewed as a scheme under the logical reflective perspective, a category is precisely such a unity of the act and so is a function of judgment. But what is a determinant judgment?

> But if I distinguish more closely the reference of given knowledge in every judgment and distinguish it, as belonging to understanding, from relationship according to laws of reproductive imagination (which has only subjective validity), I find that a judgment is nothing but the manner of bringing given knowledge to the *objective* unity of apperception. This is plain from our use of the copula **is** in the aforesaid, in order to distinguish the objective unity of given representations from the subjective. For this indicates the reference of these representations to original apperception and their *necessary unity* even though the judgment is empirical, therefore contingent . . . I do not say by this that these representations *necessarily* belong *to each other* in empirical intuition, but rather that they belong to each other by virtue *of the necessary unity* of apperception in the synthesis of intuition, i.e. according to principles of the objective determination of all our representations so far as knowledge can arise from them, these principles being all derived from the first principle of the transcendental unity of apperception. In this way alone can there arise from this relationship *a judgment*, that is, a relationship that is *objectively valid* and is perfectly distinct from the relationship of the very same representation which has only subjective validity according to the laws of association. [KANT1: B141-142]

When we come to the question of how the categories, as schemes, construct the manifold of concepts, our discussion turns to one of representation of knowledge by means of logical relationships among concepts. We are then concerned with the logical unity of the structure in this manifold, which we explain in terms of how concepts are combined by judgments.

Kant called the logical makeup of the unity by the name **logical function of understanding in judgments**. As for all representations, the representation of this makeup is expressed as a 2LAR by our four titles of Quantity, Quality, etc. with three logical momenta of synthesis under each. We can regard each of these twelve logical momenta as a logical function of the synthesis of the manifold. Figure 6.3.1 illustrates the 2LAR structure of these logical functions.

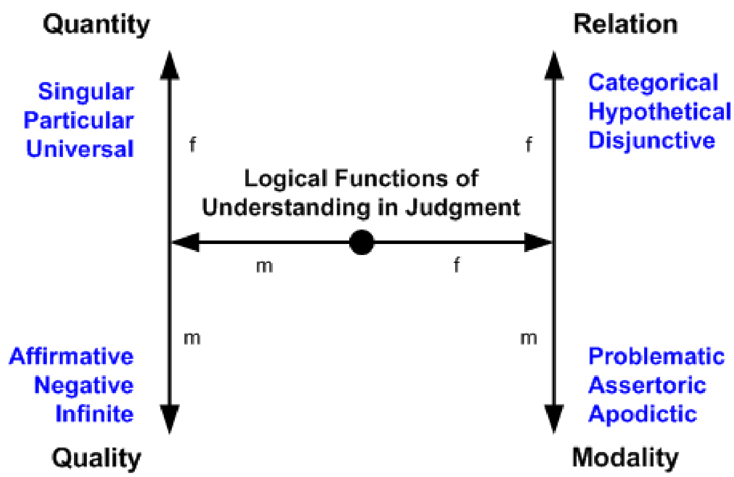

Figure 6.3.1: 2LAR of the logical functions of understanding in determinant judgment.

Kant used the language of the logic doctrine of his day to name these logical momenta but this does not signify that these momenta mean the same thing as the terms mean in conventional logic. Indeed, Kant regarded that doctrine as being fundamentally flawed. Kant's logic doctrine is epistemology-centered, while the classical doctrine of logic is not. That Kant applied what is usually called his "Copernican turn" to logic doctrine is made plain by his statement,

> The logical momenta of all judgments are so many possible ways to join representations in one state of consciousness. [KANT (**4**: 305)]

It is the task of this section to set out what Kant did mean by these terms.

§ 3.1 Manifold Terminology

The logical momenta are concise formal descriptions of how judgment combines concepts in structuring the manifold of concepts. Because a determinant judgment is itself a representation, each combination involves four logical momenta, one from each title of representation. We need a vocabulary for explaining the characteristics of these combinations and this vocabulary will be a vocabulary describing the role played *in a judgment* by a concept.

Logically considered, a concept is a representation common to many other representations. Specifically, it is a representation of a submanifold in intuition common to many intuitions since a concept is a rule for the reproduction of an intuition. Two concepts combined in a judgment have relationships to one another that are distinguished by particular technical terms. A **mark** is a concept representing a general characteristic regarded as inherent or essential in the representation of many objects. As a concept a mark represents what is contained in two or more **lower concepts** that these concepts have in common. A mark is a common ground of cognition for the objects of concepts contained under it, and these objects are said to constitute the **scope** of the mark. The concepts contained under a mark constitute the **sphere** of the mark.

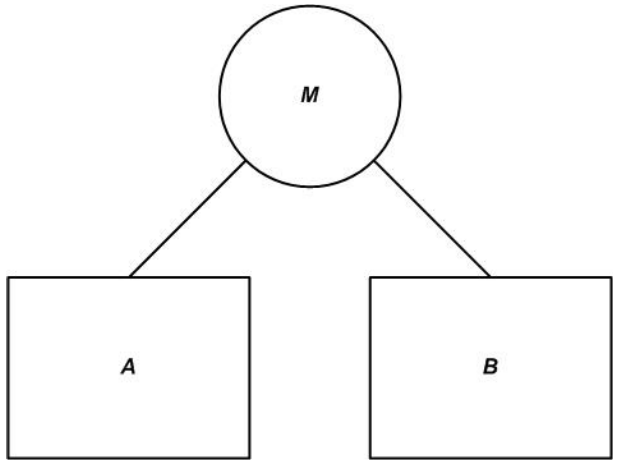

Figure 6.3.2: Graphical illustration of a mark, *M*, of two lower concepts *A* and *B*.

Figure 6.3.2 provides a graphical illustration of the relationship of a mark to the lower concepts in its sphere. Mark *M* is said to be **contained in** concepts *A* and *B* and **to understand** these concepts. Likewise, the two lower concepts are said to be **contained under** *M* and **to stand under** it. Concepts *A*, *B*, and all other concepts standing under the mark constitute the sphere of the mark. This sphere includes concepts that stand under concepts *A* and *B* as well. Note that the terms "higher concept" and "lower concept" pertain only to the relationship of the concepts in the combination of judgment and not to the concepts themselves. "Higher" and "lower" are judicially relative terms and always involve at least two combined concepts.

A mark contains what is common in the representations of two or more concepts for which it is the mark. Therefore any particular concept *A* can have a multiplicity of marks from being combined with other concepts. This is illustrated by Figure 6.3.3. Marks that immediately understand a concept *A* are called the **coordinate marks** or simply the **coordinates** of *A*. Each coordinate coordinates concept *A* with one or more other concepts at its same level.

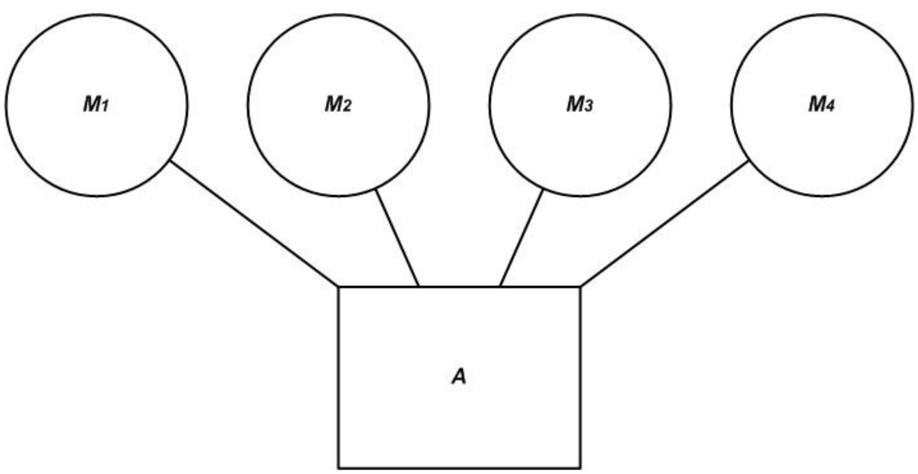

Figure 6.3.3: Graphical illustration of the coordinate marks of concept *A*.

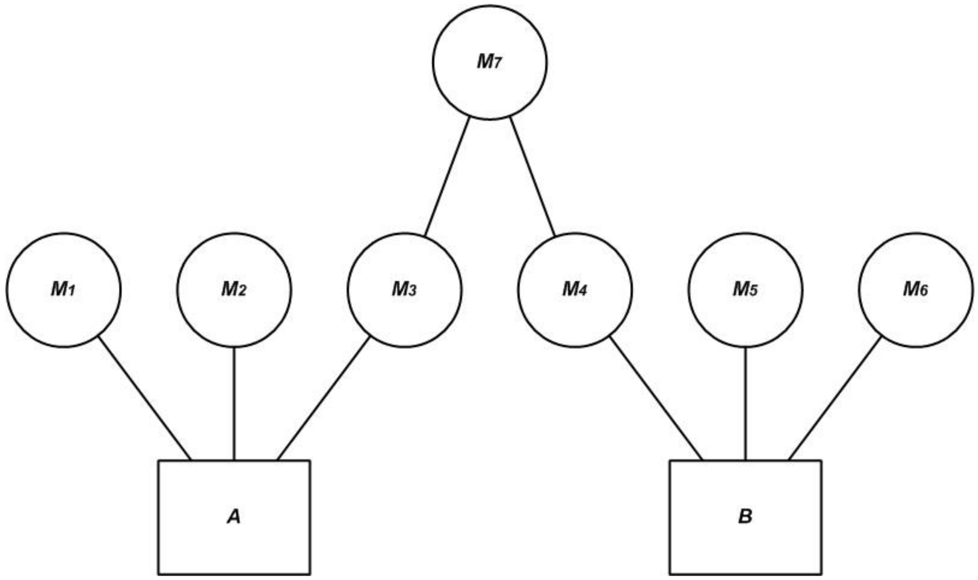

Figure 6.3.4: Combination is a series. Concept M_7 is a remote mark of concepts A and B.

The mark of a mark of a concept A is called a **remote mark** of A. This is illustrated by Figure 6.3.4. This construction illustrates a **series** running from A to the remote mark M_7 and vice versa. A mark of a remote mark is likewise called a remote mark of A. Concept A is said to be **subordinated to** its remote marks and they are said to be **subordinate marks** of concept A. In any series a higher concept is said to be a **condition of** the lower concept and the lower concept is said to be **conditioned by** the higher concept.

It was said earlier that any combination of concepts formed by determining judgment is itself a concept. This holds true for all of the diagrams illustrated in this subsection. The terminology used to describe the manifold of concepts is a relative terminology and one uses it in conjunction with the context of whatever aspect of judgment is being discussed and in the context of the Object of the judgment. It is not difficult to see why this must be so. First, the manifold of concepts, viewed as a system, is an **open system** and its structure is subject to on-going accommodations as new intuitions are re-cognized by imagination and introduced into it. The theory of determining judgment is thus an *open system theory*, and this is at present the least developed topic within the science of system theory.

Furthermore, the practical objective validity of the very idea of "concepts" is found strictly within the context of **thinking**, which we define as cognition through concepts. Psychologist and philosopher William James was rightly critical of what he saw as the "atomism" of John Locke's empiricism, and James mistakenly thought Kant's theory suffered from this same issue. He contended that "the thought" we experience is a unity. One of his favorite examples was "the-pack-of-cards-is-on-the-table" as an illustration of "thought" as a unity. We are now in a position to see that the idea of a concept in the Critical theory does not conflict with James' model of how "thoughts" are actually experienced by human beings.

§ 3.2 The Logical Momenta of Quantity

In every determinant judgment there is a determinable concept and a determining concept. We call the former the **subject concept** and the latter the **predicate concept**. (This terminology is logical terminology rather than ontological; the subject *of* a concept is its object). What a judgment adds to combine these is a connection, often called the copula, and the outcome is the **aggregate concept**. All logical compositions of Quantity are defined in terms of the sphere of the subject and its relationship to the sphere of the predicate. If the subject concept has no sphere (that is, if there are no other concepts standing under it) the composition of Quantity in judgment is **singular** [KANT (**9**: 102)]. A singular judgment terminates a series of concepts *a parte posteriori* at the subject concept.

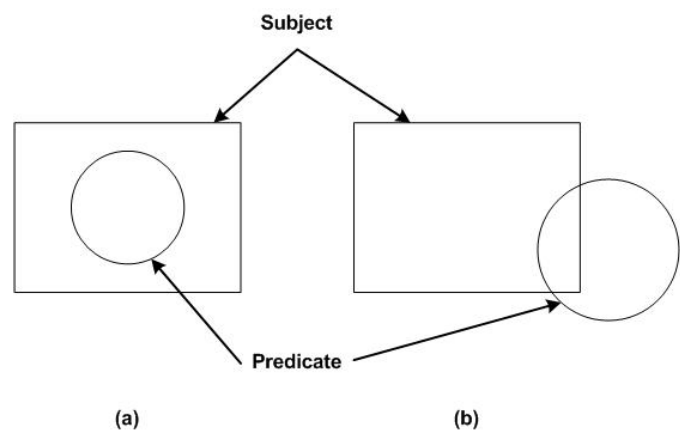

Figure 6.3.5: Euler diagrams of the particular logical momentum. (a) when the subject is the higher concept; (b) the contingently particular judgment, in which neither concept is a higher or lower concept.

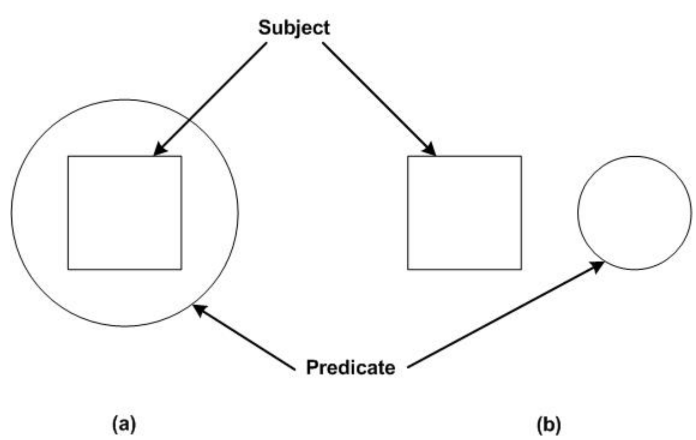

Figure 6.3.6: Euler diagrams of the universal logical momentum. (a) affirmative universal; (b) negative universal.

When the subject concept has a sphere there are two possible relationships between the spheres of the subject and the predicate. These are called the particular and the universal logical momenta. Kant used Euler diagrams to illustrate these cases. An Euler diagram represents the spheres of the concepts, i.e. represents the relationship of their extensions. Euler diagrams differ from the more familiar Venn diagram, which was invented by the English mathematician John Venn in 1881 as a topological model of Boolean algebra. Venn diagrams represent the contents (intension) of a concept without representing its sphere. Euler diagrams represent what a concept understands, Venn diagrams represent what a concept contains.

Figure 6.3.5 illustrates the two possible cases for the particular logical momentum. In a **particular judgment** part of the sphere of the subject contains part of the sphere of the predicate *and* part of the sphere of the subject is *not* part of the sphere of the predicate. Figure 6.3.5(a) illustrates the case where the sphere of the subject is greater than the sphere of the predicate and entirely contains the sphere of the predicate. In this case, the subject stands as higher concept to the predicate. Figure 6.3.5(b) illustrates the case where part of the sphere of the predicate is not contained in the sphere of the subject. This case is called the *contingently particular* [KANT (**9**: 103)] and here neither concept stands as higher concept to the other because a higher concept always contains the entire sphere of the lower concept under it.

In a **universal judgment** the sphere of the subject either is entirely contained in the sphere of the predicate or is entirely excluded from the sphere of the predicate. Figure 6.3.6 illustrates these two cases. In Figure 6.3.6(a) the predicate concept is the higher concept to the subject because the sphere of the subject is contained under it. In Figure 6.3.6(b) it is meaningless to call either concept the higher or the lower because their spheres are judged to be entirely disjoint. The universal judgment and the singular judgment differ because in the universal judgment the subject has a sphere, while in the singular judgment it does not. The subject concept in a singular judgment is always a lower concept and a terminal of intersection for the spheres of all its coordinate concepts. We cannot represent the subject of a singular judgment as a point in a Venn diagram because a Venn diagram represents what is contained in a concept and all the coordinate concepts of the subject in singular judgments are contained in the subject concept. We can, however, represent the subject concept of a singular judgment as a point in Figure 6.3.6.

§ 3.3 The Logical Momenta of Quality

Logical momenta take their entire context from the idea of the manifold of concepts. This is to say that they are merely descriptive of the organization of combinations in the manifold and are devoid of any real meaning outside this context. The three logical momenta of Quality are defined as follows [KANT (**9**: 103-104)]:

1. The **affirmative** momentum is the logical momentum of judgment by which the subject concept is thought in combination with the sphere of the predicate; its classical illustration is the predication S is P;
2. The **negative** momentum is the logical momentum of judgment by which the subject concept is set outside the sphere of the predicate and the predicate is thought as being *contradictory* to the subject; its classical illustration is the predication S is-not P;
3. The **infinite** momentum is the logical momentum of judgment by which the subject concept is set in the sphere of an undetermined concept that lies outside the sphere of the predicate; this momentum has no classical counterpart in logic but can still be illustrated by the predication S is not-P.

The form of the copula is fundamental to understanding the three momenta of Quality. A judgment S is P means the predicate P can be predicated of the subject concept S in an act of thinking. (Again, thinking is cognition through concepts). A judgment S is-not P means the predicate P cannot be predicated of the subject concept S in an act of thinking; attempting to do so sets up a real opposition in sensibility. The negative momentum sets up the reproduction of S and P during the synthesis of reproductive imagination as negative magnitudes in intuition relative to one another, whereas S is P sets them up as positive magnitudes relative to each other.

The case of the infinite logical momentum requires more discussion. S is not-P sets up a *contrary* relationship between subject and predicate (not a contradictory one). It is a positive judgment (the copula is "is") with respect to the subject but restricts the sphere *of the predicate* and bans it from having its sphere containing or contained in the sphere of the subject. Otherwise it determines *nothing whatsoever* with regard to the subject. For example, if I say, "Fred is not-German" all I have determined is that "being German" is contrary to "being Fred." Fred might be French, English, or, for that matter, a cocker spaniel or a bullfrog. In the context of the logical momenta of determining judgment, the German word for "infinite" (*unendlich*) simply means "unlimited" in the sense that *nothing positively definitive* is established in regard to the sphere of the subject concept. The infinite judgment in no way whatsoever carries any connotation related to the modern mathematical idea of "infinity" except perhaps the statement that mathematical infinity is not-a-number.

§ 3.4 The Logical Momenta of Relation

The logical momenta of Quantity and Quality speak to the composing of the manifold of concepts. The momenta of Relation speak to its connecting. When we turn to consideration of this aspect of the making of determinant judgments we find that something new must be added to our exhibition, namely the idea of "propositions." The capacity for synthesizing the manifold requires three synthetical forms of proposition, which we call the categorical, hypothetical, and disjunctive forms of proposition. A **proposition** is the aggregate concept of a determinant judgment in which the concepts of two or more Objects are connected in Relation. Of the three forms of proposition, the categorical proposition is the fundamental form and the other two are derivative forms based upon it. We will therefore begin with the categorical logical momentum.

When we turn to consideration of *nexus* in the manifold of concepts our attention shifts from the composition terms, e.g. the concepts S and P, to the form of their connection. This form of connection represents *the manner of conscious unity* in the judgment. We define the term **copula** *in the wide sense* as the form of the structure of a combination of concepts made by the process of determining judgment. This definition differs at its ontological roots from the use of that term in classical logic.[6] We also have a *narrow sense* usage of this term provided by Kant:

> In categorical judgments, subject and predicate make up their matter; the form, through which the relationship (of agreement or opposition[7]) between subject and predicate is determined and expressed, is called the *copula*. [KANT (**9**: 105)]

For notational convenience, we will denote a categorical judgment using the notation SxP and let x denote the copula in the narrow sense. The form SxP is called a **categorical proposition**. The aggregated concept SxP may be called the *copulated concept*. It is the aggregate concept of a phenomenon in which the concept of the predicate is placed in connection with the concept of the subject as *Existenz* (predicate) to *Dasein* (subject) in the conscious unity of the judgment. To illustrate the meaning of this, let us use the sentence "John married Jane" as a model.[8] In this predication the subject concept is "John" and the predicate concept is "married-Jane." In our notation, the connection of these concepts is denoted as {John x married-Jane}. If we wished to let the symbol C denote the copulated concept, we could write this as C{John x married-Jane} and perhaps by this appease our modern logicians' penchant for trying to make logic look like algebra. As a conceptual representation of a phenomenal object, C{SxP} carries the *modus* of persistence in time and the rule-scheme for its construction is the category of substance & accident.

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[6] The ontological centeredness of classical logic required a connection (the word *copula* in Latin means "a link, bond, or tie") between subject and predicate, which were regarded as "complete terms" requiring something to glue them together in a sentence. Mathematical (symbolic) logic since the time of Frege has done away with the use of the term "copula" altogether by regarding the predicate as a mathematical function requiring no connecting term to bind it to its mathematical "argument" (mathematical logic's version of the "subject").

[7] *Widerstreit*

[8] As is always the case when using analogy for an example, there is some risk attending the use of sentences as examples of logic structure because natural language is not-logic and logic is not-natural-language. This can be hard to see in one's native language, but is easier to see in other languages. For example, we can look at the Japanese sentence, *Kore wa boku no kippu desu*, which literally and word-for-word in English is "this as-for I of ticket is" and translates properly as "This is my ticket."

The categorical proposition is the most basic of the proposition types in the sense that the other two are *judgments of categorical propositions*. Our second form of Relation in the manifold of concepts is the **hypothetical proposition**, which Kant explained in the following way:

> The matter of hypothetical judgments subsists of two judgments that are connected with one another as ground and consequence. One of these judgments, which contains the ground, is the antecedent (*antecedens, prius*); the other, which is related to the former as a consequence, is the after-proposition[9] (*consequens, posterius*), and the representation of this manner of connection of both judgments under one another for the unity of the state of consciousness is called the *Consequenz*, which constitutes the *form* of hypothetical judgments.
>
> Note 1: What the copula is for categorical judgments, so the *Consequenz* is for hypotheticals – their form. [KANT (**9**: 105)]

Like the categorical proposition, the hypothetical proposition is a determined concept and it contains three parts: antecedent, "after-proposition" or **consequent proposition**, and *Consequenz*. We will use the notation AyC to denote this type of proposition. Here y denotes the *Consequenz*.

The hypothetical proposition differs substantially from the categorical both as to its matter and as to the rule that brings it to the transcendental schema of imagination. The concepts A and C must both be categorical propositions. No concept falls within the domain of the hypothetical logical momentum unless it is a determined concept that has resulted from the application of the categorical logical momentum in judgment under the rule of the category of substance & accident. If we say that SxP is a rule, then AyC is a rule about rules.

Another difference that distinguishes a hypothetical proposition from a categorical one shows up in how the concept terms in the proposition are regarded as being consciously determined by the act of judgment. In the categorical proposition the subject concept is regarded as determinable and the predicate concept is regarded as a determination. For the hypothetical proposition the consequent proposition C is regarded as conditioned, the antecedent A is regarded as the condition. The proposition as a whole is regarded as a determination. Its rule-scheme is the category of causality & dependency.

For the hypothetical proposition the connection is not directed at the object of the judgment as a substance but rather to the establishment of the truth of a relationship between the antecedent and the consequent. The form of the judgment corresponds to the external Relation in our general 2LAR. This form has a two-fold character with regard to the *Consequenz*. Borrowing from the language of classical logic, the first of these can be called the "positing connection" or *modus ponens*. The second can be called the "rescinding connection" or *modus tollens*. The connection

[9] *Nachsatz*, the proposition that "follows after" the antecedent. Note well that Kant is making a number of very fine distinctions here. One does not normally find this level of distinction in formal logic, but Kant's logic must take concepts *and* objects into account. We will call this "the consequent proposition."

made in a hypothetical judgment is a connection to the state of consciousness regarding the criterion under which the connection of the judgment is held-to-be-true according to our *a priori* norms from the previous section. For the *modus ponens* the norm is: if the antecedent is true then the consequent is also true. For the *modus tollens* the norm is: if the consequent is false then the antecedent is also false. Thus, the hypothetical logical momentum finds its nearest equivalent in classical logic in the construct called the "implication," which is usually represented by the form "If *A* then *C*" or, alternatively, "*A* implies *C*."

Having said this, however, we must not neglect an important detail that brings on a subtle distinction between the hypothetical logical momentum and implication in traditional logic. The positive act of the judgment is the positing of the connection according to *modus ponens*. If, in the overall structure of a judgment, we have a hypothetical proposition in which the momentum of Quality is the negative logical momentum, this corresponds to a statement in traditional logic of "*A* does-not-imply *C*." *The real ground* for making such a judgment is best described as "discovering a mistake" – i.e., encountering in experience an appearance where an *anticipation* of *C* from an actual perception of *A* (based on a prior judgment AyC) is *thwarted* by actual events *as they turn out this time*. This amounts overall to a violation of the condition of truth expressed in *modus tollens*. *Modus ponens* and *modus tollens* pertain to truth conditions, not products of judgments. It is this distinction that leads us to say, "the form of connection in hypothetical propositions is twofold" [KANT (**9**: 106)] rather than "the form of connection in hypothetical propositions is of two kinds." We call *modus tollens* a "rescinding connection" because thwarted anticipation is a ground for discovering an error of Relation in empirical judgment *and adding conditions to* ("rescinding") the prior judgment AyC by altering y ("changing one's mind"). Such a judgment makes an accommodation in the manifold of concepts.

The **disjunctive proposition** is a proposition that divides the sphere of a concept *H* into reciprocally determined and mutually exclusive subspheres. By mutually exclusive it is meant that if subsphere s_1 is asserted (re-presented in an intuition at a moment in time as part of the cognition of an appearance) then all the other subspheres s_2, s_3, etc. of *H* are held-to-be-not-true of this same object at this same moment in time. If the-pack-of-cards-{is}-on-the-table then the-pack-of-cards-{is-not}-in-the-drawer, although clearly at some other moment in time the opposites of this pair of assertions could be true of the concept the-pack-of-cards.

Relatively simple examples such as this one are useful for formal explanations of the disjunctive proposition, but its importance in structuring the manifold of concepts is perhaps better illustrated by a much more complex example. Let us suppose the two photographs in Figure 6.3.7 represent appearances re-produced in cognition (remembered). Further suppose these

(a) (b)

Figure 6.3.7: Different re-presented appearances of the same Object ("Richard"). (a) age 7 (b) age 13

appearances are appearances of one and the same Object, a specific individual boy we will call Richard. Figure 6.3.7(a) is an appearance of this boy at age 7 years; (b) is an appearance of the same boy at age 13 years. If an Organized Being knew this boy and could remember these appearances *as appearances of the very same boy*, this implies that the concepts going into the different reproductions are united under a common Object ("Richard") but belong to *different subspheres* of the sphere of the Object concept "Richard." The ability of an Organized Being to recall either of these appearances through the synthesis of reproductive imagination without getting the reproduced concepts mixed up – e.g. placing the image of the head in (a) on the shoulders of the body in (b) – is a demonstration of the *Existenz* of a manifold structure that makes possible the reproduction of pairs of complex memories such as those Figure 6.3.7 illustrates. This structure is the structure of the disjunctive proposition. The rule-scheme of the disjunctive proposition is the category of community.

There are two activities we must distinguish in the making of a disjunctive proposition. The first is the logical division of concept *H* into mutually exclusive subspheres. A concept P brings into its sphere other concepts S_i in categorical propositions, e.g. S_1xP, S_2xP, etc. In hypothetical propositions the antecedent A brings consequents C_i into its sphere, e.g. AyC_1, AyC_2, etc. The consequents, in turn, can be used as antecedents of still lower concepts, thus making A a remote mark of these still lower concepts. Of these concepts, some or all of their spheres are contained in the sphere of A. The composition of this sphere can also have for its matter a mixture of different relational types of propositions, including other disjunctive propositions. The first act in the disjunctive logical momentum is the division of the sphere of *H*, which we can symbolically represent as $s_H = \{s_1, s_2, . . ., s_n\}$ for a sphere s_H divided into *n* parts. This logical division is mutually exclusive, i.e., no subsphere can contain part of another subsphere involved in the disjunction.

But this is not all. In making the logical division, the disjunctive proposition does so in such a way that if some concept, say that of s_1, is asserted in thinking as being true at a specific moment in time, then the propositions of the other subspheres are held not-to-be-true *at this same moment*. The real significance of this is the following: When s_1 is used to form some part of an intuition *J* during the synthesis of apprehension, none of the other subspheres can participate in forming this same intuition *J*.

Thus, while the matters in a disjunctive judgment are all given (i.e., we have a representation of the sphere of *H* before this sphere can be divided), something new is added to the *nexus* of the manifold of concepts by the disjunctive logical momentum, namely *reciprocal determination* among the subspheres of *H*. The effect of the disjunctive proposition is to coordinate *under* the higher concept *H* all the other concepts in its sphere. This is the significance of the idea of sub-spheres of a concept. This is fundamentally different from the coordination of marks of a concept (Figure 6.3.3) since *H* stands under its coordinate marks but its subspheres stand under *H*.

Now, there are *metaphysical considerations* that become readily apparent when we look more closely at the making of a disjunctive proposition. One of them is the following. Not all propositions in the sphere of *H* must be categorical and, in particular, some of them may be hypothetical. Hypothetical propositions always contain two categorical propositions, and each of these in turn contain the representation, by S, of a substance. Suppose a hypothetical proposition $(S_1xP_1)y(S_2xP_2)$ is made and then a disjunctive proposition is made immediately after in a prosyllogism (synthesis *a parte priori*). This disjunctive proposition cannot place S_1 and S_2 in different subspheres because of the *nexus* established between them by the hypothetical proposition. The disjunctive proposition is co-determining, not subordinating, division. It cannot "draw a boundary" between the antecedent and the consequent of a hypothetical proposition.

Which, then, of the two substance concepts (S_1 and S_2) is the *basis* of a possible division of a sphere? The answer to this question is provided in Rational Physics by the third Analogy of Experience and it is this: The object of the consequent proposition is the object "present" at the moment in time during thinking when the disjunctive proposition is formed in intuition (because, by definition, the antecedent is prior in time relative to the consequent). It is therefore the substance concept of the consequent (S_2) around which a division of the sphere can nucleate and not that of the antecedent (S_1).

This example serves to alert us to an important facet of Logic. The *sequence* in which propositions are formed affects the manner of the thinking that produces propositions. There is, in other words, a *temporal dynamics* factor in the actions of determining judgment. In formal logic no such dynamic is recognized. The logician takes whatever logical statements he wishes in whatever order he wishes to place them and operates mechanically on the static situation he has just posed. Thinking does not have this freedom of action because it is bound by the epistemology of Critical metaphysics. This temporal dimension in Kant's Logic is something both classical logic and modern mathematical logic completely miss. And this is one reason why modern logic is impoverished and stands in need of major reconstruction on fundamental theoretical grounds just as much from the practical need to deal with complexity that von Neumann foresaw.

§ 3.5 The Logical Momenta of Modality

Modality in judgment is a judgment of a judgment. In determinant judgments it adds nothing at all to the concept of an object *as* object and concerns nothing but the manner in which the concept is used in cognitions. Kant tells us,

> The modality of judgments is a quite special function of them, which has the distinction that it contributes nothing to the content of the judgment (for besides magnitude, quality, and relationship there is nothing more that makes up the content of a judgment), but rather only has to do with the value of the copula in reference to thinking in general. Problematic judgments are those where one accepts the assent or denial as merely possible (arbitrary); assertoric [judgments] are those regarded as actual (true); apodictic [judgments] are those one sees as necessary. [KANT1: B99-100]

Elsewhere he writes,

> Possibility, actuality, and necessity are of course logical but not metaphysical (real) predicates, i.e. determinations. We know through them not object-matters but the relationship of our concepts to the capacity of the mind to posit and rescind. [KANT (**18**: 125-126)]

Possibility & impossibility, actuality & non-being, and necessity & contingency are, of course, the categories of Modality. The real meanings of possibility, actuality, and necessity are provided by the transcendental Ideas of Rational Physics (specifically, by the Postulates of Empirical Thinking in General). These categories are the rule-schemes for the logical momenta of the problematic, assertoric, and apodictic judgments, respectively.

The **problematic** logical momentum underlies the ability of the power of spontaneity to exhibit *creativity*. Creativity is the ability to think concepts of things one has never experienced and to *know* that these concepts do not spring from external experience but rather are fruits of one's own spontaneity. In a problematic judgment the Organized Being's holding-to-be-true of the judgment is provisional and tentative. The practical *application* of this momentum can be illustrated for the three types of propositions as follows:

Problematic categorical - *What if* SxP*?*
Problematic hypothetical - *What if* AyC*?*
Problematic disjunctive - *What if* $s_H = \{s_1, s_2\}$*?*

This is not to say cognition in the Organized Being's thinking takes on this appearance. Indeed, one must hold this is not so because problematic judgments are possible for a child still too young to formulate cogent thoughts with these specific appearances. If we speculate that a baby can be curious at all, it would be nothing more than a kind of practical curiosity and not a theoretical or introspective curiosity.

However, inherent in all acts of thinking is the connection of the cognition to the *I* of transcendental apperception in the form Kant described as *I think*. Cast in this *logical* form, the **problematic** proposition is described by

Problematic categorical - *I think maybe* SxP;
Problematic hypothetical - *I think maybe* AyC;
Problematic disjunctive - *I think maybe* $s_H = \{s_1, s_2\}$.

There is a tentative, uncertain, and *intellective* character in how problematic judgments are held in consciousness. Concrete behavioral evidence of a baby exhibiting problematic judgmentation comes relatively late in the sensorimotor period of development and first occurs in what Piaget called Stage V ("Discovery of new means through active experimentation") at around age ten months to one year. The capacity for making problematic judgments is, therefore, a rather advanced capability in comparison to the capabilities of the newborn. It requires a sufficiently developed and complex construction of the manifold of concepts be achieved to provide *materia* for this character of thinking to be possible in the free play of understanding and imagination:

> The faculty of imagining is the capacity for intuition of objects of past time, the faculty of anticipating is the capacity for intuition of objects of future time. The capacity for intuition itself insofar as it is not entirely bound to time is called the fictive faculty. All three capacities have their laws. The first law is the law of the association of ideas. The law of the power of imagination as a capacity for seeing in advance is the law of expectation of similar occasions. The law of the fictive faculty is the law of compatibility of ideas. [An idea] is to be conceived according to the law of compatibility, it is to be reproduced according to the law of the association of ideas. [KANT (**28**: 585)]

While the problematic momentum imputes no commitment to holding a problematic judgment to be *empirically* true, the situation is otherwise for the other modal momenta. In experience we are each in possession of a great number of cognitions that we hold, in varying and often quite high degree, to be *factual*. What all concepts of such cognitions share in common with each other is that each is regarded as being actually true *on at least one real occasion*. Such a judgment belongs to the **assertoric** logical momentum. The propositions are:

Assertoric categorical - *I think* SxP;
Assertoric hypothetical - *I think* AyC;
Assertoric disjunctive - *I think* $s_H = \{s_1, s_2\}$.

All empirical evidence gathered by major studies in developmental psychology indicates the assertoric Modality of judgment utterly dominates the earliest life of the infant, perhaps even exclusively at first. Assertoric judgments are, in a manner of speaking, the first workmen at the construction site of the manifold of concepts. All human beings start life as uncritical realists and the assertoric nature of one's earliest object concepts, concepts that form a nucleating framework for the manifold, may well be the reason the great majority of people cling to ontology-centered maxims of thinking even in the teeth of the legionary contradictions and paradoxes this failed metaphysic structure confronts us with in physics, in biological neuroscience, in psychology, in the teaching sciences, and in philosophy.

Finally, there are some cognitions that, although never presented to us directly in sensual experience, are nonetheless held-to-be-true *a priori*. For example, I hold it to be true that atoms are made of "something." I don't flatter myself that I know what this something "is essentially" and I content myself with the understanding that the role of physics is to understand the *Existenz* of this "something" rather than its "primordial essence of *Dasein*." Nonetheless, I think it must be true that "atoms are made of something." Propositions such as this carry an *a priori* conviction of truth. This is the **apodictic** Modality of judgment. The forms are:

Apodictic categorical - *I think it must be* SxP;
Apodictic hypothetical - *I think it must be* AyC;
Apodictic disjunctive - *I think it must be* $s_H = \{s_1, s_2\}$.

While problematic judgments are the artists of understanding, apodictic judgments are the fine craftsmen of reasoning. When Piaget and his collaborators undertook to study the psychology of possibility and necessity, they found (translating their words into Critical terminology) that problematic judgments played a differentiating role in the construction of concepts and apodictic judgments played the integrating role of a model [PIAG8-9]. The unitary role is, of course, played by assertoric judgment.

§ 4. Remarks

The process of determining judgment and its logical functions for structuring the manifold of concepts has the practical characteristics of what psychologists sometimes call "cold cognition." The acroamatic principle of determining judgment is the principle of **conformity to law**. It is a rule-making faculty of judgmentation, its *materia* are concepts, and the objects of its actions are intuitions. A **cognition** is an objective perception involving the representation of both an intuition and a concept. **Thinking** is cognition through concepts. Determining judgment does not determine its own employment – that belongs to pure speculative Reason and its regulative principles – nor does it determine that *this* representation in sensibility is a cognition and *that* representation is not (that action belongs to the process of reflective judgment). But within the scope of its narrow jurisdiction, determining judgment operates according to its own laws (the categories of understanding) and in communion with the power of imagination. The immediate outcome of all this judicial labor is the manifold of concepts and it is from this manifold that we find the context for the idea of *understanding*. Understanding is an outcome, judgmentation is the process producing this outcome.

The logical functions of understanding in judgment are not primitives. If they were we could give no *Realerklärung* of the logical momenta as we have done in this chapter because a primitive admits only a *Realdefinition*. The *Realerklärung* of the logical momenta is an explanation within the logical reflective perspective of the theoretical Standpoint in Critical epistemology. From this *Realerklärung* we obtain the schemata of the transcendental logic of concept structuring, and it is from this that science must take its point of departure in developing an objectively valid Critical doctrine of formal logic as a language and tool for the practice of science.

Modern logic largely eschews the use of modality in its propositions, predications, and other formal constructs. It is true that from time to time some researchers will venture into the realm of formal modal logics; this happens today in computer science, for instance. However, the modal operators introduced in these logics are always[10] aimed squarely at an objective role. But the logical momenta of Modality do not have this character. The connection they provide for the manifold of concepts is connection *with the apperception* of the Organized Being. This is the source of their objective validity, and if one protests that "this means we must mix the idea of consciousness in with logic" the practical answer to this objection is that consciousness is the representation that another representation is presentable and is to be attended to. When such a presentable representation actually affects the actions of the Organized Being, then it is being attended to. Modal logic in the formal logico-mathematical context of a tool for science is the logic of attending to representations – which representations and how they are to affect the Organized Being insofar as the free play of understanding and imagination is concerned.

The structure of the manifold of concepts is the structure of an open system. In its structuring the problematic propositions provide the flexibility required for generalizing assimilation, and for accommodation of this structure under the press of new experiences, to be possible. The assertoric and apodictic propositions lack this flexibility in thinking new aggregate concepts and without problematic propositions thinking and learning would be rigid, dogmatic, and very dull-witted. Indeed, an apodictic proposition can be regarded as a problematic proposition held-to-be assertoric and so without the problematic momentum human beings would be incapable of the intellectual achievements that demonstrate the most striking difference between humankind and

[10] or, at least, almost always; I make no claim to having read 100% of the journal articles published on this subject.

the lower animals.

But it is also precisely the capacity for accommodation brought to the structure of the manifold of concepts that magnifies and multiplies the complexity of this manifold as experience grows. Indeed, the inherent *uncertainty* of problematic propositions may well be the starting point for the *development* of theoretical ideas of randomness and the *noumenal* idea of probability itself. We already know these ideas are absent in young children [PIAG10].

Von Neumann might not have been entirely correct in conjecturing that a new "probabilistic logic" must be developed in order to deal scientifically with complexity in automata and "natural automata" (i.e., *us*), but he was not altogether wrong either. The logic of problematic propositions is the *creative factor* in thinking and it is this capacity for creativity that brings with it the amazing complexity of the human capacity for thinking, understanding, and reasoning beyond immediate sensual experience. Furthermore, we must not neglect to take into account temporal dynamics as a factor in the construction of the manifold of concepts because this factor brings the contingency of experiential *order* into play in the determination of the manifold. If we nudge von Neumann's word "probabilistic" just enough to make it mean "unanticipated" then the new logic he called for takes on the appearance of a *logic of unpredictability*. It does not seem to be a risky conjecture to say that "complexity" would be a pretty likely outcome of this.

Next, we must not forget the fundamental role that subjectivity plays in the construction of the manifold of concepts. Generalizing concepts arise through the imaginative re-cognition of intuitions that are themselves the products of inferences of judgment. The inference of judgment is an effect reflective judgment has on determining judgment and all reflective judgments are non-objective and take their determinable judicial *materia* from affective perception. The subjective factor is the *personal* factor in judgmentation and is involved in the temporal dynamics of thinking. And this leads us directly into the topic of our next chapter.

Finally, let us recall that I spoke earlier (Chapter 1) of the "Mother structures" the Bourbaki mathematicians held to be the basis for all of mathematics. These are topological structure, order structure, and algebraic structure. The Critical grounds for the first two lie with the topological structuring of the intuition of space and the order structuring of the intuition of time, as we saw earlier. To this it can now be added: ***the Critical ground for algebraic structure is the logical structuring of the manifold of concepts***. I promised you that we would discuss the topic of the Mother structures; this has now been done. It is a hopeful conjecture that with this understanding we might achieve a happier outcome for mathematics' "crisis in the foundations" than was achieved when the mathematics community last confronted the issue. If so, the outcome will be Critical mathematics and Critical Logic.